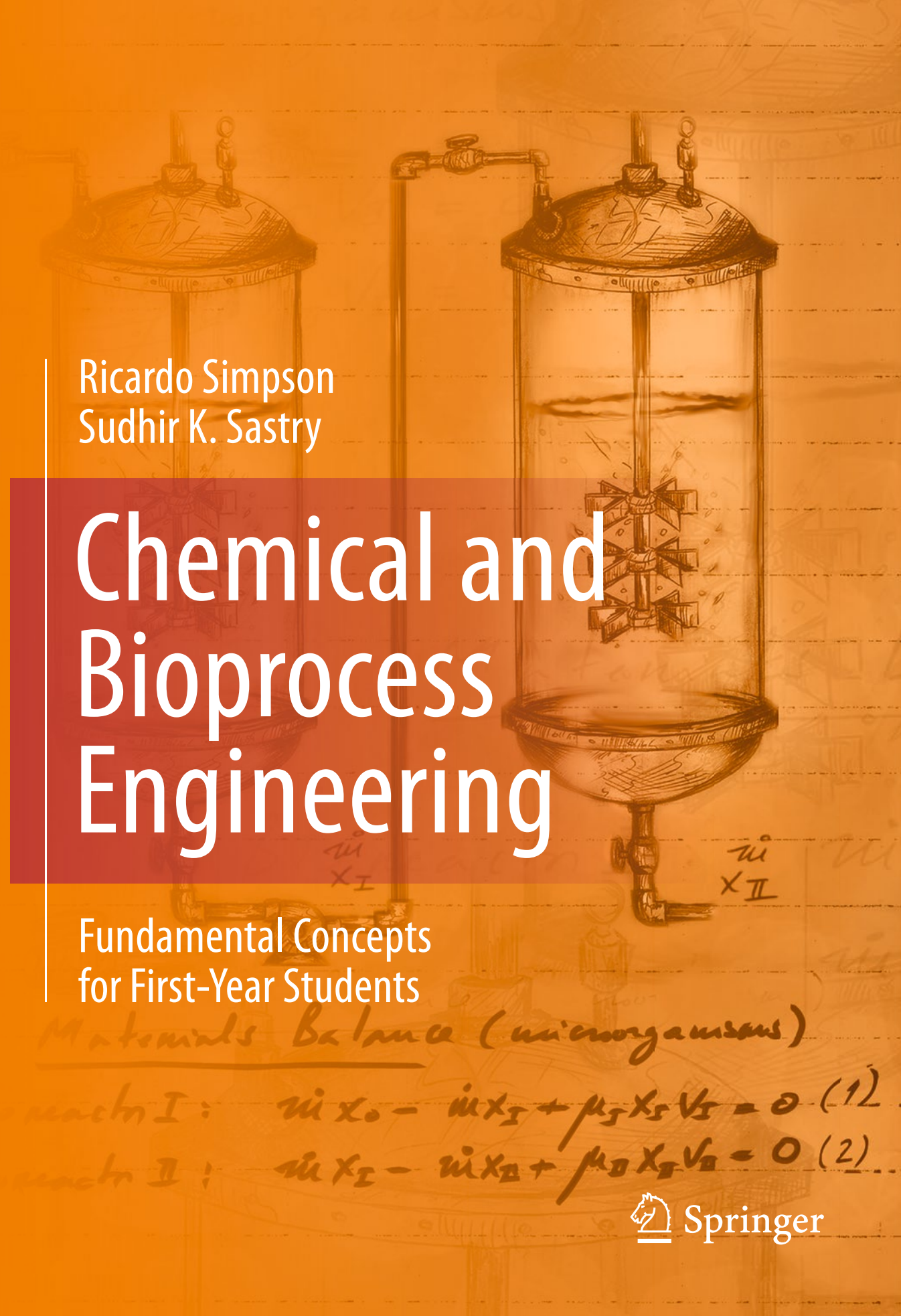

Ricardo Simpson
Sudhir K. Sastry

Chemical and Bioprocess Engineering

Fundamental Concepts for First-Year Students

Springer

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This book is dedicated to our wives, Anita and Padma, and family, José Ignacio, María Jesús, Enrique and Amit and Nevin.

Ricardo Simpson and Sudhir K. Sastry

Preface

To give anything less than your best, is to sacrifice the gift.

Steve Prefontaine

There are many excellent introductory books for future chemical and bioprocess engineers. So what motivates us to write this book?

Why and How?

No introductory chemical engineering book covers all the branches related to chemical engineering in the twenty-first century. In addition, existing books, independent of content, generally cover only chemical engineering or biochemical engineering, but not both chemical and bioprocess engineering. Bioprocess engineering is broader than biochemical engineering, covering other areas such as food engineering, environmental engineering, biochemical processing, and biological engineering.

Many books offer broad-based introductions to the subject but do not reveal the underpinnings of future knowledge. One key motivation in our case is the approach of Robert H. Frank (http://www.robert-h-frank.com/), wherein the building of understanding of the fundamentals via practice with relevant real-world examples takes precedence over the coverage of large amounts of material. Herein, we identify specific areas within which specific levels of competencies are to be attained. To facilitate and orient professors and students, we will classify each chapter following Bloom's taxonomy (BT). Each chapter will be identified in relation to the cognitive domains of BT: (1) knowledge, (2) comprehension, (3) application, (4) analysis and synthesis, (5) evaluation, and (6) creation.

Our approach involves a thorough grounding in the fundamentals so that the background may be used in future work. Here we present numerous solved and proposed exercises (more than 400). In each chapter, problems will be classified by level of difficulty (from 1 to 10^+, were 10^+ is the most difficult). Each chapter will have references and selected Web pages to vividly illustrate all the examples. Since this is intended to be an introductory book, some concepts will receive closer attention than others. Therefore, as mentioned, each chapter will be clearly marked according to BT. A note to students using this book: we are not pretending that you will be an expert in

everything, but you will be fully prepared as a problem solver and with probably the most important knowledge of a future chemical and bioprocess engineer, which is to say you will receive a rigorous preparation in material balances. As stated, you will not learn a large quantity of material, but you will be armed with the necessary tools for success. Just follow our recommendations firmly and consistently. As we repeatedly tell our students, you need to be dutiful.

Motivation is a key aspect and component in your first steps as an engineer. There is a need to engage students with interesting material, especially in a subject matter like units and dimensions, which are critically important but somewhat dry. Thus, many of the examples involve real-world situations to which students may be able to relate readily. In this book, you will be exposed to all of the chemical and bioprocess engineering areas, so you will understand your future role in society, and, most importantly, we will do our best to communicate to you the enchantment we feel with this marvelous profession.

What Is an Engineer?

First and foremost, an engineer is a problem solver, independent of his or her specialization. Therefore, in Chap. 5, we develop a method and strategy to give you the necessary tools to start on your long journey to becoming a strong and solid problem solver. Experience has shown us that with the correct methodology and the necessary background, you will be prepared to face intricate problems. As a strong problem solver in chemical and bioprocess engineering, you should acquire a good knowledge of basic sciences, mainly mathematics, chemistry, biochemistry, microbiology, economics, and physics.

Secondly, your main mission and role as an engineer will be to connect or channel the discoveries made by scientists at the laboratory scale to transform them into products made at the factory level and, finally, to scale them up as a profitable end product for society. This text will familiarize you with the whole range of areas that comprise this field. Its multidisciplinary characteristics will imply that most of the time you will have to work in teams, meaning that you should acquire the ability and the language to communicate with persons from different fields.

What Do We Expect from You and Why?

In the mid-1980s, psychologists discovered and elaborated what is called the theory of cumulative advantages. In simple terms, if you practice continuously and systematically, at the end you will achieve a high standard on this specific topic or field of study. (The concept has other applications, too.) The interesting and robust message behind this theory is that, more important than your IQ, it is your attitude and effort put forth in what you are pursuing that matter. We are convinced that if you follow our advice rigorously and work

with effort, effort, and more effort, at the end you will be entirely compensated. If you want to excel or, more importantly, to be satisfied with your achievements, you need to understand that it mostly depends on you and your attitude. We will do our best to be good guides, but always remember that we need your collaboration, and, of course, you need to cooperate with your professors and advisers at your college/university.

What You Will Learn?

You will need to master the basic skills of chemical and bioprocess engineers. Most notably, we are committed to having an impact on you. We are committed to showing you all aspects of this fascinating field. Ultimately, we expect to teach you to be a much better problem solver, to motivate and captivate you with the enormous diversity and brilliant future of this profession, and to enable you to acquire one of the most important gems of knowledge required for a chemical and bioprocess engineer: a strong and solid preparation in material balances and economy fundamentals to introduce you to the first stages in project evaluation. As Albert Einstein said, *"Imagination is more important than knowledge*."

In summary, attitude, questioning, effort, a sound method and strategy to solve problems (mainly Chap. 5), material balances (Chaps. 7 and 8), and basic economic principles (Chap. 12) will be the main topics and patterns to be learned. In addition, you will be given a clear overview of this profession.

Valparaíso, Chile — Ricardo Simpson
Columbus, OH, USA — Sudhir K. Sastry

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We are grateful to all those who contributed to improving this book; they are too numerous to mention individually—our apologies in advance. We would like to mention those individuals and institutions that were directly involved in the publication of this book. First, our sincere gratitude goes to our universities, Universidad Técnica Federico Santa María (UTFSM), Valparaiso, Chile, and The Ohio State University (OSU), Columbus, OH, USA. Second, many individuals helped us and provided their critical opinion. Among them we would like to mention Professor Fernando Acevedo from Catholic University (Valparaíso, Chile), who patiently and critically reviewed Chaps. 4 and 10. Suzanne Kulshrestha read, revised, and improved virtually all chapters of the book. Students Daruska Miric, Alfonso Flóres, and Andrés Ceballos checked all the problems and their answers. Iván Cornejo advised us on the development of Chaps. 3 and 9, and Professor Luis Bergh critically reviewed the same chapters. Professor Enrique Norero (Universidad Santo Tomás, Chile) contributed to and reviewed Chap. 11 and was kind enough to provide us some material and exercises of his own. Professor Mario Ollino from the Department of Chemistry at UTFSM, Rodrigo Solís and Cristián Antonucci critically reviewed Chaps. 7 and 8, which form the core of this book. Christopher Gepp, a young biochemical engineer, provided us with interesting exercises to Chaps. 2, 7, 8, 11, and 12. In addition, we thank UTFSM colleagues Alejandra Urtubia, Carolyn Palma, Alonso Jaques, Luis Borzone, Pedro Valencia, and Juan de la Fuente for their contributions and critical comments on Chap. 6. Research assistants (UTFSM) Marlene Pinto and M.Sc., Helena Nuñez are gratefully acknowledged for their cooperation and excellent disposition in editing and designing most of the figures of this book. We deeply regret the omissions, but we cannot forget to mention all colleagues of the Department of Chemical and Environmental Engineering (UTFSM) for their continued support and rave reviews. We are most grateful for the advice, help, and patience of Susan Safren and Rita Beck from Springer. Last but not least, our deepest gratitude goes to all our freshmen students and particularly to Carlos Gardella, Valentina Torres, Andrés Arriagada, and Camila Salvo.

To Our Students, Colleagues and Tutors

The only thing worse than being blind is having sight but no vision.

Helen Keller

Where there is no vision, there is no hope.

George Washington Carver

Leadership is the capacity to translate vision into reality.

Warren G. Bennis

Engaging Freshmen Engineering Students

As was stressed in the preface, our main goal is to enchant freshman students with the broad field of chemical and bioprocess engineering. We have learned over the years that the most effective way of engaging freshman students is by motivating them. Normally, when we have been invited to share our experience with freshman engineering students, we say that the three most important characteristics are motivation, motivation, and motivation. Most will probably agree, but the key question is how? First, experience has shown us that accomplishments are normally accompanied by coherent foundational beliefs. One of the intentional, subtle, guiding beliefs of this book is that with a clear method and procedure, problems are normally easier to solve. We have long experience with freshmen who have internalized, simultaneously, this guiding belief and the proposed methodology with extraordinary success. We attribute this success primarily to the fact that, from the very beginning, students feel that problems can be solved, and they develop the necessary confidence to do so. In addition, students are introduced to a well-founded methodology and procedures to solve engineering problems. Further, we have attempted to structure the book so as to attract students to the broad field of chemical and bioprocess engineering. As an example, we have included chapters that, while not essential for the book, will help us engage freshmen and, in addition, help these young students have a better and a broader comprehension of process engineering. We specifically refer to Chaps. 4 and 10, "Learning from Nature" (biomimetics) and "Scale-Up."

Bloom's Taxonomy

We have classified each chapter in accordance with Bloom's taxonomy (from 1 to 5) to guide the user regarding the relevance of each topic for this freshman course. In addition, we would like to emphasize that our main goal is to empower students to formulate problems (Chap. 5) and teach them material balance (Chaps. 7 and 8). Although the book is composed of 12 chapters, some of them could be disregarded at the instructor's discretion. Nevertheless, the key object remains: to teach problem formulation and material balance as key aspects for freshman students, always keeping in mind the importance of

enchanting and delighting students with the exciting and broad field of chemical and bioprocess engineering.

Key Concepts

In what follows, we would like to share with you, colleagues and instructors, some ideas and concepts on how to better understand and seize the advantages of this book.

Problem Formulation

First, as was already mentioned, an engineer is a problem solver. As you will see throughout the book, our main objective is to empower students with a method and strategy for correctly formulating problems. The first steps in problem formulation are presented in Chap. 5, and these concepts are applied to chemical and bioprocess engineering problems throughout Chaps. 6, 7, 8, 11, and 12. It is very clear that during their future engineering careers, students will be trained in problem resolution. From our vantage point, problem formulation is a critical point because in our experience, normally small amounts of time and effort are dedicated to teaching students to correctly formulate problems compared to the prodigious amounts of time and energy devoted to teaching mathematical methods on how to solve equations. This only gets worse with high school students, who are normally very poorly trained in problem formulation.

Method of Problem Formulation

As a strategy, we have chosen first to present in each chapter examples and solved problems, from introductory to advanced levels. We then provide a long list of proposed problems (with answers). In these proposed problems, we indicate the degree of difficulty (from 1 to 10^+, with 10^+ being the hardest) according to our experience and our students' feedback. In some proposed problems that we consider most difficult (8 to 10^+ on our scale) we include hints to facilitate their resolution. Finally, in specific cases, we considered it relevant to include references and background details. In Chap. 5, we present a method on solving problems, and then, in Chap. 7, the method is expanded and detailed for the solution of material balance problems. Experience has shown us repeatedly that freshmen can be empowered to formulate and solve interesting and intricate material balance problems and, at the same time, enjoy chemical and bioprocess engineering.

Material Balance

Chapters 7 and 8 form the core of the book. As we say to our students, half-jokingly: "If you learn material balance, you are 50 % of a process engineer." Experience has shown that without much background knowledge, students can reach a high level of comprehension and feel empowered to formulate and solve material balance problems. Furthermore, material balance gives professors the opportunity to show, in a fruitful way, the breadth of chemical and bioprocess engineering.

Optimization and Alternative Decisions

An important question is whether freshmen, just out of high school, have the mathematical background to tackle optimization problems. Our answer is no, in most cases they do not. Nevertheless, with the help of spreadsheets we will be able to introduce freshmen to the fascinating world of process engineering. Two objectives are expected to be accomplished simultaneously. First, familiarize and delight freshmen with the applications of optimization in a career in chemical and bioprocess engineering and, second, provide them with the elementary tools (graphics and spreadsheets) to solve interesting and challenging optimization problems.

Using the Book

As you can appreciate, the book is a bit long for one semester. Although it is possible to cover all the topics in one semester, we suggest an alternative course of action. First, depending on the course objectives, you may decide to skip some chapters (possibly Chaps. 4, 9, and 10) or just cover some chapters in one session. Because of the relevance that we assign to this course for freshmen, our suggestion is to cover Chaps. 1, 2, 4, 5, 7, 8, and 10 in the first semester and leave Chaps. 3, 6, 9, 11, and 12 for the second semester. Why? Because in the first semester you can focus on problem solving and material balance (reactive and nonreactive systems) with elementary tools from high school and then in the following semester cover the engineering topics. By the second semester, students will have acquired some basic tools of mathematics, physics, and chemistry, and it will be easier at that point to approach and teach the engineering chapters. As you may have noticed, in conformance with our emphasis on motivating freshman engineering students, we have assigned Chaps. 4 (biomimetics) and 10 (scale-up) to the first semester.

In addition, an advantage of dividing the course into two semesters is the observation that normally chemical engineering students do not become familiar with process engineering courses until their third and fourth semesters. Normally, the first three semesters of college are devoted to giving them the necessary tools of mathematics, physics, chemistry, and biology. Dividing this critical course into two semesters will ease their initial preparation for process engineering courses.

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Introduction 1[2]

It does not matter how slowly you go as long as you do not stop.

Confucius

It is not enough that we do our best; sometimes we must do what is required.

Winston Churchill

I am a slow walker, but I never walk back.

Abraham Lincoln

1.1 What Is an Engineer?

Engineering is defined by the Accreditation Board for Engineering and Technology (ABET) as *the creative application of scientific principles to design or develop structures, machines, apparatus, or manufacturing processes, or works utilizing them singly or in combination; or to construct or operate the same with full cognizance of their design; or to forecast their behavior under specific operating conditions; all as respects an intended function, economics of operation and safety to life and property.*

Engineering, in some form, has existed as long as humankind has been building devices for specific purposes. Examples of early forms of engineering include the development of the wheel, the use of fire, and the development of tools from bronze and iron. However, the term *engineering* is of more recent origin, and the application of scientific principles had to await the development of science and scientific methods as we know them today.

While science has been vital and important in the development of engineering, it does not account for everything. Engineers often encounter situations where scientific information is unavailable or simply missing. In such situations, human creativity, resourcefulness, and experience can play a key role; this is the "art" of engineering. At the same time, we must emphasize that engineering science provides a critical base of knowledge; without it, we might reinvent the wheel! Today, developments in computer hardware and software have enabled engineers to predict outcomes in situations where measurements are impossible, enabling them to accomplish safe design in "blind" or difficult situations. Consider the situation faced by a pilot who must land a plane in heavy fog with limited visibility. It is here that the genius of design engineers, with their ability to design automatic controls

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enabling instrument landings, comes into play. Another situation is the landing of exploration vehicles on Mars: this is happening even as this book goes to press, yet humans have never been on the surface of Mars. How can we engineer devices and systems that respond to our commands from Earth and do our bidding at extreme distances? It is the interplay of art and science under the constraints of economics, safety, and other relevant considerations that makes engineering the amazing and exciting profession that it is. We welcome you to join and enjoy a satisfying career in engineering.

1.2 Is Engineering for Me?

Some tests:

- myfuture.edu.au provides a simple profile test that may help you gain a better understanding of your interests, values, and goals.
- Take the free test at the Career FAQs Web site: www.careerfaqs.com.au.
  Visit http://www.engineeryourcareer.org.au/?page_id=55 for

1.3 What Are Process, Chemical, Bioprocess, and Biochemical Engineers?

We use the term *process* engineers to refer to a broad class of engineers that includes chemical engineers. However, the term *process engineering* is likely less familiar to most laypeople than the better-known *chemical engineering*, and hence we have chosen chemical engineering for this book's title. Process engineers typically are involved in the design, operation, control, and optimization of chemical, physical, and biological processes.

A number of definitional statements exist for chemical engineering. The one adopted by the Institution of Chemical Engineers (UK) in 1924 stated essentially that *a chemical engineer is a professional experienced in the design, construction and operation of plant and works in which matter undergoes a change of state and composition*.

Historically, chemical engineering has been at the heart of major parts of the chemical, petroleum, pharmaceutical, and electronics industries. However, more recently, many chemical engineering and other departments have diversified to encompass biochemical (or biomolecular) engineering.

Biochemical engineering involves the design, construction, and operation of unit processes that involve biological organisms or molecules and is involved in the biotechnology, pharmaceutical, food, environmental, and waste treatment industries. Many similar names (and flavors) exist, including biomolecular (focused on the manipulation of molecules) and biomedical (focused on medical applications) engineering. Nevertheless, a common theme is the central focus on biology as a core discipline. As the world's critical resources will require management and stewardship in the twenty-first century, the discipline of biochemical engineering will take on increasing importance.

While chemical and biochemical engineering are well recognized and represented in department and journal names, we would like to focus specifically on processes, or process engineering, which encompasses a vast class of operations that include chemical and biochemical engineering. Bioprocess engineering, which may be considered a subset of process engineering, deals with the design and development of equipment and processes for the manufacture of products such as food, feed, pharmaceuticals, nutraceuticals, chemicals, and polymers and paper from biological materials.

1.4 History

1.4.1 Why Is History Important?

History is important because by knowing our past we can better understand our present and, thus, better predict the future. It has been said that man is the only animal that trips twice over the same stone[1]; thus knowing history can help us avoid this characteristically human trait.

We shall take a brief look at history to understand how engineering was born within human society and thus facilitate our understanding, especially for young people just starting their studies. Moreover, we will start discussing, briefly, prehistory, because we may well discover that we were born engineers!

1.4.2 Were We Born Engineers?

Prehistory involves the study of the development of human societies before the existence of writing and is based primarily on the analysis of tool artifacts. Thus, we have an opportunity to witness the activities of prehistoric humans through their emerging engineering works; these small works facilitate our understanding of ancient times. As long as humans have existed, they have sought ways to solve problems, and in prehistoric times the basic problems were food, clothing, defense, and habitat. Thus, early engineering works involved manufacturing tools to solve and improve their chances of obtaining food, hides for cover, and weapons for defense and to build appropriate places for shelter (e.g., caves and small huts). In fact, humans are born engineers because we have always sought ways to solve problems, initially to improve our standards of living. Initially ancient engineers solved their problems only through trial and error, and millennia would pass before the development of sciences like mathematics, physics, chemistry, and biology. Over time and to the extent of their understanding of their environment, these ancient engineers slowly incorporated empirical "prescientific" knowledge to solve problems. Around approximately 8,000 years B.C. humans ceased leading a hunter-gatherer (nomadic) lifestyle and created the conditions (first crops and livestock) to live in a fixed location. It is perhaps at this time that ancient engineers merged architecture and engineering. Their main work was the construction of buildings and walls to defend their communities.

From the standpoint of food and food preparation, the development of various cuisines related to practical, engineering-type considerations. For example, Chinese cuisine evolved from the need to conserve fuel during cooking; thus food was cut into small pieces and quickly stir-fried to completion. This represents an early application of heat transfer from energy conservation considerations (Wilson 2012).

Another key operation in chemical engineering involves size reduction. This entailed chopping, grating, and grinding, which were all difficult and labor-intensive tasks historically performed by laborers for the benefit of the rich. Wilson (2012), in a very interesting book, describes the history of this process. When abundant servant labor was available, there was no motivation to improve upon such processes; indeed wealthy people served highly refined food to guests, often in an attempt to show how many servants they could afford. In the present era, we are very concerned with obesity due to highly refined foods, perhaps because of the dietary habits we inherited from our wealthy ancestors!

[1] "El hombre es el único animal que tropieza dos veces en la misma piedra"—Spanish proverb.

1.4.3 Industrial Revolution

Until the seventeenth century, engineering developments were very slow, mainly because scientific advances were also slow. The Industrial Revolution began in the second half of the seventeenth century and continued to the early nineteenth century. Manual labor was gradually replaced by mechanization, mainly in the textile industry. We could say that over the last three centuries, scientific advances have been significant, if not explosive. Improved transportation systems, the invention of the steam engine, and the invention of the railroad facilitated even greater advances, with the steam engine being perhaps the most important invention of the time (http://en.wikipedia.org/wiki/Steam_engine). By this time the production of goods and labor began to organize in factories. Later, engineering started to diversify, a trend that continues to this day and may well continue into the foreseeable future. The first branches of engineering were military engineering, civil and construction engineering, metallurgical engineering, mechanical engineering, electrical engineering, and chemical engineering.

Before going into the descriptive and conceptual issues of engineering as we currently know it, students are encouraged to internalize the information on engineering works in ancient Egypt, Mesopotamia, Greece, Rome, and the East. There is abundant information on the Internet. A good place to start is http://humweb.ucsc.edu/gweltaz/courses/techno/biblio.html.

1.4.4 A Brief History of Chemical and Biochemical Engineering

Many fine descriptions exist for the histories of chemical and biochemical engineering (e.g., Kim 2002; Katzen and Tsao 1999). We will merely present some of the highlights here.

1.4.4.1 Chemical Engineering

Humans have been attempting or performing chemical transformation throughout history. Early efforts included the cooking of food and the production of metals from ores. An early goal of alchemy (the precursor of chemistry) was to convert base metals into gold! The attempt was not successful, but it was not until the advent of our understanding of atomic-level chemistry that we understood why. Various other theories regarding the composition of materials were popular at various times in history—for example, the phlogiston theory, where it was believed that all substances contained phlogiston, a component of all materials that left when the material was burned. It was not until Lavoisier, in the eighteenth century, showed, through careful accounting of mass balances, that the theory was flawed. We could go on indefinitely, but the history of science and its many detours is a fascinating, though separate, subject.

Chemical engineering in its current form probably has its origins in the Industrial Revolution of the nineteenth century. As industrial production accelerated at an ever increasing rate, certain chemicals became necessary to sustain this growth. One critical chemical was sulfuric acid; indeed it was thought that a nation's industrial prowess could be measured by its sulfuric acid output. Both sulfuric acid and alkali were produced on a large scale in Germany and England. However, products were prepared in batch mode, i.e., in individual vats in batches. Batch production was slow and tedious, but rapid production required continuous flow reactors. This in turn greatly increased the complexity of the process since engineering considerations of flow and control became critical. Thus, engineering entered the domain of what had hitherto been industrial chemistry. Among the improvements in this process was the recovery of nitric oxide via a mass transfer tower by John Glover in 1859.

Soap making had also been practiced since the eighteenth century due to demand for washing clothes using sodium carbonate. The LeBlanc process accomplished this from salt using sulfuric acid, limestone, and coal, which produced hydrochloric acid as a byproduct. However, the process had severe environmental consequences and was eventually replaced in 1873 by the Solvay process.

In 1887, George Davis, an alkali inspector, developed a series of 12 lectures on chemical operations for the Manchester Technical School. These eventually came to be known as unit operations, one of the pillars of a chemical engineering education. In 1888, the first 4-year bachelor's program in chemical engineering was created at the Massachusetts Institute of Technology (MIT). The program was largely descriptive at this stage.

Many developments occurred in the chemical industry over the years, including the development of optimization methods, continuously operating reactors, recycling and recovery, and purification technology. These required knowledge of plumbing systems (then unknown to chemists) and physical chemistry (then unknown to mechanical engineers). The study of unit operations, which focused on underlying processes, became the domain of the chemical engineer.

The next major paradigm shift came about with the publication of the textbook *Transport Phenomena* by Bird et al. (1960) and *Mathematical Methods in Chemical Engineering* by Amundson and Aris (1966). These works constituted the scientific pillars of chemical engineering as a discipline and led to a turning away from older, more empirical work. Thus, the core curriculum of chemical engineering consists of unit operations and transport phenomena.

1.4.4.2 Biochemical Engineering

Commercial-scale biomass processing dates back to ancient times. The process of fermentation to produce cheeses, beer, and wine have existed for thousands of years. However, the nature of the process was not understood until Pasteur's development of the science of microbiology. Thereafter, developments in this area proceeded apace, gaining commercial value from the development of valuable medicines such as antibiotics, most notably penicillin.

The term *biochemical engineering* came into use in the 1940s with the development of aerobic submerged culture in response to the need to produce penicillin in large quantities (Katzen and Tsao 1999). This led to the need to improve gas-liquid interfacial mass transfer and the development of engineering science necessary for understanding the technology. Since the energy crises of the 1970s, the need for renewable energy sources has led to work in renewable energy from biomass. Large-scale developments in molecular biology led to an expansion in biochemical engineering to fill the need to produce pharmaceutical and other bioproducts. Developments in this field continue to this day; indeed, many chemical engineering departments have now changed their names to include bioprocess, biological or biomolecular engineering.

1.5 Why Integrate Chemical and Bioprocess Engineering Fundamentals in One Book?

Currently, most available books at the introductory level focus exclusively on the needs of either chemical engineers or biochemical engineers, but not both. With the consolidation of chemical and biochemical engineering programs, it is justified, in our view, to develop a unified textbook that enables the beginning student (whether in chemical or biochemical engineering) to gain the necessary basic skills for further development. In principle, the underlying engineering sciences are the same, although the specifics of the chemistry and process constraints are different in the two cases.

1.6 Chemical and Bioprocess Engineering in the Twenty-First Century

The twentieth century witnessed dramatic growth in the chemical industry, particularly industrial chemicals, petroleum, plastics, and polymers. The twenty-first century promises an increased focus on environmental issues, clean air, water, food security, global warming, and human health and wellness. It is clear that developments in one industry cannot continue unabated without an understanding of the consequences of these developments on humans and biological species on the planet. For this reason, we see the need for an increasingly integrated treatment of chemical and bioprocess engineering.

1.6.1 Required Basic Knowledge: Basic Tools of Mathematics, Chemistry, Physics, and Biology

In developing this book, we were inspired by Robert Frank (Frank 2006), who studied how students learn economy and found that many of them had difficulty with fundamental concepts even after they graduated. Professor Frank attributes that state of affairs to an overemphasis on covering large amounts of material, with the result that many students lack a grasp of basic concepts. It is suggested that a better approach would involve deeper learning, with a less broad coverage of material.

In this connection, we have endeavored to develop and present problems that are interesting, demonstrate most of the applications of chemical and bioprocess engineering, and in turn require a basic knowledge of mathematics, chemistry, physics, and biology.

Our main goal is not to try to teach you too much information but to familiarize and enchant you with chemical and bioprocess engineering and, basically, expose you, in some depth, to one of the most useful tools for a process engineer: material balance. Fortunately, you can learn, and learn well, material balance with the knowledge that you bring with you from high school. As presented and discussed in Chap. 11, we will address some complex mathematical problems that will be solved with the help of a spreadsheet (if you are not accustomed to spreadsheets, several examples will familiarize you with them). In addition, in some specific cases the goal is not necessarily to solve problems and obtain final results but to formulate and understand the problems.

But what is basic knowledge? What do we mean when we say that the knowledge that you bring from high school is enough? More specifically, for example, in mathematics, we can state that you should know algebra at the level of solving systems of equations where normally each equation is a first-order equation. As you will notice, the stress and goal of the majority of, if not all, the problems is to place an emphasis on the formulation and not on your math skills for solving them. We are not saying that math skills are unnecessary; of course they are, but not yet. You will have several math courses to prepare you to solve real problems. The same goes for chemistry, physics, and biology.

Experience has shown us that using this approach students learn more than they expected and acquire a solid understanding of chemical and bioprocess engineering. As one student expressed to us: *The problems are fun, intellectually challenging, and above all they excited and motivated us regarding the career that we were just starting to study."*

1.7 Cognitive Domain: What Will We Comprehend and Learn?

There is not just one mode or way of learning. According to psychologist Benjamin Bloom, there are three intersecting domains of psychology: cognitive, psychomotor, and affective. In this book, the prevailing domain is cognitive, but also important—very important—is the affective domain. The cognitive domain prevails because it relates directly to knowledge and the development of

intellectual skills. But why is the affective domain important? Because the affective domain relates to enthusiasm, motivation, and attitude. In our experience, with freshmen, the three most important things for teachers to keep in mind are: motivation, motivation, and motivation.

Our primary goal is to touch you in the affective domain. Then, depending on your interests and motivations, you will start reading and assimilating the topics of this book.

A glimpse at Bloom's taxonomy: Bloom's cognitive domain is related to knowledge and the development of intellectual skills. Bloom's taxonomy is divided into six categories. Each subsequent category implies a higher level of difficulty, and so each one is listed in ascending order. The six categories are as follows:

1. Remembering
2. Understanding
3. Applying
4. Analyzing
5. Evaluating
6. Creating

Each chapter of the book is identified by its own number but also by a number in braquets that relates the chapter to Bloom's cognitive level it is expected to reach. For example, Chaps. 2 and 7 have a Bloom cognitive level of 5, then on the first page of the chapter you will see Chaps. 2[5] and 7[5], respectively, indicating that both chapters are expected to reach the level of evaluation. As you will see, most chapters are expected to reach a level 2 understanding, meaning that most of the book is focused on motivation rather than learning. We would like to motivate you (mainly Chaps. 3[2], 4[2], and 9[2]), to teach you something and illustrate the potential applications of this career (Chaps. 11[3] and 12[3]), and, finally, go into more depth in Chaps. 2[5], 5[4], 7[5], and 8[4].

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A Closer Look at Chemical engineering http://www.youtube.com/watch?v=iAWLMMZlkq4

What is Chemical and Bioprocess Engineering all about http://www.youtube.com/watch?v=Ga2m1TEoDSo ?

Bioprocess Engineering-career guidance http://www.youtube.com/watch?v=mnVe4aF6JZY

Chemical Engineer - Profiles of Scientists and Engineers http://www.youtube.com/watch?v=k-7B_YfHWXQ

Fundamentals of Magnitudes, Unit Systems, and Their Applications in Process Engineering

2[5]

The mind, once expanded to the dimensions of larger ideas, never returns to its original size

Oliver Wendell Holmes

According to String Theory, what appears to be empty space is actually a tumultuous ocean of strings vibrating at the precise frequencies that create the 4 dimensions you and I call height, width, depth and time

Roy H. Williams

2.1 Chapter Purpose and Scope

Most students, at various levels, are probably familiar with unit systems. In this chapter we will provide some details on the International System of Units and its relation to the English Engineering system. Our main purpose is to ensure that students cannot only use the different systems but also convert units among different systems. In addition, students will gain an understanding of the different approximate dimensions and sizes that occur frequently in the field of chemical and bioprocess engineering.

2.2 Introduction

2.2.1 Brief History

A long time ago, many kings, including Charlemagne, wanted to establish a common system of measurements. However, it was only from the eighteenth century (several centuries later) that humans began to converge on a unified system of units. History suggests that France had a large number of systems that grew to over 700 different units at the end of the eighteenth century. Overall, measures represented adaptations based on the human body. For example, consider the etymology of the word *digit* (which means *finger*); other examples include foot and breaststroke. As an example, in sixth-century Egypt, a cubit measured length and was equivalent to the distance from the elbow to the middle finger, but whose? Unfortunately, these units of measure varied by location and were at that time not standardized. Therefore, calculations were complex and complicated scientific advances and commercial relations between peoples, communities, and nations. Undoubtedly harmonization and standardization into a universal system was much needed and greatly facilitated the development of communities and nations. In the nineteenth century, France established the metric system as a legal system of measures.

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2.2.2 Metric and SI Systems

The metric system was established at the General Conference on Weights and Measures in Paris in 1889. The main idea was to launch a universal system to encourage and improve scientific, commercial, and cultural exchanges among nations. Then, in 1960, at the General Conference on Weights and Measures, the International System of Units (SI) was created; it initially defined six fundamental magnitudes: length, mass, time, electric current, thermodynamic temperature, and luminous intensity.

In 1971 the seventh fundamental magnitude was added: amount of substance. From the combination of these seven fundamental units are obtained all derived units. Since then, the SI system has been dominant around the world.

2.2.3 Basic Units (Table 2.1)

Table 2.1 International system of units (SI)

Fundamental magnitude	Symbol	Basic unit	Symbol	Definition
Length	*l, x, r*	Meter	m	A meter (m) is the distance light travels in vacuum in 1/299,792,458 of a second; thus, the speed of light is exactly 299,792,458 m/s
Mass	*m*	Kilogram	kg	A kilogram (kg) is equal to the mass of an international prototype of mass. The prototype is a cylinder of platinum-iridium with a height and diameter of 0.039 m. Mass is the only unit defined in terms of an arbitrary artifact instead of a natural phenomenon
Time	*t*	Second	s	A second (s) is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of a cesium 133 atom
Electric current	*I, i*	ampere	A	An ampere is the intensity of a constant current that, if maintained in two rectilinear parallel conductors of infinite length, of negligible circular section, and placed at a distance of 1 m from one another in vacuum, would produce a force equal to 2×10^{-7} N per meter of length; according to this definition, the value of the constant μ_0 is exactly equal to $4\pi \times 10^{-7}$ N/A^2
Thermodynamic temperature	*T*	Kelvin	K	Kelvin (K) is the thermodynamic temperature unit and is a fraction (/273.15) of the thermodynamic temperature of the triple point of water Observation: also used in addition to the thermodynamic temperature (symbol T) expressed in Kelvin is the temperature in Celsius (symbol t) defined by the equation $t = T - T_0$, where $T_0 = 273.15$ K by definition
Luminous intensity	I_v	Candela	Cd	A candela (cd) is the luminous intensity, in a specific direction, from a source that emits a monochromatic frequency of $540 \times 1{,}012$ Hz and whose energetic intensity in this specific direction is 1/683 W per steradian
Amount of substance	*n*	Mole	Mol	A mole (mol) is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kg of carbon 12

Table 2.2 Derived units and parallels between English Engineering system units and SI units

Magnitude	SI system		English Engineering	
	Name	SI units	Name	English units
Area	Square meters	m^2	Square feet	ft^2
Volume	Cubic meter	m^3	Cubic feet	ft^3
Speed	Meters per second	m/s	Feet per second	ft/s
Acceleration	Meters per second squared	m/s^2	Feet per second squared	ft/s^2
Density	Kilograms per cubic meter	kg/m^3	Pounds (mass) per cubic foot	lb/ft^3
Specific volume	Cubic meters per kilogram	m^3/kg	Cubic feet per pound (mass)	ft^3/lb
Amount of substance concentration	Moles per cubic meter	mol/m^3	Pound moles per cubic foot	$lbmol/ft^3$
Mass concentration	Kilograms per cubic meter	kg/m^3	Pounds per cubic foot	lb/ft^3
Mass flow rate	Kilograms per second	kg/s	Pounds per second	lb/s
Viscosity (absolute)	Kilograms per second meter	kg/s m	Pounds per second foot	lb/s m
Frequency	Cycles per second, Hertz (Hz)	1/s	Cycles per second, Hertz (Hz)	1/s
Force	Newton (N)	m kg/s^2	Pound force	lbf
Energy	Joule (J)	m^2 kg/s^2, N m	British thermal unit	BTU
Heat capacity	Joule per kilogram Kelvin	m^2/s^2 K, J/kg K	British thermal unit per pound Rankine	BTU/lb R
Power	Watts (W)	m^2 kg/s^3, J/s	Horse power, BTU per second	hp, BTU/s
Pressure	Pascal, Newton per square meter	Pa, N/m^2	Pound-force per square inch	psi

2.2.4 Derived Units

The above table (Table 2.2) was constructed considering the most relevant derived units at this stage of your engineering studies and depicting the parallels between SI and English Engineering system units. It is not exhaustive but will be very useful for all situations and problems presented in this book.

2.3 Equivalence Among Different Systems, Conversion Factor g_c, and Prefixes

2.3.1 Equivalence Among Different Systems

The following table (Table 2.3) presents some equivalences between the SI and English Engineering system units.

2.3.2 Conversion Factor g_c

In the SI system, where length, mass, and time are defined as primary dimensions and force a secondary dimension, g_c is equal to 1. In the English Engineering system mass and force are defined

Table 2.3 Unit measurements in SI and English Engineering system and its corresponding equivalences

Magnitude	SI	English Engineering	Equivalence
Length	1 m = 100 cm 1 cm = 10 mm 1 km = 1,000 m	1 ft = 12 in 1 mi = 5,280 ft	1 ft = 0.3048 m 1 mi = 1.609.344 km
Mass	1 kg = 1,000 g	1 lb = 16 oz	1 lb = 0.453592 kg
Volume	$1\ m^3 = 1{,}000$ L	ft^3	$1\ ft^3 = 28.317$ L
Temperature	K = °C + 273	R = °F + 460	°C = (5/9)(°F − 32)
Amount of substance	1 kg mol = 1,000 mol		1 lbmol = 453.6 mol
Density	$1\ kg/m^3 = 1\ g/L$	$1\ lb/in^3 = 144\ lb/ft^3$	$1\ kg/m^3 = 0.06243\ lb/ft^3$
Mass flow rate	1 kg/s = 3,600 kg/h	1 lb/s = 3,600 lb/h	1 lb/s = 0.453592 kg/s 1 lb/s = 1,632.93 kg/h
Viscosity	kg/m s (1 Poise (P) = 0.1 kg/m s); 1 P = 100 cP (cP, centipoise)	lb/ft s	1 lb/ft s = 14.88163 P 1 lb/ft s = 1,488.2 cP
Force	N	lbf	1 lbf = 4.4482 N
Energy	J (N m) = 0.23884 cal	BTU	1 BTU = 1,055.056 J 1 BTU = 778.17 lbf ft 1 lbf ft = 1.3558 J
Heat capacity	J/kg K	BTU/lb R	1 BTU/lb R ~4,184 J/kg K
Power	J/s (W)	1 hp = 0.706787 BTU/s	1 BTU/s = 1,055.056 W
Pressure	Pascal (Pa) (N/m^2)	psi	1 psi = 6,894.75 Pa 1 atm = 14.69 psi 1 atm = 760 mmHg (0 °C) 1 Bar = 100,000 Pa

as primary dimensions, and g_c is not equal to 1. To reconcile the mass and force units in the English Engineering system, 1 pound force ≡ 1 pound mass

$$\frac{g}{g_C} = \frac{1\ lb_f}{1\ lb_m}.$$

Therefore,

$$g_c = 32.174 \frac{\text{lbm-ft}}{\text{lbf-s}^2}.$$

In the English Engineering system, g_c should always be used when mass and force are related. It is normally present in calculations of, for example, work and power.

2.3.3 Multiples and Submultiples in SI System

Factor	Prefix	Symbol
10^{12}	Tera	T
10^9	Giga	G
10^6	Mega	M
10^3	Kilo	K
10^2	Hecto	H

(continued)

(continued)

Factor	Prefix	Symbol
10^{1}	Deca	Da
10^{-1}	Deci	D
10^{-2}	Centi	C
10^{-3}	Milli	M
10^{-6}	Micro	μ
10^{-9}	Nano	N
10^{-12}	Pico	P

2.4 Basic Concepts and Process Variables

Process variables are chemical and physical properties involved in the streams of different processes of a complete system. The most common process variables at this stage of your development are mass, volume, density, moles, pressure, and temperature and will be discussed in some detail in terms of the main units and conversions between SI and English Engineering systems.

2.4.1 Mass, Volume, Density (ρ), and Specific Volume (1/ρ)

As shown in Table 2.2, density is a derived unit, and its basic unit is mass per length cubic, i.e., mass per unit volume. In general, the density of solids is not temperature dependent and can be measured by immersing a known mass of a solid in a liquid and measuring the displaced volume. As we will see in Chap. 6, an estimate of the density of air, at atmospheric pressure, can be obtained through the ideal gas law. From the ideal gas law we can obtain the following expression to estimate density:

$$PV = nRT,$$

then

$$PV = \frac{m}{MW}RT,$$

where P is pressure, V is volume, n is moles, m is mass, R is the ideal gas constant, and T is temperature.

Finally:

$$\rho = \frac{m}{V} = \frac{PMW}{RT}.$$

In the SI system, mass is measured in kilograms (kg) and in the English Engineering system in pounds (lb). As presented in Table 2.3, the equivalence between these systems is

$$1\text{lb} = 0.453592\ \text{kg}.$$

Warm-up Example 1: The density of water at 20 °C is 1.0 kg/L. Express the density of water in (a) kg/m^3 and (b) lb/ft^3.

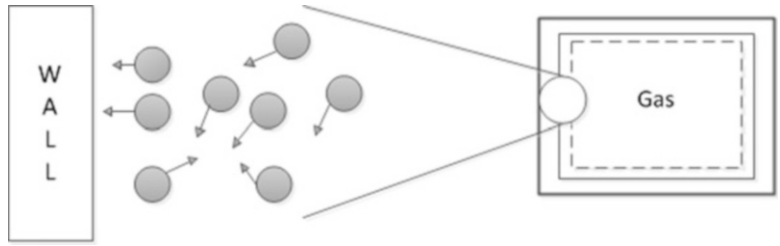

Fig. 2.1 Gas particles exchanging momentum through collisions with the molecules of the container wall

An easy and practical way to convert units is by multiplying by 1, as developed in the following solution:

(a) $\rho = 1\dfrac{\text{kg}}{\text{L}} * \dfrac{1,000\text{L}}{1\text{m}^3} = 1.0 * 10^3 \dfrac{\text{kg}}{\text{m}^3}$,

(b) $\rho = 1\dfrac{\text{kg}}{\text{L}} * \dfrac{1,000\ \text{L}}{1\ \text{m}^3} * \dfrac{(0.3048)^3\text{m}^3}{1\text{ft}^3} * \dfrac{1\ \text{lb}}{0.453592\ \text{kg}} = 62\dfrac{\text{lb}}{\text{ft}^3}$.

2.4.2 Moles

This represents a unit to measure the amount of a substance. A mole is defined as 6.022×10^{23} molecules, atoms, or any elementary entity. For example, 1 mol of water has 6.022×10^{23} molecules of water and weighs approximately 18 g (remember that the molecular weight of water is 18):

$$n_S = \frac{m_S}{MW_S},$$

where n_s is the number of moles of substance S, m_s is the mass of substance S, and MW_s is the molecular weight of substance S.

Why are moles used in chemistry most of the time instead of mass? Because when chemical reactions occur, the calculations on material balance are greatly simplified. Remember that the stoichiometry in chemical reactions is balanced or equilibrated in moles.

2.4.3 Pressure (P)

The pressure of a fluid on a surface is defined as the normal force exerted by the fluid per unit of surface area. Assuming a gas within a cylinder, gas particles exchange momentum through collisions and interactions with the molecules of the container walls (Fig. 2.1).

The momentum exchange is represented by a force exerted by each molecule at the point of collision with the wall. Given a large number of collisions over a large area (A), the macroscopic effect can be represented as force (F) acting on the entire area:

$$P = \frac{F}{A},$$

The basic unit in the SI pressure system is a Pascal (Pa), which represents the force exerted by a Newton (N) on a square meter (m^2) of surface: N/m^2. The most common units of measure are atmosphere and bar (0.98693 atm $\equiv$ 1 bar). The basic unit of pressure in the English Engineering

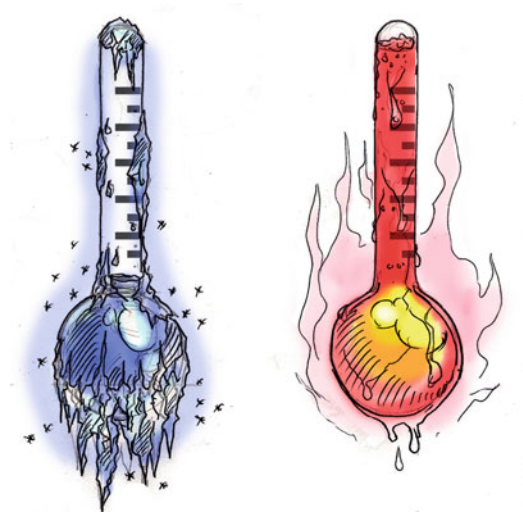

Fig. 2.2 Thermometers showing "cold" and "hot" temperatures

system is psi (lbf/in^2). More specific units are psia = pounds force/in^2 (absolute) and psig = pounds force/in^2 gage (relative):

$$\text{Ppsia} = \text{Ppsig} + 14.7(1\ \text{atm} = 14.7\ \text{psia}).$$

Warm-Up Example 2: Convert 50.0 psia to (a) mmHg (0 °C) and (b) atm.
As presented in the example for the conversion of the density of water, again, and as will apply throughout the text, we will use the procedure to multiply by 1, as follows:

(a) $50\ \text{psia} * \dfrac{1\ \text{atm}}{14.7\ \text{psia}} * \dfrac{760\ \text{mmHg}}{1\ \text{atm}} = 2.58 * 10^{3}\ \text{mmHg},$

(b) $50\ \text{psia} * \dfrac{1\ \text{atm}}{14.7\ \text{psia}} = 3.4\ \text{atm}.$

2.4.4 Temperature (T)

Temperature is the property that expresses the energy state or level of inner energy of the molecules of a substance. It is intuitively associated with the feeling of hot or cold of an object (Fig. 2.2).

The most common method of measuring temperature is with a thermometer, which has a fluid inside that expands when heated. In the International System of Units (SI), temperature is measured on an absolute scale. The unit is called Kelvin (K), and the ideal gas is considered a thermometric fluid. A Kelvin corresponds to approximately 1.38×10^{-23} J per particle. Degrees Celsius are related to the Kelvin absolute scale as

$$\text{T[K]} = \text{T[°C]} + 273.$$

In the English system, temperature is measured on the Fahrenheit scale. The unit is called a degree Fahrenheit (°F). The absolute scale of this system corresponds to the Rankine scale (R):

$$\text{T[R]} = \text{T[°F]} + 460.$$

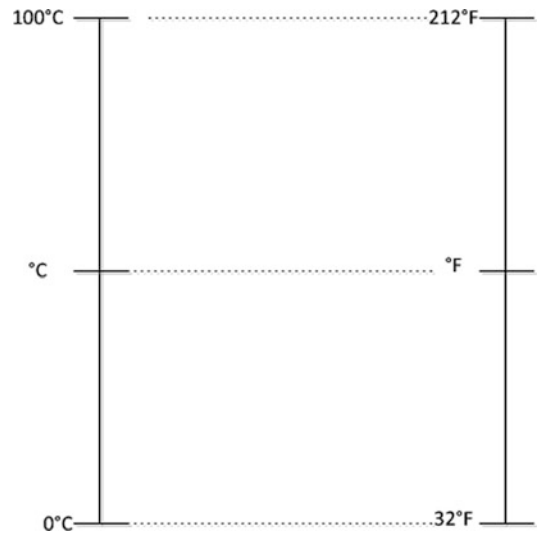

Fig. 2.3 Schematic representation of Celsius and Fahrenheit temperature scales

Celsius degrees and Fahrenheit degrees are related by

$$[^{\circ}\mathrm{C}] = (5/9)^{*}([^{\circ}\mathrm{F}] - 32).$$

The relationship between Celsius degrees and Fahrenheit degrees can be obtained if one knows the freezing point and boiling point of water at 1 atm in each scale. Figure 2.3 presents a schematic representation of the freezing and boiling points of water in each scale.

From this representation we can formulate the following equation:

$$\frac{^{\circ}\mathrm{C} - 0}{100 - 0} = \frac{^{\circ}\mathrm{F} - 32}{212 - 32}.$$

Therefore:

$$^{\circ}\mathrm{C} = \frac{5}{9}(^{\circ}\mathrm{F} - 32).$$

Warm-Up Example 3: Convert 50 °C to a temperature in degrees Rankine.
First we will convert 50 °C to degrees Fahrenheit, and then from Fahrenheit to Rankine.

From: $^{\circ}\mathrm{C} = \frac{5}{9}(^{\circ}\mathrm{F} - 32)$ we get that $^{\circ}\mathrm{F} = \frac{9}{5}{}^{\circ}\mathrm{C} + 32$; therefore, 50 °C = 122 °F.

The equation that relates Rankine to Fahrenheit is R = °F + 460. Finally, 50 °C on the Rankine scale is R = 122 + 460 = 582.

2.5 Dimensionless (Nondimensional) Numbers and Their Importance in Process Engineering

Here we present and briefly discuss why dimensionless numbers (DNs) are important in chemical and bioprocess process engineering and in addition present the most common DNs encountered in chemical and bioprocess engineering.

2.5.1 Why Are Dimensionless Numbers Important?

1. Because they reduce the number of variables needed to describe a phenomenon and minimize the number of experiments to fit the data.
2. Because they simplify physical understanding of a phenomenon.
3. Because they can guide us in understanding which processes are dominant.

2.5.2 Dimensionless Numbers

Here we introduce you to the most common DNs with a brief explanation of their relevance. Although we employ SI units, you can use other units as long as you are consistent. The calculation of a DN does not depend on the unit system used.

2.5.2.1 Reynolds Number (*Re*)

The Reynolds number is probably the most common DN in process engineering. One of its characteristics is that you can determine if the flow is laminar or turbulent:

$$R_e = \frac{Dv\rho}{\mu},$$

where, D is the characteristic dimension (m) (e.g., diameter in a pipe), v is the velocity of the fluid (m/s), ρ is the density of the fluid (kg/m^3), and μ is the dynamic viscosity of the fluid (kg/m s) or (Pa s).

The regime of the flow can be characterized according to the following *Re* numbers:
$Re < 2{,}100$ laminar flow,
$2{,}100 < Re < 4{,}000$ transition,
$Re > 4{,}000$ turbulent flow.

2.5.2.2 Nusselt Number (*Nu*)

The Nusselt number represents the ratio of the convective and conductive heat transfer of a fluid in a certain length:

$$N_u = \frac{hl}{k},$$

where h is the local heat transfer coefficient (W/m^2 K), l is the characteristic length (m) (e.g., diameter in a pipe), and k is the thermal conductivity of the fluid phase (W/m K).

The *Biot* (*Bi*) DN is used when the heat transfer is in unsteady state:

$$B_i = \frac{hl}{k} = \frac{\text{Conductive Resistance}}{\text{Convective Resistance}}$$

where h is the local heat transfer coefficient (W/m^2 K), l is the characteristic length (m) (minimum conduction path length), and k is the thermal conductivity of the solid (W/m K).

The following Biot numbers indicate which resistance is dominant:
$Bi < 0.1$ convection,
$0.1 < Bi < 40$ both convection and conduction,
$Bi > 40$ conduction.

2.5.2.3 Sherwood Number (*Sh*)

The Sherwood number is similar to the Nusselt number but for mass transfer. It represents the ratio of convective to diffusive mass transfer:

$$S_h = \frac{h_D l}{D},$$

where h_D is the mass transfer coefficient (m/s), l is the characteristic length (m), and D is the mass diffusivity (m^2/s).

2.5.2.4 Prandtl Number (Pr)

The Prandtl number is normally used in heat transfer and represents the ratio of the kinematic viscosity and thermal diffusivity:

$$P_r = \frac{C_p \mu}{k},$$

where C_p is the specific heat of the fluid (J/kg K), μ is the dynamic viscosity of the fluid (kg/m s or Pa s), and k is the thermal conductivity of the fluid (W/m K).

2.5.2.5 Froude Number (Fr)

The Froude number is normally used to investigate the fluid flow in free surfaces:

$$F_r = \frac{v}{(gl)^{0.5}},$$

where v is the velocity of the fluid (m/s), g is the acceleration gravity (m/s^2), and l is the characteristic length (m).

There is a plethora of DNs. Some additional DNs are the Grashof number (Gr), Mach number (Ma), Peclet number (Pe), Schmidt number (Sc), Lewis number (Le), Rayleigh number (Ra), Stanton number (Sn), Thiele number (Th), and Weber number (We).

2.6 Solved Problems

1. **Acceleration [3]**. Convert the acceleration of gravity (g) 9.81 m/s^2 to (a) mi/h^2, (b) km/h^2, (c) ft/min^2, and (d) ft/s^2.

   (a) $g = 9.8 \frac{\text{m}}{\text{s}^2} * \frac{1 \text{ mi}}{1,609.344 \text{ m}} * \frac{3,600^2 \text{ s}^2}{1 \text{ h}^2} = 7.89 * 10^4 \frac{\text{mi}}{\text{h}^2},$

   (b) $g = 9.8 \frac{\text{m}}{\text{s}^2} * \frac{1 \text{ km}}{1,000 \text{ m}} * \frac{3,600^2 \text{ s}^2}{1 \text{ h}^2} = 1.27 * 10^5 \frac{\text{km}}{\text{h}^2},$

   (c) $g = 9.8 \frac{\text{m}}{\text{s}^2} * \frac{1 \text{ ft}}{0.3048 \text{ m}} * \frac{60^2 \text{s}^2}{1 \text{min}^2} = 1.16 * 10^5 \frac{\text{ft}}{\text{min}^2},$

   (d) $g = 9.8 \frac{\text{m}}{\text{s}^2} * \frac{1 \text{ ft}}{0.3048 \text{ m}} = 32.2 \frac{\text{ft}}{\text{s}^2}.$

2. **Speed [3]**. A car is traveling at 90 km/h and then changes its speed to 60 mi/h. Did the car increase or decrease its speed?

First, we will convert 90 km/h to a speed in mi/h and then compare the result with 60 mi/h:

$$90\frac{km}{h} * \frac{1mi}{1.609344\ km} = 56\frac{mi}{h}$$

Thus the driver increased the speed of the car from 56 to 60 mi/h.

3. **Specific Heat [5]**. If the specific heat of water is 4,184 J/kg K, then express the specific heat in (a) cal/g °C and (b) BTU/lb °F.

(a) $4,184\frac{\text{J}}{\text{kgK}} * \frac{0.238846\ \text{cal}}{1\ \text{J}} * \frac{1\ \text{K}}{1°\text{C}} * \frac{1\ \text{kg}}{1,000\ \text{g}} \cong 1.000\frac{\text{cal}}{\text{g°C}}$.

Although we learned that K = °C + 273, in this case, because it is a ratio, from the relationship from Kelvin to Celsius we obtain

$$\Delta K = \Delta°\text{C}.$$

(b) $4,184\frac{\text{J}}{\text{kgK}} * \frac{1\ \text{Btu}}{1,055.056\ \text{J}} * \frac{0.453592\ \text{kg}}{1\ \text{lb}} * \frac{1\ \text{K}}{1°\text{C}} * \frac{1°\text{C}}{1.8°\text{F}} = 1.000\frac{\text{Btu}}{\text{lb°F}}$.

Again, the approach to use is not to transform degrees Celsius to Fahrenheit but to use the concept presented in answer (a), that is, in this case: a difference of 1 °C is equivalent to a difference of 1.8 °F.

4. **Flow rate [4]**. The flow rate in a certain pipe is 40.0 m^3/h. (a) What is the flow rate in ft^3/s? (b) What is the flow rate in L/s?

(a) $40\frac{\text{m}^3}{\text{h}} * \frac{1\ \text{h}}{3,600\ \text{s}} * \frac{1\ \text{ft}^3}{(0.3048)^3\text{m}^3} = 0.392\frac{\text{ft}^3}{\text{s}}$,

(b) $40\frac{\text{m}^3}{\text{h}} * \frac{1\ \text{h}}{3,600\ \text{s}} * \frac{1,000\ \text{L}}{1\ \text{m}^3} = 11.1\frac{\text{L}}{\text{s}}$.

5. **Reynolds number [4]**. The speed of the fluid (water at 20 °C) in a pipe is 1.0 m/s, and the diameter of the pipe is 2.0 in. What is the Reynolds number?

To calculate the Reynolds number, we need to know the diameter of the pipe (D), the speed of the fluid (v), the density of the fluid (ρ), and the viscosity of the fluid (μ)—and all in the same unit system. So far we know the speed of the fluid and the diameter of the pipe, but the units are not consistent. First we will express the diameter in SI units as follows:

$$D = 2\ \text{in} * \frac{1\ \text{ft}}{12\ \text{in}} * \frac{0.3048\ \text{m}}{1\ \text{ft}} = 0.051\ \text{m}.$$

Now we need to search for the density (ρ) and viscosity (μ) of water (20 °C). A good option is to search in Perry's Chemical Engineering Handbook. We will use the following approximate values: $\rho = 1.0$ kg/L and $\mu = 0.010$ Poise.

Expressing density and viscosity in the SI system we obtain

$$\rho = 1\frac{\text{kg}}{\text{L}} * \frac{1,000\ \text{L}}{1\ \text{m}^3} = 1.0 * 10^3\frac{\text{kg}}{\text{m}^3},$$

$$\mu = 0.01\ \text{Poise} * \frac{0.1\frac{\text{kg}}{\text{m}-\text{s}}}{1\ \text{Poise}} = 0.0010\frac{\text{kg}}{\text{m}-\text{s}}.$$

Therefore,

$$R_e = \frac{Dv\rho}{\mu} = \frac{0.0508\ \text{m} * 1\frac{\text{m}}{\text{s}} * 1,000\frac{\text{kg}}{\text{m}^3}}{0.001\frac{\text{kg}}{\text{m}-\text{s}}} = 51 * 10^4$$

Note that the Reynolds number is a DN. In addition, you can check and obtain the same result working with the English Engineering system.

6. **Planet Verck [8]**. The name of the basic units on the planets Earth and Verck are presented in the following table.

Fundamental magnitude	Earth (basic unit)	Verck (basic unit)
Length	Meter (m)	Meverck (mv)
Mass	Kilogram (kg)	Kiloverck (kv)
Time	Second (s)	Severck (sv)
Temperature	Kelvin (K)	Kelverck (Kv)
Amount of substance	Mole (mol)	Molverck (molv)

where the equivalencies are as follows: 1 m = 3 mv, 1 kg = 5 kv, 1 s = 0.1 sv, and 1 mol = 2.5 molv.

In relation to temperature, the freezing point of water (0 °C) measured in Kelverck grades is 500 °Kv, and the boiling point of water (100 °C) measured in Kelverck grades is 1,000 °Kv.

(a) Find an equation to relate the Celsius scale to the Kelverck scale. (b) Find an equation to relate the Fahrenheit scale to the Kelverck scale. (c) If the acceleration of gravity (g) is 9.81 m/s^2, how much is it in Verck units? (d) If the specific heat of water (Cp) is 4,184 J/kg K, how much is it in Verck units? (e) Convert 10 lbmol to the Verck scale. (f) The ideal gas constant is approximately 8.3144621 J/mol K. Express the ideal gas constant in Verck units.

(a) The following scheme (Fig. 2.4) shows a parallel between the Celsius and Kelverck temperature scales.

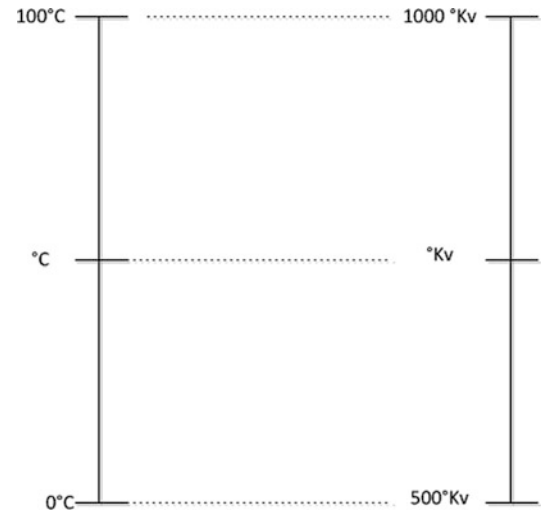

Fig. 2.4 Schematic representation of Celsius and Kelverck temperature scales

From the scheme we can formulate the following equation:

$$\frac{^{\circ}\mathrm{C}-0}{100-0}=\frac{^{\circ}\mathrm{Kv}-500}{1000-500}.$$

Therefore, rearranging we obtain

$$^{\circ}\mathrm{C}=\frac{1}{5}(^{\circ}Kv-500).$$

(b) Knowing that

$$^{\circ}\mathrm{C}=\frac{1}{5}(^{\circ}\mathrm{Kv}-500)$$

and

$$^{\circ}\mathrm{C}=\frac{5}{9}(^{\circ}\mathrm{F}-32)$$

we obtain

$$^{\circ}\mathrm{F}=32+\frac{9}{25}(^{\circ}\mathrm{Kv}-500).$$

You can check the validity of this equation using the freezing and boiling points of water and obtain 32 °F for 500 °Kv and 212 °F for 1,000 °Kv.

(c) To convert g from SI to the Verck scale, we will proceed multiplying by 1 as follows:

$$g=9.8\frac{\mathrm{m}}{\mathrm{s}^2}*\frac{3\ \mathrm{mv}}{1\ \mathrm{m}}*\frac{1\ \mathrm{s}^2}{0.1^2\ \mathrm{sv}^2}=2.94*10^3\frac{\mathrm{mv}}{\mathrm{sv}^2}.$$

(d) Again, to convert the specific heat of water to the Verck scale, we will proceed multiplying by 1.

From Table 2.2 we know that Joule (J) in SI units is $\mathrm{m}^2\,\mathrm{kg/s}^2$. In addition, the ratio between °C and K is one and the ratio between °C and Kv is 1 °C = 5 °Kv; therefore, 1 K = 5 °Kv:

$$Cp=4,184\frac{J}{kgK}*\frac{1\frac{\mathrm{m}^2\mathrm{kg}}{\mathrm{s}^2}}{1\ \mathrm{J}}=4,184*\frac{1\ \mathrm{K}}{5^{\circ}\mathrm{Kv}}*\frac{3^2\ \mathrm{mv}^2}{1\ \mathrm{m}^2}*\frac{1\mathrm{s}^2}{0.1^2\ \mathrm{sv}^2}=7.531*10^5\frac{\mathrm{mv}^2}{\mathrm{sv}^2{}^{\circ}\mathrm{Kv}},$$

(e) $10\ \mathrm{lbmol}*\frac{453.6\ \mathrm{mol}}{1\ \mathrm{lbmol}}*\frac{2.5\ \mathrm{molv}}{1\ \mathrm{mol}}=1.1*10^4\ \mathrm{molv},$

(f) $R=8.3144621\frac{\mathrm{J}}{\mathrm{mol\ K}}*\frac{1\frac{\mathrm{m}^2\mathrm{kg}}{\mathrm{s}^2}}{1\ \mathrm{j}}=8.3144621\frac{\mathrm{m}^2\ \mathrm{kg}}{\mathrm{mol\ s}^2\ \mathrm{K}}*\frac{1\ \mathrm{mol}}{2.5\ \mathrm{molv}}*\frac{3^2\ \mathrm{mv}^2}{1\ \mathrm{m}^2}*\frac{1\ \mathrm{K}}{5^{\circ}\mathrm{Kv}}*\frac{1\ \mathrm{s}^2}{0.1^2\ \mathrm{sv}^2}*\frac{5\ \mathrm{kv}}{1\ \mathrm{kg}}=2,993.21\frac{\mathrm{mv}^2\ \mathrm{kv}}{\mathrm{molvs}^{\circ}\mathrm{Kv}}.$

2.7 Proposed Problems

The first nine problems are simple and meant to help you familiarize yourself with most of the units used throughout the book. They are not representative of all units that you will be using as a chemical and bioprocess engineer, but they will allow you to handle unit conversions and in addition become familiar with some dimensions in process engineering and in other systems.

Then the rest of problems are presented to test your ability in more difficult situations where the main subject is the handling of unit conversions.

2.7.1 Warm-Up

1. **Speed conversion [3]**. Complete the following table:

Speed	To speed in
100 km/h	mi/h: 62.1 mi/h
65 mi/h	km/h: 1.05×10^2 km/h
10 m/s	km/h and mi/h: 36 km/h and 22 mi/h
1,000 cm/s	m/s, km/h, and mi/h: 10.00 m/s, 36.00 km/h, and 22.37 mi/h

2. **Length conversion [3]**. Complete the following table:

Length	To length in:
100 m	ft, in, and mm: 328 ft, 3.93×10^3 in, and 100×10^3 mm
60 in	m, mm, and ft: 1.5 m, 1.5×10^3 mm, and 5.0 ft
10 μm	m and ft: 10^{-5} m and 3.28×10^{-5} ft
1,000 cm	in and m: 393.7 in and 10.00 m
50 nm	in, μm and ft: 2.0×10^{-6} in, 0.050 μm and 1.64×10^{-7} ft

3. **Area conversion [3]**. Complete the following table:

Area	To area in:
100 m^2	ft^2, in^2 and mm^2: 1.08×10^3 ft^2, 155×10^5 in^2 and 1.00×10^8 mm^2
60 in^2	m^2, mm^2, and ft^2: 0.039 m^2, 3.9×10^4 mm^2, and 0.42 ft^2
10 μm^2	m^2 and ft^2: 10^{-11} m^2 and 1.1×10^{-10} ft^2
1,000 cm^2	in^2 and m^2: 155.0 in^2 and 1.000×10^{-1} m^2
50 nm^2	in^2 and μm^2: 7.8×10^{-14} in^2 and 5.0×10^{-5} μm^2

4. **Volume conversion [3]**. Complete the following table:

Volume	To volume in:
1 ft^3	L: 28.3 L
100 m^3	L, ft^3, and in^3: 1.00×10^5 L, 3.53×10^3 ft^3, and 6.1×10^6 in^3
60 in^3	L, m^3, and ft^3: 0.98 L, 9.8×10^{-4} m^3, and 0.030 ft^3
10 μm^3	m^3 and ft^3: 10^{-17} m^3 and 3.5×10^{-16} ft^3
1,000 cm^3	in^3 and m^3: 61.02 in^3 and 1.0×10^{-3} m^3
50 nm^3	in^3: 3.1×10^{-21} in^3

5. **Density conversion [4]**. Complete the following table:

Density	To density in:
62.4 lb/ft^3	kg/L: 1.00 kg/L
1,000 kg/m^3	kg/L, lb/ft^3, and lb/in^3: 1.000 kg/L, 62.43 lb/ft^3, and 0.036 lb/in^3
960 g/L	kg/L, kg/m^3, and lb/ft^3: 0.960 kg/L, 960 kg/m^3, and 59.9 lb/ft^3
10 mg/μm^3	kg/m^3 and lb/ft^3: 10^{13} kg/m^3 and 6.2×10^{11} lb/ft^3
1,000 g/cm^3	lb/in^3 and kg/m^3: 36.13 lb/in^3 and 10^6 kg/m^3
50 μg/nm^3	μg/in^3, μg/μm^3, and kg/L: 8.2×10^{23} μg/in^3, 5.0×10^{10} μg/μm^3, and 5.0×10^{16} kg/L

6. **Temperature conversion [4]**. Complete the following table:

Temperature	To temperature in:
37 °C	K, °F: 310 K, 98.6 °F
68 °F	R, °C: 528 R, 20 °C
–44 °C	°F: –47.2 °F
100 °C	°F: 212 °F
32 °F	°C: 0 °C

7. **Pressure conversion [5]**. Complete the following table:

Pressure	To pressure in:
1.00 Pa	atm, lb$_f$/in^2 (psi), and mmHg (0 °C): 9.87×10^{-6} atm, 1.45×10^{-4} psi, and 7.50×10^{-3} mmHg (0 °C)
450 mmHg (0 °C)	atm, psi: 0.592 atm and 8.7 psi
14.69 psi	atm: 1.000 atm
6 atm	psi: 88 psi

8. **Mole conversion [4]**. Complete the following table:

Moles	To moles in:
2.0 kg mol	lb mol and g mol: 4.4 lb mol and 2.0×10^3 g mol
453.6 g mol	lb mol: 1.000 lb mol
10.0 lb mol	kg mol: 4.54 kg mol

9. **Energy conversion [4]**. Complete the following table:

Energy	To energy in:
12.0 J	BTU: 0.0114 BTU
10.0 Kw h	BTU, cal, J: 3.41×10^4 BTU, 8.6×10^6 cal, 3.6×10^7 J
10.0 ft-lb$_f$	BTU, J, Kw h, and cal: 0.0129 BTU, 13.6 J, 3.77×10^{-6} Kw h, and 3.24 cal

2.7.2 Conversions and Calculations

1. **Cylinder-piston system [4]**. A gas is confined in a cylinder-piston system. What is the pressure of the gas (atm) if the piston has a mass of 100 kg and a surface area of 0.009672 m^2? g = 9.8 m/s^2.
   A: 2 atm; $P_{GAS} = P_{PISTON} + P_{ATMOSPHERIC} = 1 + 1 = 2$ atm [the pressure of the gas corresponds to the pressure of the piston (1 atm) plus the atmospheric pressure (1 atm)].

2. **Higher temperature [4]**. What is the increment in degrees Celsius if the temperature increases by 10 K?
   A: 10 °C
3. **Higher temperature 2 [6]**. What is the increment in degrees Fahrenheit if the temperature increases by 10 °C?
   A: 18 °F
4. **Equalizing [6]**. At what temperature are the Celsius and Fahrenheit scales numerically equal?
   A: –40 °C = –40 °F
5. **Density [6]**. What is the density of a cylinder of radius R m, height H m, and mass of M kg in kg/m^3 and lb/ft^3?
   A: M/(πR^{2}H) kg/m^3 and ~0.0624 M/(πR^{2}H) lb/ft^3
6. **Density 2 [4]**. (a) What is the density (SI system) of a cube with a mass of a kg and a length of a m? (b) What is the density of the cube in lb/ft^3?
   A: (a) 1/a^2 kg/m^3, (b) ~0.06243/a^2 lb/ft^3
7. **Mass [5]**. By what percentage will the mass of a cylinder increase if the radius increases by 50.0 %? Assume that density is constant.
   A: 125 %
8. **Measuring mass [5]**. An object has been measured five times in a balance giving an average of 50.40 kg. Another student adds a new measurement of 112.6 lb. What is the average mass of these six measurements?
   A: ~50.51 kg
9. **Measuring mass 2 [7]**. An object has been measured ten times, and the average weight is 1.04 kg. If the average weight of the first eight measurements was 2.315 lb, what is the total weight of the last two measurements?
   A: ~2 kg
10. **Copper and iron [7]**. A wire of radius r is made from C kg of copper ($\rho_{Cu} = 8.96$ g/cm^3). Another wire of radius r is made from C kg of iron ($\rho_{Fe} = 491.3$ lb/ft^3). (a) Which one is longer? (b) How much longer is the wire in (a) as a percentage of the shorter one?
    A: (a) The iron wire, (b) 13.9 %
11. **Air density [4]**. If the mean molecular weight of air is approximately 29 g/mol, then estimate the density of air in (g/L) using the ideal gas law ($PV = nRT$) at an absolute pressure of 14.69 psi and 0 °C. Remember that the R constant is equal to 10.7316 ft^3 psi/R lb mol.
    A: 1.3 g/L

2.7.3 Equivalences and Conversions on Planet Morc

1. **Planet Morc [8]**. The name of the basic units on planets Earth and Morc are presented in the following table:

Fundamental magnitude	Earth (basic unit)	Morc (basic unit)
Length	Meter (m)	Metorc (mt)
Mass	Kilogram (kg)	Kilotorc (kt)
Time	Second (s)	Secorc (sc)
Temperature	Kelvin (K)	Keltorc (Kt)
Amount of substance	Mole (mol)	Molrce (molrc)

where the equivalencies are 1 m = 5 mt, 1 kg = 4.5 kt, 1 s = 10 sc, and 1 mol = 0.5 molrc.

In relation to temperature, the freezing point of water (0 °C) measured in keltorc grades is 1,000 Kt, and the boiling point of water (100 °C) measured in keltorc grades is 2,000 Kt.

(a) Convert 122 °F to the keltorc scale. (b) Express the force of 10 N in Morc units. (c) Express the speed of 100 km/h in Morc units. (d) Express the pressure of 5.0 atm in Morc units. (e) Express the volume of a tank of 100,000 L in Morc units. (f) Express 10.0 BTU in Morc units. (g) Express 10.0 cal in Morc units. (h) Convert 1,500 Kt to the Rankine scale. (i) Convert 73.45 psi to Morc units.

A: (a) 1.5×10^3 Kt, (b) 2.2 kt mt/sc^2, (c) ~13.9 mt/sc, (d) 4.6×10^4 kt/sc^2 mt , (e) 12,500 mt^3, (f) 1.19×10^4 kt mt^2/sc^2, (g) 47.1 kt mt^2/sc^2, (h) 582 R, (i) 4.495×10^3 kt/sc^2 mt

2. **Mount Everest [3]**. The highest mountain range on Morc is the Morcian Mountains, and the highest peak is at 44,000 mt. How do you compare the altitude of the highest peak of the Morcian Mountains with Mount Everest in the Himalayas?
   A: Mount Everest is taller (8,848 m), equivalent to 44,240 mt (taller than 44,000 mt)
3. **Adding masses [4]**. 2,400 kg are mixed with 2,400 kt. What is the total mass in Morc units and in SI units?
   A: 1.32×10^4 kt and 2.933×10^3 kg
4. **Gravity [7]**. Morcians have informed us that the acceleration due to gravity on their planet is 0.60 mt/sc^2. How do you compare the gravity on planet Morc with the gravity on Earth?
   A: The gravity on Morc is higher, 0.60 mt/sc^2 = 12 m/s^2 (on Earth it is ~9.8 m/s^2)
5. **Density [6]**. What are the mass and density of the cylinder in problem 5 (2.7.2 Conversions and Calculations) in Morc units?
   A: 4.5*M kt and 0.036*M/(πR^{2}H) kt/mt^3
6. **Ideal gas constant (*R*) [8]**. The ideal gas constant is approximately 8.3144621 J/mol K. Express the ideal gas constant in (a) Morc units, (b) atm L/mol K, and (c) ft^3-psi/R-lbmol
   A: (a) 187.075 kt mt^2/sc^2 Kt molrc, (b) 0.08206 atm L/mol K, and (c) 10.73159 ft^3 psi/R-lbmol

2.7.4 Engineering Problems

1. **Planet *abc* [6]**. On planet ***abc*** temperature is measured on the ***abc*** scale. If the boiling point of water (100 °C) corresponds to *c* °*abc* and the freezing point of water (0 °C) corresponds to *a* °*abc*, then find an equation that relates the Celsius scale with the *abc* scale.
   A: °C = 100*(°abc – *a*)/(*c* – *a*). If $a = 32$ and $c = 212$, then the ***abc*** scale will be equal to °F and then you can check that the expression that relates °C to °abc will be °C = (5/9)*(°abc – 32) or °C = (5/9)*(°F – 32)
2. **Planet *abc* 2 [6]**. On planet *abc* temperature is measured on the *abc* scale. If the boiling point of water (212 °F) corresponds to *c* °*abc* and the freezing point of water (32 °F) corresponds to *a* °*abc*, then find an equation that relates the Fahrenheit scale to the *abc* scale.
   A: °F = [180*(°abc – a)/(c – a)] + 32. Again, you can test this equation using a = 0 and c = 100 (Celsius scale)
3. **Reynolds number [5]**. The Reynolds number is a DN utilized in fluid mechanics. The Reynolds number is defined as $Re = Dv\rho/\mu$, where D is the diameter of a pipe, v the linear velocity of a fluid, ρ the density of the fluid, and μ the dynamic viscosity of the fluid. (a) Determine Re if $D = 3.0$ in, $\rho = 62.4$ lb/ft^3, $v = 3.0$ m/s, and $\mu = 0.0020$ lb$_f$ s/ft^2. (b) Will the calculation of the Reynolds number depend on the unit system used? Why?
   A: (a) 2.4×10^3 , (b) No, because the Reynolds number is dimensionless.
4. **Antoine's equation [8]**. Antoine's equation relates the vapor pressure (mm Hg 0 °C) to temperature (°C) and is expressed as follows: $P = 10^{A-\frac{B}{C+T}}$,

where A, B, and C are constant and T is the temperature in degrees Celsius. (a) What are the fundamental magnitudes and units of A, B, and C? (b) If, for water, the values are $A = 8.07131$, $B = 1{,}730.63$, and $C = 233.426$, what is the vapor pressure of water at 100 °C according to Antoine's equation?

A: (a) A is dimensionless, B has a temperature dimension and in this case degrees Celsius units, and C has temperature as a fundamental magnitude and degrees Celsius units. (b) 760.086 mmHg (0 °C) [recall that the vapor pressure of water at 100 °C is 760 mmHg (0 °C) (1 atm)].

5. **Apparent volumetric specific heat [5]**. A semiempirical equation to estimate the apparent specific heat (C) of foods at freezing temperatures is as follows: $C = C_1 + \frac{C_2}{(T_{SW}-T)^{C_3}}$, $T < 0$ °C, where C_1, C_2, and C_3 are constants, T_{SW} is the freezing point of water (0 °C), and T is the temperature of the food material in Celsius. In addition, the corresponding values for tylose MH-1000 (cellulose gel, edible food simulant) are $C_1 = 1.19 \times 10^6$, $C_2 = 1.13 \times 10^8$, and $C_3 = 1.66$. (a) If the units for the apparent specific heat are J/m^3 °C and C_3 is dimensionless, then what are the units of C_1 and C_2? (b) Calculate the apparent specific heat of tylose at –10 °C. (c) Calculate the apparent specific heat of tylose at –10 °C in BTU/ft^3 °F units.

   A: (a) C_1 (J/m^3 °C) and C_2 (J °C$^{0.66}$/m^3), (b) 3.67×10^6 (J/m^3 °C), (c) 54.7 BTU/ft^3 °F

6. **Apparent volumetric specific heat-2 [10]**. Express the equation for apparent specific heat in BTU/ft^3 °F units for C and °F for Tsw and T.

   A: $C = 17.743 + \dfrac{4{,}470.1574}{(32-T)^{1.66}}$, where C is in BTU/ft^3 °F and T is in degrees Fahrenheit.

7. **Specific heat of water [7]**. The specific heat of water (J/kg °C) as a function of temperature T (°C) in the range of 0–150 °C has been reported as:

$$C_p = 4{,}176.2 - 0.0909T + 5.4731 * 10^{-3}T^2.$$

   Express this equation to get C_p in J/kg °C but replacing temperature in °F (t).

   A: $C_p = 4{,}176.2 - 0.0505 * (t-32) + 1.689 * 10^{-3}(t-32)^2$

8. **Flow of a falling film [6]**. The velocity (length/time) profile of a fluid in an inclined flat surface can be expressed as

$$v = A\frac{\rho\delta}{\mu}\left(1 - \left(\frac{x}{\delta}\right)^2\right)$$

   where ρ is the fluid density, δ the width of the film, and μ the viscosity. What are the dimensions of A if v has dimensions of length/time? The fundamental magnitude for x is length.

   A: Length^2/time^2

9. **Draining a tank [7]***. One of your best friends has derived a semiempirical equation to predict the time required to drain a tank. Given that you are an expert in analyzing dimensions, please help your friend and tell him what the fundamental magnitudes of the constant K are:

$$t = K\frac{\mu L R^2}{\rho R_0}\ln\left(1 + \frac{H}{L}\right),$$

   where μ is the fluid viscosity, L the length of the pipe (Fig. 2.5), R the radius of the tank, H the height of the tank, R_0 the radius of the pipe, and ρ the density of the fluid.

   A: Time^2/length^4

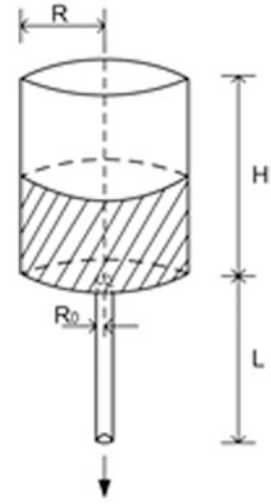

Fig. 2.5 Schematic representation of a draining tank

10. **Ideal gas constant, R [9]**. What are the fundamental magnitudes of the ideal gas constant (R) in the equation $PV = n\,RT$?
    A: Mass length^2/time^2 –amount of substance– temperature (energy/amount of substance –temperature)
11. **Stefan-Boltzmann constant [7]**. The Stefan–Boltzmann constant has a value of 0.1712×10^{-8} BTU/h ft^2 R^4. (a) What is the value of the constant in cal/s cm^2 K^4? (b) What is the value of the constant in J/s m^2 K^4?
    A: (a) 1.354×10^{-12} cal/s cm^2 K^4, (b) 5.67×10^{-8} J/s m^2 K^4
12. **Dimensionless temperature [6]**. A simplified analytical solution for homogeneous solids confined in a finite cylinder is presented in the following equation. If R is the radius of the finite cylinder and L the height of the cylinder, what are the fundamental magnitudes of α (thermal diffusivity)?

$$\frac{TP - T_{CP}}{TP - IT} = 2.0396\ \exp\left[-\left(\frac{\left(2.4048^2\right)}{R^2} + \frac{\pi^2}{L^2}\right) \cdot \alpha \cdot t\right].$$

    A: Length^2/time
13. **Dimensionless temperature 2 [6]**. What will be the critical point temperature (T_{CP}) of a canned food (cylinder can) after 90 min if the initial temperature (IT) of the canned food was 40 °C and the process temperature (TP) is maintained at 120 °C? In addition, the cylinder has a radius of 4 cm and height of 10 cm, and the thermal diffusivity of the food is 6.12×10^{-4} m^2/h. To solve the problem, you can use the dimensionless equation provided in problem 12.
    A: $T_{CP} = 117.6$ °C
14. **Vapor speed in an autoclave bleeder [7]**. Considering an isentropic steam flow in an autoclave bleeder that obeys the ideal gas law, the following equation could be written as

$$v_b^2 = \frac{2g_c P_{sv}}{\rho_{sv}}\left(\frac{\gamma}{\gamma - 1}\right)\left(\left(\frac{P_b}{P_{sv}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} - 1\right).$$

    If density (ρ_{sv}) is expressed in kilograms per cubic meter (kg/m^3), what are the units of P_{sv} to get v_b in meters per second (m/s)?
    A: Pa (N/m^2)

15. **Enzyme kinetics [6]**. The initial rate of a reaction in the presence of substrate inhibition can be written as

$$v = \frac{V_{\max} \cdot [S]}{K_m + [S] + \frac{[S]^2}{K_S}},$$

where normally v, the reaction rate, is expressed in mol/L s and $[S]$, the substrate concentration, is expressed in moles per liter (mol/L). What are the units of V_{max}, K_m, and K_S in SI units?
A: V_{max} in mol/m^3 s, K_m in mol/m^3 and K_S in mol/m^3

16. **Heat transfer [5]**. The following equation is used to estimate the rate of heat transfer:

$$Q = UA\Delta T,$$

where Q is the energy per unit time, A is the heat transfer area, and ΔT is the temperature difference. What are the units for U (global heat transfer coefficient) in the SI system?
A: J/m^2 s K

17. **Heating up a mass of water [4]**. To estimate the energy required (Q) to heat up some mass of water (m) you can use the equation $Q = m\ Cp\ \Delta T$, where Cp is the specific heat of water and ΔT is the temperature difference (final − initial). If the energy required to heat up 2 kg of water from 20 to 60 °C is 334.72 kJ, then calculate the specific heat of water in (a) English Engineering units and (b) SI units.
A: (a) 4,184 J/kg K, (b) 1 BTU/lb R

18. **Viscosity of a pure liquid [6]**. Estimate the viscosity of liquid benzene (25 °C) in centipoise (cP) using the following equation and data:

$$\mu = \frac{Nh}{V} * e^{\frac{3.8T_b}{T}},$$

where $N = 6.023 \times 10^{23}$ (1/mol), $h = 6.624 \times 10^{-27}$ (g cm^2/s), $V = 89.0$ (cm^3/mol), and $T_b = 353.3$ K.
A: 0.4047 cP (4.047×10^{-3} g/cm s)

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Math Skills Review Dimensional Analysis http://www.chem.tamu.edu/class/fyp/mathrev/mr-da.html
Fun with dimensional analysis http://www.alysion.org/dimensional/fun.htm
Unit Analysis http://mrcauseysworld.com/chemistry/Unit-Analysis.html
Dimensional Analysis/Factor Label Method - Chemistry Tutorial http://www.youtube.com/watch?v=DsTg1CeWchc
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3[2] Fundamentals of Process Control, Communication, and Instrumentation

The ideal engineer is a composite . . . He is not a scientist, he is not a mathematician, he is not a sociologist or a writer; but he may use the knowledge and techniques of any or all of these disciplines in solving engineering problems.

N. W. Dougherty

Engineering is the professional art of applying science to the optimum conversion of natural resources to the benefit of man.

Ralph J. Smith

3.1 Introduction

There are a number of situations in nature where the principles that govern process control are applied. More specifically, in many human actions, such as walking or putting a spoon to one's mouth, we can define our procedure as measurement, comparison, decision, and action. Such is the case of a person who stumbles and loses his balance; certain ear sensors send information to the brain, which compares that information to the baseline information on what it means to be upright and in balance. Using this comparison, the brain makes decisions to correct the imbalance through the coordinated movement of the limbs. The brain continues to receive information about the situation and keeps moving the limbs until it confirms that the person has recovered equilibrium. In process control, this strategy is called feedback control. Independent of the cause of the imbalance, the brain will take the actions necessary to recover equilibrium.

Throughout history and in nature, we see many examples of feedback control, almost instinctively used by various living creatures. One of the first scientific mechanisms of measurement and control was developed for use in the Watt steam engine. Much later, in the 1940s, industrialization forced companies to increase productivity and maintain optimal performance, which ultimately led to the appearance of the first electronic automatic controllers. This was a historic step where the operator was replaced by a machine. Of course, the first thought was that this would create massive unemployment; however, in the 1960s, data showed that many more and skilled jobs had been created. Furthermore, it was possible to significantly reduce the number of accidents, either by reducing human interaction with dangerous equipment or toxic and hazardous materials. In this way, process control began to emerge as one of the most comprehensive disciplines of chemical and bioprocess engineering.

In the field of process control, most of the techniques that are used to describe processes are applied. As mentioned previously, through basic tools, such as material balances, we can determine

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the conditions under which a piece of equipment is able to perform a specific role. The designs are made under certain assumptions, for example, steady state. In such cases, the equipment has a feed mass flow rate equal to the output mass flow rate, so there is no accumulation (steady state). However, changes in the feed flow or in its concentration will result in an imbalance in the process. If no control action is taken to prevent the accumulation or deaccumulation of matter, the equipment will eventually be overloaded or emptied.

Control of the process means mastering the process, maintaining it in peak condition, and keeping it safe for people and equipment. How are processes controlled? How does one detect that a process is out of control? What is a smart way to take control of a process? These are some of the questions to be addressed in this section.

3.2 Understanding What Process Control Is

Although it may seem strange, there are certain forms of process control that we execute on a daily basis. Suppose you are taking a shower, in which the flow rate of hot water is fixed, and we can adjust the flow rate of cold water to achieve a pleasant water temperature. Most likely, you would proceed as follows:

1. Through our skin we are continuously sensing (feeling) the temperature of the water.
2. (a) If the water is warmer than we expect, then we will increase the flow rate of cold water. (b) If the water is colder than we expect, then we will reduce the flow rate of cold water.
3. After changing the flow rate of cold water, given that the skin is constantly sensing water temperature (step 1), then depending on the outcome, we will return to step 2(a) or (b) until we feel a pleasant water temperature.

This procedure could be schematized as depicted in Fig. 3.1.

Using this everyday example, we can identify several elements that constitute a controlled process as follows.

3.2.1 Objective of Control

Creating a pleasant shower. We start with the target because in this case it is easy to define and evaluate, as it is in many cases. But in some situations, defining the goal, or identifying the desired value, can be a very complex problem. To know what is best involves the resolution of an optimization problem. There are other cases where more than one objective could be involved, and some of them could be in conflict. What if we would like to have a pleasant temperature but also a certain flow rate of water, not too low, not too high? Later, we will revisit such problems (multipurpose or multivariable).

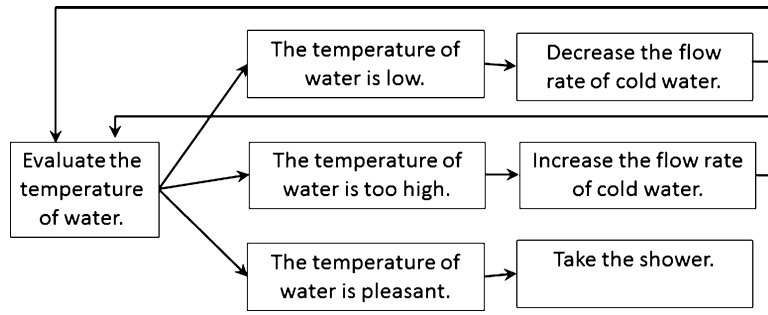

Evaluate the temperature of water.
The temperature of water is low.
Decrease the flow rate of cold water.
The temperature of water is too high.
Increase the flow rate of cold water.
The temperature of water is pleasant.
Take the shower.

Fig. 3.1 Schematic representation of the temperature control of the shower

3.2.2 Meter

Skin as temperature meter. What we did in the shower example was to use our skin directly to see if the water temperature was above or below our feeling of pleasure. The meter is very important: it provides the necessary information for us to take action. Meters are part of what is, by convention, called the instrumentation. Some instruments are capable of measuring a wide variety of magnitudes, such as, for example, temperature, pressure, humidity, and weight. Considering several measurements at different times, the instrument will also provide information on how the system evolves. Later, we will further analyze instrumentation. For now, it should be noted that we are measuring temperature, and the meter is digital (e.g., single digit, 0 or 1, on or off). Since through our skin we cannot know the exact temperature of the water, we just perceive whether the water temperature is higher or lower than the desired pleasant level.

3.2.3 The Process to Be Controlled

The shower example, although illustrative, is very simple. Real control problems in chemical and bioprocess engineering could range from simple mixers to a large set of reactors/bioreactors, columns, tanks, or other process equipment. In general, transforming a raw material into a finished product often involves several stages of physical, chemical, and biological changes. Also, keep in mind that no matter how complex a process is, it can be addressed using the same fundamental tools, i.e., material balance, energy balance, and the like. With strict effort, discipline, and patience, we will be able to model complex systems.

3.2.4 Manipulated Resource

Flow rate of cold water. While this flow rate allows us to change the temperature of the water, by convention, it is common to refer to the actuator (valve) rather than the resource (water flow) as part of the control system. This is justified because in practice, we adjust the cold water valve, which, as a result, will change the flow rate of the cold water. We must be clear that while the resource is the flow rate of cold water, it was the water that which was manipulated, by convention called an actuator. Speaking of actuators, we are not only talking about valves: it suffices to mention the case of a fan, which varies the air flow rate to obtain a pleasant temperature.

3.2.5 Action of Control

Increase or decrease the flow rate of cold water. In this case, the action of control is based on a comparison of the desired pleasant temperature and the temperature of the shower as "measured" by the skin. The element responsible for deciding how much and in what direction the resource will change is called the controller. The controller is the “brain” of any control structure, and in this case our real brain calculates the magnitude of the change, i.e., the flow rate of cold water. The brain processes the information provided by the meter and responds according to its internal configuration. At this stage, we would like to highlight that we have implemented a control strategy called feedback based on the result of a trial-and-error procedure.

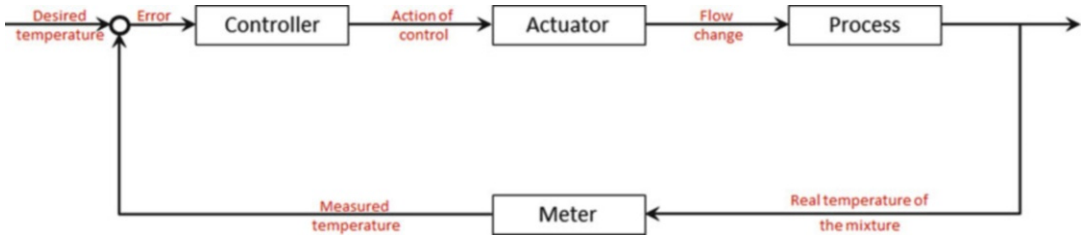


Fig. 3.2 Control loop of the temperature control of the shower

3.2.6 Control Loop

The four elements described previously are the main components of what by convention is called a control loop, depicted in Fig. 3.2, as follows:

Observing Fig. 3.2, we see that each component (box) receives information of one nature and delivers information of a different nature, so that it is useful for the next component (box). Walking through the loop, starting from the meter, we note that it is exposed to the process temperature, registers it, then compares it to the desired temperature (or set point). The difference between the two is what we call the error. Then the controller acts on the error, setting the control action. The action could be to open, close, or leave as is the cold water valve. The actuator, in this case the cold water valve, will change according to the action dictated by the controller; this will result in a change in the flow rate of cold water. The process, i.e., the mixture of hot and cold water, receives a different amount of cold water, and thus the mixture will have a different temperature. The meter will record the new temperature, restarting the cycle.

In formal terms, what happens to the information within each block of the control loop is a mathematical transformation that we call a transfer function. Suppose that in the example of the shower, it is not the skin that is used for measuring temperature but a thermometer that is able to deliver the exact temperature of the water. With this assumption, it is easy to explain the transfer function of the comparator (represented by a circle in Fig. 3.2), which is given by the following equation:

$$e = T_m - T_{SP} \tag{3.1}$$

where e is the positive feedback temperature error (°C), T_m is the temperature measurement (°C), and T_{SP} is the desired or set point temperature (°C).

To understand this more clearly, consider that the desired temperature, the set point, is 40 °C. If the measured temperature is 44 °C, then the error will be 44 – 40 = +4 °C, i.e., 4 °C above the desired temperature (Fig. 3.3). Now, if the measured temperature was 38 °C, then the error would be 38 – 40 = –2 °C, or 2 °C below the desired temperature (Fig. 3.3). In this simple case, it turns out that the transfer function is a straight line, as shown in Fig. 3.3, where the x-axis is the measured temperature and the y-axis is the error. Transfer functions of the other elements of the control loop are a bit more complex; therefore they will be addressed in Chap. 9. As in this example, it is easier to use positive feedback error. However, in practice, it is more common to use negative feedback error, i.e., the set point (T_{SP}) minus the measured value (T_m).

The strategy of formulating a control problem through a loop has tremendous potential because it is a highly standardized, orderly, and robust procedure to address control problems. It divides a process into elements with clear boundaries and makes it easy to recognize what should work and how to assemble the parts.

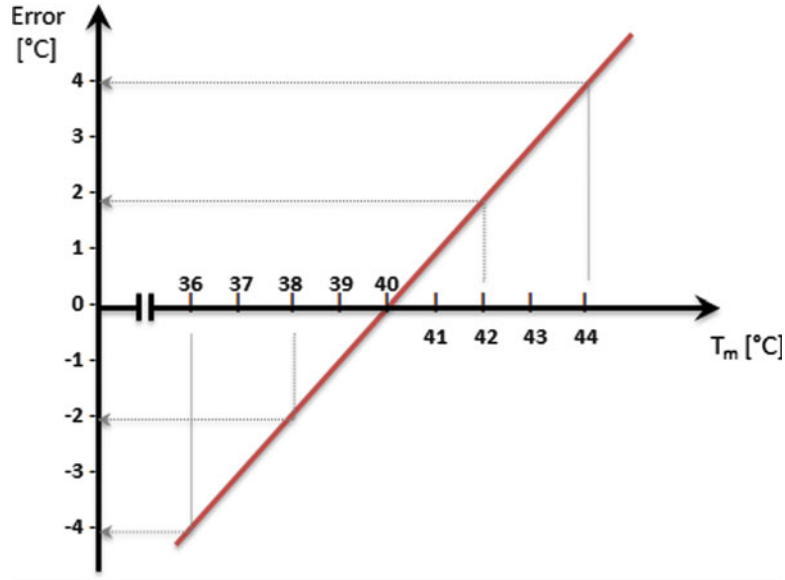


Fig. 3.3 Graphic representation of the temperature error

3.3 Instrumentation

Instrumentation refers to all electronic, electrical, mechanical, or any kind of equipment used in a process for control purposes. That is, meters record the state of a process, actuators modify the control resources, and controllers command the action of the actuators.

Consider the case shown in Fig. 3.4, a tank of 10 m height (E-101), which is fed by a flow of water from several production plants. To prevent the tank from overflowing or emptying, we have arranged a control system composed of a level controller (LC-101), a level meter (LT-101), and a valve (LICv-101) that regulates the output flow of water. The chosen notations are not coincidental; the international community has agreed to use a standard symbology for instrumentation to facilitate the transfer of knowledge among different research groups. The international instrumentation notation will be examined in the appropriate section.

The shaded portion of Fig. 3.4 is usually written in summary as indicated on the right-hand side of the figure.

3.3.1 Sensors

The function of meters is to collect information on the current status and progress of a process. In the case of Fig. 3.4, this is the liquid level in the tank. This information (level of the tank) will be used by the controller, the alarm system, or the supervisor to make decisions about the process.

One of the requirements of the meter is to measure the required amounts, which in the case of a level meter for the tank in Fig. 3.4 is to be capable of indicating measurements from 0 to 10 m. The difference between the maximum and the minimum range of work or operation is called the span.

Keep in mind that in this case, the measurement is in the rage of 0 to 10 m. In other situations, it may be necessary to measure between 20 and 50 m, in others between 1 and 20 mm. So the question is whether it is necessary to manufacture devices that measure different magnitudes. Sometime a

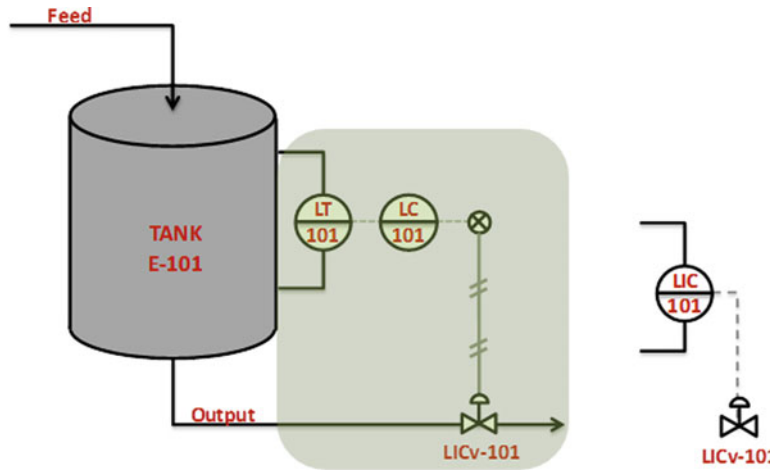


Fig. 3.4 Instrumentation of a tank

device can measure in a wide range, for example from 0 to 50 m, but be configured to measure in a desired range.

This is accomplished by setting the zero and the span at the minimum and the maximum values that we are measuring. A critical point is the resolution of the instrument, that is, its ability to identify two measurements as being distinct. For example, let us construct a level meter that delivers its format measurement as 0.0 m, to one decimal place. In addition, it is desired to measure two levels of liquid: the first one 151 cm and the second one 154 cm. If in the first case the meter indicates 1.5 m and in the second measurement 1.5 m, in other words the same measurement, then the meter does not have the resolution to distinguish these two measurements. That is why when selecting a meter, you must consider its working range and its resolution. Finally, it is important to mention that most modern electronic instruments allow you to configure the zero, the span, and the resolution, giving them much greater versatility.

3.3.1.1 Accuracy and Precision

When a physical entity is repeatedly measured, we rarely get the same value for each test. This difference is determined by several factors, some inherent in what is being measured, including by the fact that the measured quantity changes over time. In addition, fluctuations can result from the properties of the meter, such as its precision and accuracy. Precision refers to the dispersion of the values obtained, whereas accuracy means how close the measured value is to the actual value. Examples of groups of accurate or precise measurements are shown in Fig. 3.5.

The most common meters are used to measure temperature, pressure, flow, and level. However, various physical principles allow us to measure, directly or indirectly, the density of a liquid, the concentration of a pollutant in a gas, or the volatility of certain compounds, for example. Tables 3.1, 3.2, 3.3 and 3.4 present descriptions of some devices.

3.3.1.2 Biosensors

Recently, the field of bioprocessing has seen the development of a new range of meters called biosensors. These are instruments used to record the presence of biological or chemical agents. They are based on an immobilized enzyme that catalyzes a highly specific reaction and detects what is sought, the catalyzed reaction product. For example, using the urease enzyme, we can detect the presence of urea, and with glucose oxidase/catalase, we can detect glucose. These sensors represent a breakthrough in allowing online measurements and indicate, in real time, the status of a biochemical

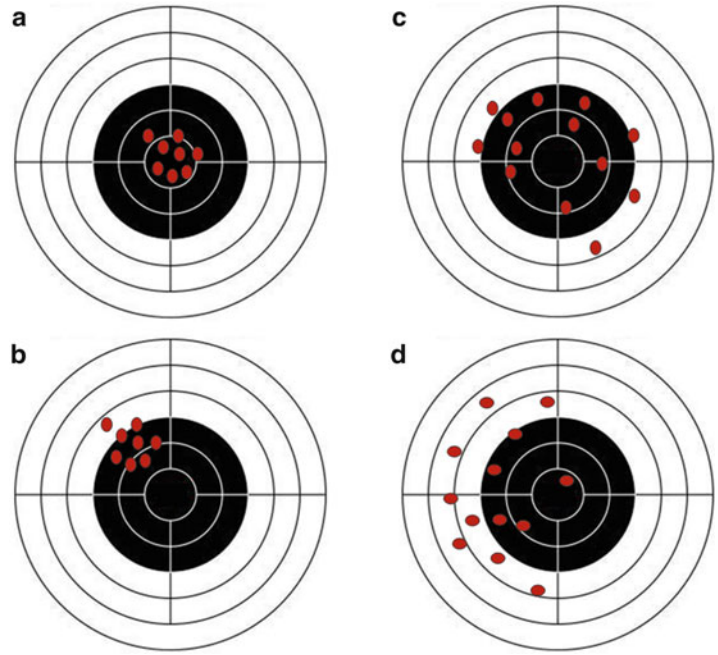


Fig. 3.5 Examples of types of measurements. (**a**) Accurate and precise. (**b**) Inaccurate and precise. (**c**) Accurate but imprecise. (**d**) Inaccurate and imprecise

Table 3.1 Level sensors

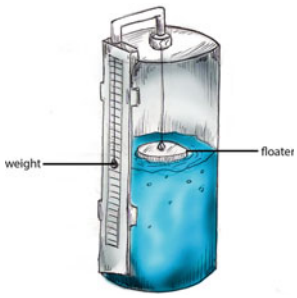
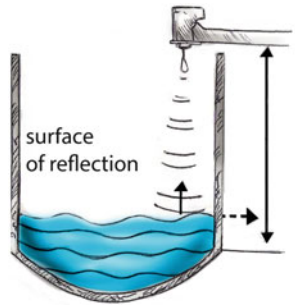
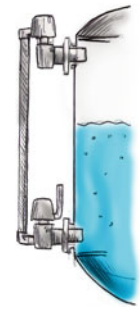
Floater	Ultrasonic	DpCell
		
Principle: the floater remains at the liquid surface. It has a rope tied to it and, depending on how full the tank is, will indicate the level on a scale.	Principle: the meter emits an ultrasonic wave that strikes the surface of the liquid and returns to the meter. The level is a function of the time it takes the signal to return to the instrument.	Principle: from fluid mechanics it is known that a column of fluid exerts a pressure equivalent to its height. Measuring the pressure, the level is calculated indirectly.
Comment: there are various ranges of measurements. As shown in the figure, the floater delivers no signal but can be attached to the end of a device that allows the rope to transmit the measurement.	Comment: such meters are expensive, but they deliver a signal and require external power. They can be installed in various ways, including being submerged in liquid	Comment: widely used in the industry. Level is not measured directly, but is very reliable. They can deliver bad measurements if the fluid has a varying density over time.

Table 3.2 Flow meters

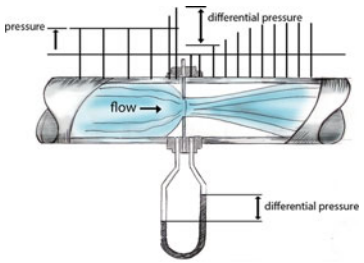
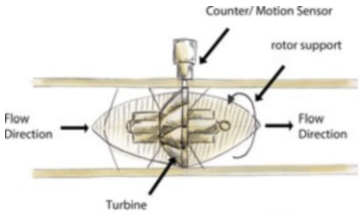
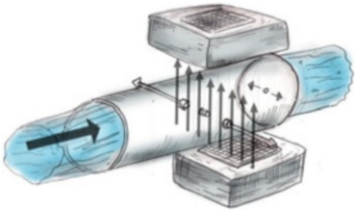
Orifice plate	Turbine	Magnetic
		
Principle: a plate with a hole in the middle is inserted in the pipe through which the fluid passes. The friction on the plate generates a pressure drop in the fluid, making it possible to estimate the average flow.	Principle: a turbine is inserted in the same direction as the fluid. As the fluid picks up speed, the turbine blades will rotate faster. The flow rate will be a function of the rotational speed of the blades.	Principle: magnetic flow meters are based on Faraday's law, which says that the speed of a conductor is proportional to the voltage induced in a magnetic field. The fluid should be conductive.
Comment: some orifice plate deliver signals, others not. Generates permanent pressure drops. They are widely used due to their low cost but require intervention on pipes for installation.	Comment: turbines are modern and are being increasingly used; they deliver a signal. Pipes should be modified for installation. Different price ranges exist.	Comment: they are very expensive but versatile. They deliver a signal and require no intervention on pipes. They have a wide measurement range. Used especially with corrosive fluids.

Table 3.3 Temperature meters

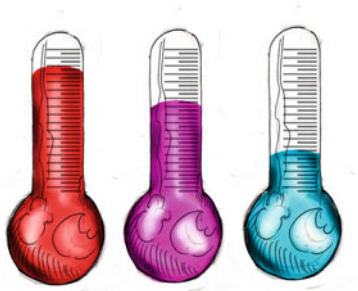
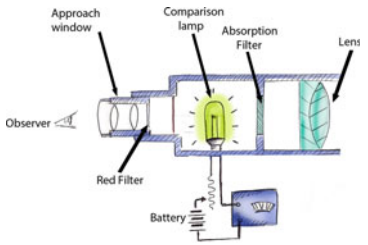
Bulb	Variable resistance	Radiation pyrometer
		
Principle: a sealed tube containing a liquid that significantly changes its volume depending on temperature. The volume is directly related to temperature.	Principle: this is an electric circuit that has a component with an electrical resistance that varies depending on the temperature. Measuring the electric current it can estimate the temperature.	Principle: They are based on the Stefan-Boltzmann law, which relates the radiation from a body with the temperature. By measuring the radiation emitted it can estimate the temperature of the emitting body.
Comment: this is a typical home thermometer. It is very cheap but does not deliver a signal. Not used for control purposes.	Comment: this is the most commonly used temperature meter in the industry (Pt-100). It has a platinum resistance of 100 ohm at 0 °C. Moderately priced and delivers a signal.	Comment: these meters are required not to contact with the medium being measured, are used to measure high temperatures, above 500 °C. The measurement can contain large errors caused by gases or anything between the meter and the element that can shield the radiation.

Table 3.4 Pressure meters

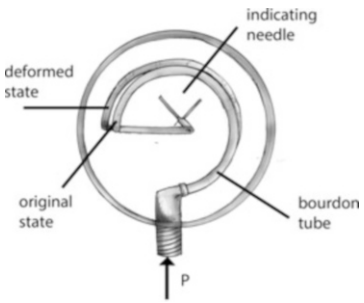
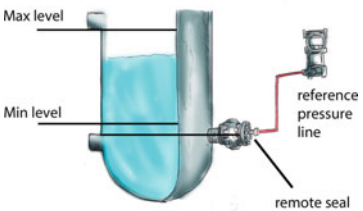
Bourdon	Piezoelectric	Pressure transmitter
		
Principle: a Bourdon meter has a tube with a sealed open end, and the other end is open and fixed. The latter end is brought into contact with the fluid, causing a distortion in the free end that is proportional to the pressure.	Principle: a piezoelectric meter has a piece of quartz that changes its electrical resistance when it changes shape. When the quartz is exposed to pressure on one side, it is deformed in proportion to the pressure.	Principle: pressure transmitters function based on fluid mechanics. It is the same principle as described in differential pressure cells.
Comment: the Bourdon meter is probably the most widely used meter; it delivers no signal but exists in different ranges and sizes. It is inserted into a line where it will measure pressure. It can be used for gas and liquid pressure.	Comment: piezoelectric meters deliver signals and have a medium price range. They are capable of measuring from 0.1 to nearly 1,000 bar. Therefore, one of its applications is to detect landslides in mine tunnels.	Comment: pressure transmitters deliver signals but require power. You can set the zero, span, measurement scale and range, and more. They are pricey but very versatile

Table 3.5 Substances detected by biosensors

Detected substance	Immobilized enzyme	Range mmol/L
Ascorbic acid	Oxidase of Asc. a.	0.05–0.6
ATP	Hexokinase	1–8
Cholesterol	Cholesterol oxidase	0.03–0.15
Ethanol	Alcohol oxidase	0.01–1
Glucose	Glucose oxidase	0.002–0.8
Insecticide (pharation)	Acetylcholinesterase	0.005
Insulin	Antigen immob.	0.1–1 unity/L
Triglyceride	Lipase	0.1–5
Urea	Urease	0.01–500
Uric acid	Uricase	0.5–4

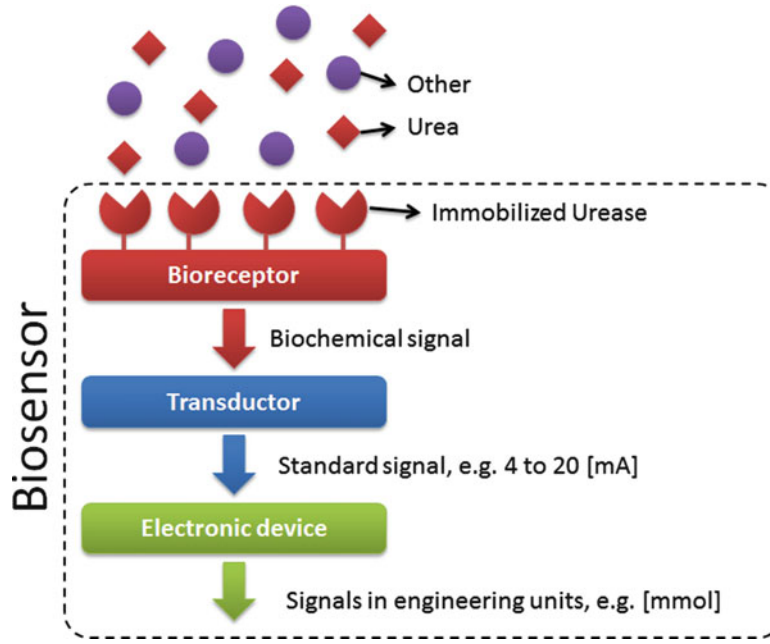


Fig. 3.6 Schematic representation of the operation of a biosensor

process, measuring and registering properties that previously had to be analyzed in the laboratory, thereby saving much time and money. Table 3.5 presents some substances detected by biosensors.

A biosensor is composed mainly of three parts. A sensitive bed is the tissue culture, enzyme, etc. where the substance detection occurs. A transducer converts the signal produced by the detection in an understandable signal that is quantifiable and measurable by electronic or computer equipment. Finally, the electronic device understands the transducer signal and is able to translate it into engineering units. A scheme of the preceding explanation is shown in Fig. 3.6.

In addition, there are also biothermistors that are used to detect the energy released by a reaction catalyzed by an enzyme. In biotransistors, the product of a catalyzed reaction changes the properties of the electronic device, making it possible to detect the presence of those products.

We must differentiate between two groups of meters, transmitters and nontransmitters. This is one of the most important features of a meter because the value should be reported in a way that is understandable to the controller that will be using the measurement. By convention, the most common way for an instrument to communicate is through a linear electrical signal, which delivers

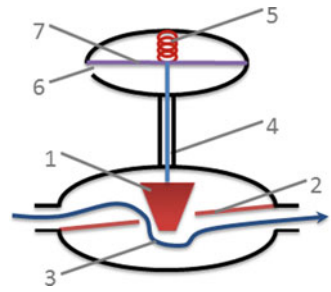


Fig. 3.7 Control valve with pneumatic positioner: (1) plug, (2) seat, (3) flow-through, (4) stem, (5) spring, (6) air intake, and (7) diaphragm

4 mA when the instrument is measuring the minimum value and 20 mA when it is measuring the maximum of its span. The choice of the sensor signal when the instrument is measuring the minimum value is not fortuitous: it shows whether the instrument is properly connected since, even if you are measuring the minimum value, it will receive 4 mA. Another standard is 0–5 V, this standard has the disadvantage of not delivering a signal when it is measuring zero and does not have the problem of a current falling through the circuit when they are connected to the instruments and signal contaminations. Instruments that do not deliver a signal are used not to implement automatic control loops but as indicators of not so relevant amounts since they are usually less expensive.

3.3.2 Actuators

In processes, by control actuators we mean any mechanism that transforms an electrical signal into a hydraulic or pneumatic change over the process variables, typically material flow. The most common actuators are control valves and servomotors, which execute commands like open, close, or similar operations as indicated by the controller.

Control valves consist essentially of three parts. The valve body is used to obstruct the flow to a desired degree to obtain a specified fluid passage. Figure 3.7 shows a balloon-type valve, where the amount of flow that passes the valve is determined by how far inside the seat the cap is, indicated by numbers 1 and 2.

The valve type is defined by the role it must play within the process. For example, a globe-type valve, as depicted in Fig. 3.7, is commonly used to deliver flows in a continuous range. That is, if the process requires a water flow between 0 and 10 L/s, then the valve must be able to deliver 0, 2, 5, 7 L/s or any number between 0 and 10 L/s. Other valves, such as emergency valves, have only two positions, open and closed, as they seek relief in case of an alarm in the process, removing material from the equipment or adding a fire retardant. Another type of valve with a very specific function is a check valve. In the circulatory system of the human body, there are multiple check valves that prevent blood from flowing through the arteries and veins by gravity in the opposite direction to that desired. Process and bioprocess industries also require check valves. Figure 3.8 shows two pumps operating in parallel. It should be noted that if one pump stops its operation, the fluid may be returned through it. A check valve avoids this undesirable situation.

The position of the plug relative to the valve seat, and therefore the magnitude of the flow, is defined by the rod that is attached to the plug. At the other end of the stem is the valve positioner. Figure 3.7 shows how the rod is connected on one side to the plug and on other side to the positioner, which in this case is pneumatic. Within the positioner there is a flexible diaphragm that divides the

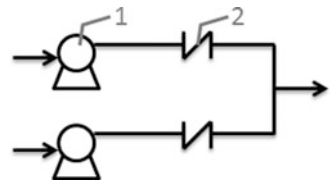


Fig. 3.8 Arrangement of parallel pumps. 1: Pump. 2: Check valve

positioner into a lower and an upper part. In this case, the lower part has an air inlet, and when the air is injected at a pressure that overcomes the force of the compressed spring across the diaphragm, the spring rises, bringing the stem and, therefore, the plug with it. If because of some defect the air supply to the positioner fails, the spring will not expand, and then the plug will fall completely, thus closing the valve. That is, the valve "closes to failure," but if the air intake was above and the string below the valve, it would "open to failure." This is an important feature within the specifications of the actuator because, depending on the process, under certain failures, some valves should close and others open. International standards indicate that the positioner should be calibrated to be fully open at an air pressure of 3 psig and fully closed at a pressure of 15 psig. Although pneumatic positioners are widely used, when the valves reach large dimensions, for example 2 or 3 m in diameter, it is necessary to use motorized positioners, which differ from pneumatic positioners in that an electric motor moves the stem instead of the assembly, air, diaphragm, and spring.

We can see that the plug transforms the position in a certain flow and the positioner transforms the pneumatic pressure to a certain plug position. But the nucleus of the actuation, i.e., the element that converts the electrical signal of the controller into a physical motion, is the transducer, in this case an I/P transducer. Once the controller has calculated a position for the actuator, it sends an electric current to the transducer, typically from 4 to 20 mA, which transforms it into a pressure between 3 and 15 psig. Commonly, the transducer is part of the valve and therefore is omitted in diagrams, which explains why normal and summary forms (Fig. 3.4) are equivalent. There are cases where the resource to be used is not linked to a valve, as in the case of a fan in a gas pipeline or a pump that delivers a certain flow. In these cases, it is common to use an engine with variable revolutions per minute (rpm), also known as a servomotor. These types of actuators can receive a standard signal from a controller or other similar source.

3.3.3 Controllers

Physically, controllers are usually represented by a programmable logic controller (PLC), which is a computer dedicated solely to control tasks. Although sometimes there are special rooms for PLCs, they are usually installed in the field, near the equipment to be controlled, making it essential that they be resistant to shock, dust, electrical noise, and other harsh environments they may be exposed to. Recall that the task of a controller is to use information from sensors to determine the action of the actuators. It does this through logical or mathematical relationships called control algorithms. To understand this logic, we will use Fig. 3.9, which shows an E-102 accumulator tank whose mission is to ensure the availability of compressed air to a process laboratory. Whenever users require compressed air, they connect a hose to the manual valve HS-02 and open it. Because users utilize variable quantities of air, a PIC-102 controller is installed, which reads the pressure inside the tank and commands the PICV-102 valve to fill it. To provide additional security, an alarm is installed (PAH-102) that is scheduled to open the PAHv-102 valve fully if the tank pressure exceeds 100 bar.

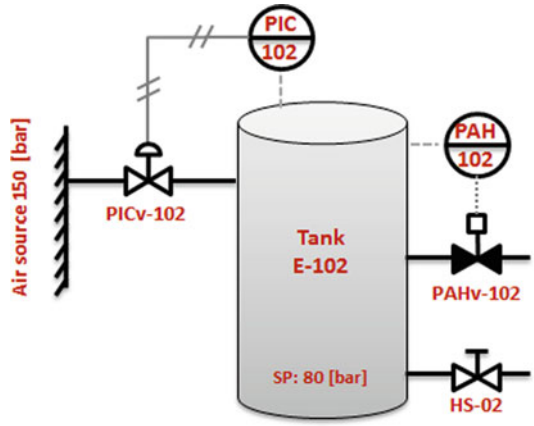


Fig. 3.9 Gas storage tank under automatic control

Starting from a steady state, each time there is a deviation from the desired pressure value inside the tank, the controller will react, calculating a compensating action for that deviation. That is, it will make a change to the current position of the valve based on the error calculated. For this case, we can define the following mathematical expression:

$$c_{(t)} = c_e + \Delta c_{(t)} \tag{3.2}$$

where c (t) is the actuator position over time (%), c_e is the position of the actuator at steady state (%), and $\Delta c_{(t)}$ is the calculated compensatory action taken by the controller (%).

3.4 Process Diagrams, PFDs, and P&ID

Chemical and bioprocessing plants may involve tens, even hundreds, of different pieces of equipment. Among these, the most common are tanks, reactors, bioreactors, distillation columns, furnaces, and so forth, as well as minor equipment such as pumps, compressors, and elevators. Even a simple process can involve recirculation, bypass, and derivations of process fluids, aside from electrical power lines and similar electronic signals. Given this complexity, it is not feasible to include all the information of a plant in the same diagram. It is more convenient to generate various functional diagrams, grouping the details of a specific project. It is reasonable to assume that if an electrical failure is detected, the team in charge of repair will want to be clear about the details of all power lines in the plant and may want access to a diagram that does not include all the details of pipelines for liquids and gases and other streams that bear no relationship to electricity.

3.4.1 PFD

The process flow diagram is known by its English acronym, PFD, but it is also known as a flowsheet. It is used by process engineers to indicate the relevant flows and equipment in a processing plant. Minor details are omitted, such as, for example, pipe diameters and heights from the floor. Generally, PFDs contain everything necessary to understand a process and are often accompanied by additional

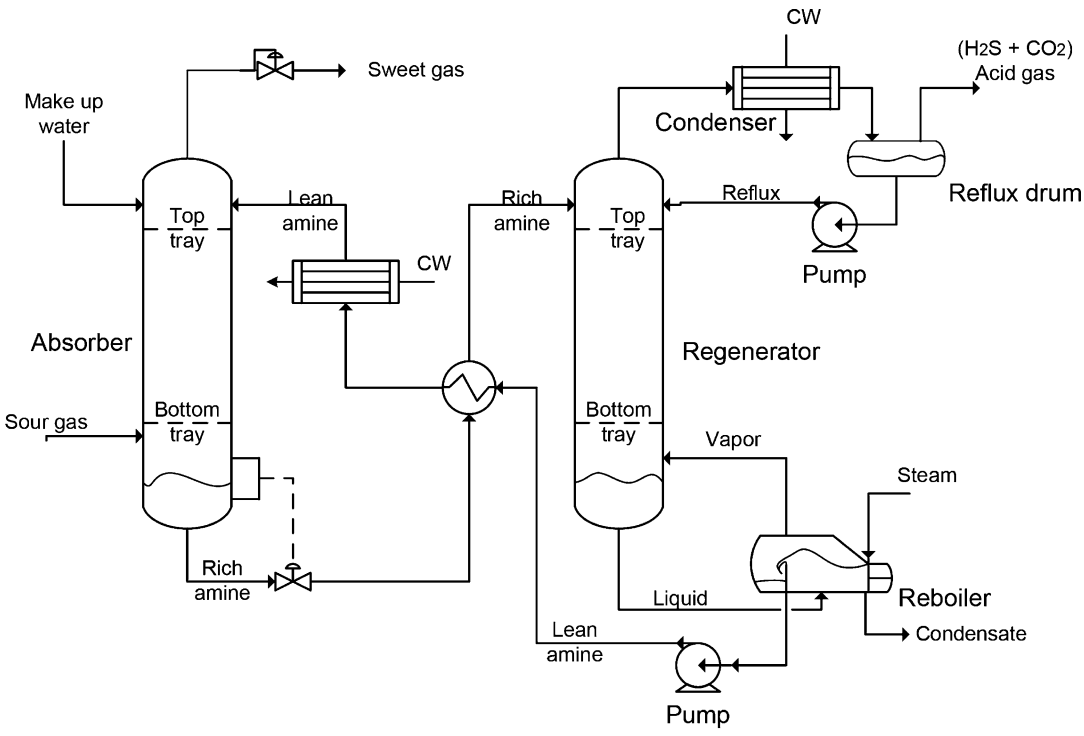


Fig. 3.10 A PDF diagram

documents. If necessary, they will include certain relevant elements of the implementation, such as control valves and pumps. An example of a PFD is shown in Fig. 3.10.

It is important that the information contained in a PFD be clear and understandable to all who will need it, so the PFD should be drawn up in conformance with international standards. The most widely used standard is *ISO 10628: Flow diagrams for process plants - General rules*. Some of the most commonly used symbols are depicted in Fig. 3.11.

Large databases of PFDs that can be consulted to learn about specific processes. In addition, some commercial versions of computer-aided design (CAD) have ready-to-use embedded symbol libraries.

3.4.2 P&ID

This is a diagram that summarizes the instrumentation dedicated to process control, along with its most critical parts. It takes its name from the English term: piping and instrumentation diagram/drawing. It is a bit more complex than a PFD since it includes controllers, actuators, and sensors. An example of a P&ID is shown in Fig. 3.12.

The P&ID plays a vital role in maintaining processes as described. All instrumentation should be reviewed periodically to ensure the functionality of the control and the safety of the systems. Therefore, drawings must be clear and understandable. In such drawings like the PFD, it is preferable to follow the relevant international standards. The most widely used standard is the ANSI/ISA of the

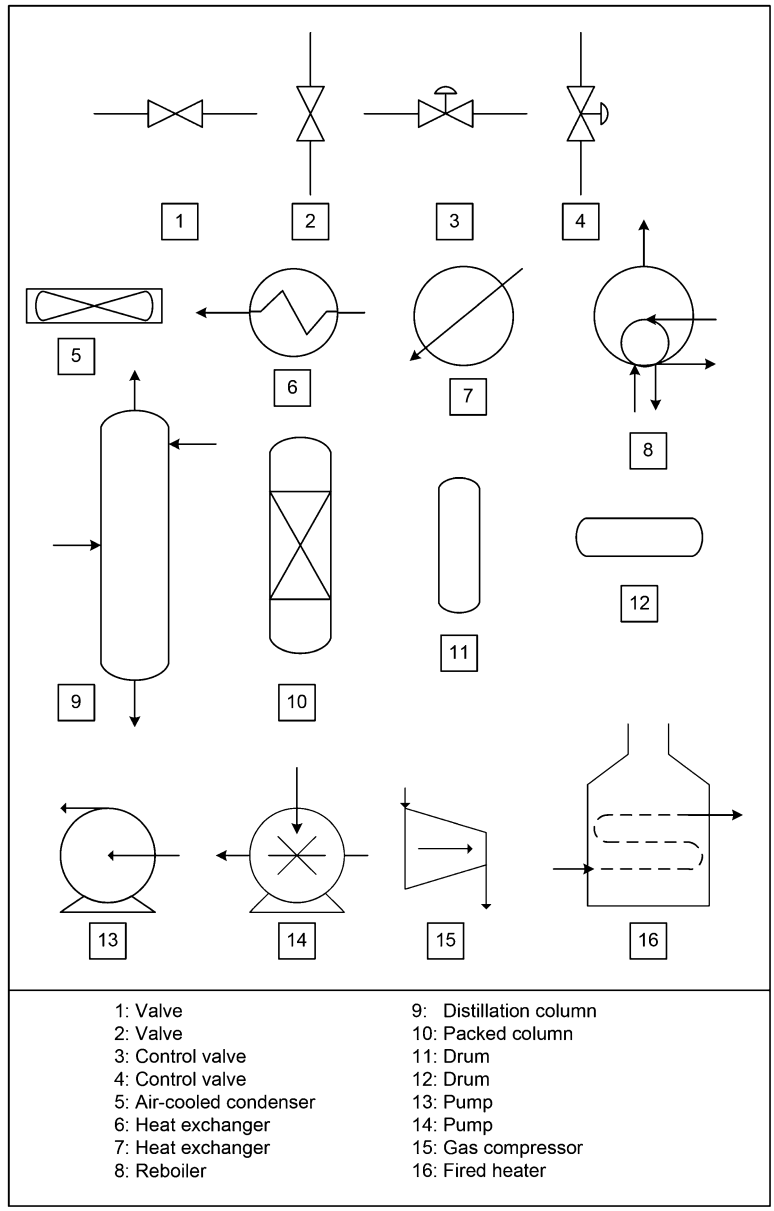


Fig. 3.11 Most commonly used ISO symbols in PFDs

International Society of Automation, which describes in detail how various types of valves, controllers of all types, gauges, etc., should be represented in diagrams. Figures 3.13, 3.14, 3.15 and 3.16 show some of the symbols used in ANSI/ISA.

The framework outlined in Fig. 3.12 can contain two, three, or four letters. Most commonly, there are three letters, where the first letter indicates the variable, the second a secondary function, and the third the main function. A description of the most commonly used letters is given in Table 3.6.

Thus, as can be seen in Fig. 3.8, PIC-102 represents the pressure indicator controller number 102, and PAH is a high-pressure alarm number 102. PAHv valve-102 is a normally closed valve, activated by a solenoid that receives an electrical signal, while PICV-102 is a pneumatic valve normally opened with an actuator.

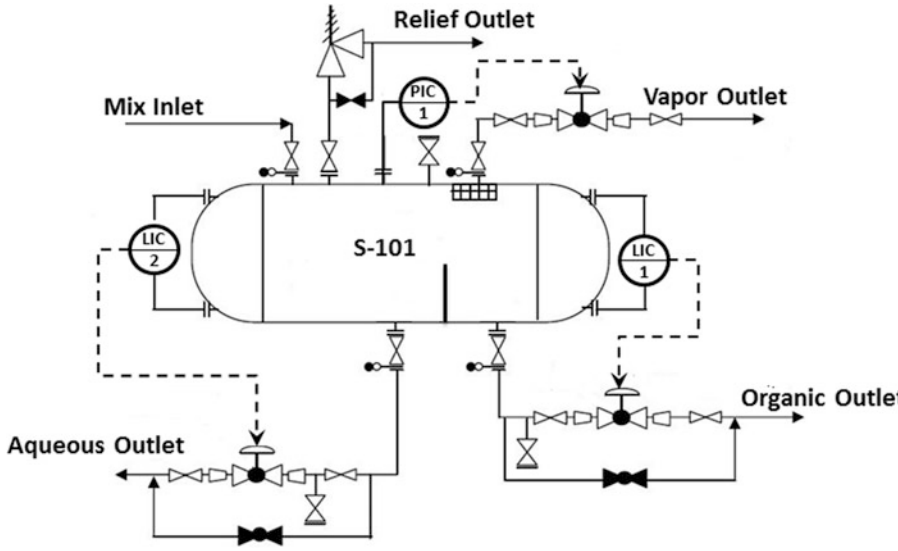


Fig. 3.12 A P&ID

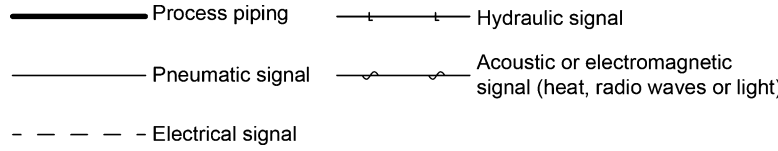


Fig. 3.13 Different line representations

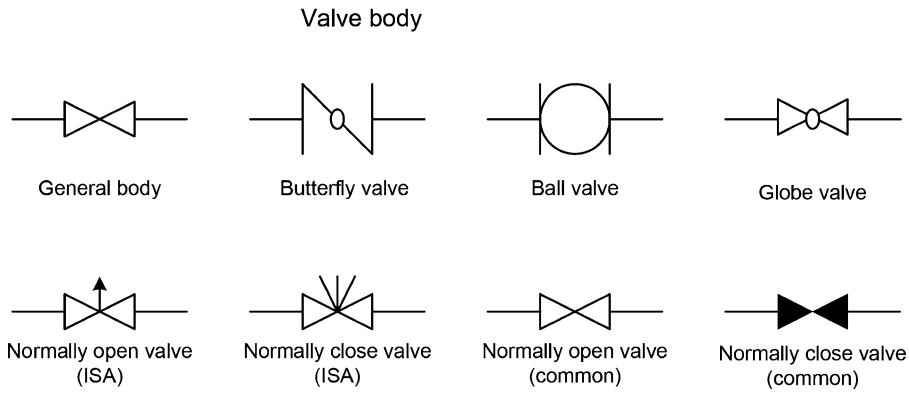


Fig. 3.14 Different types of valves

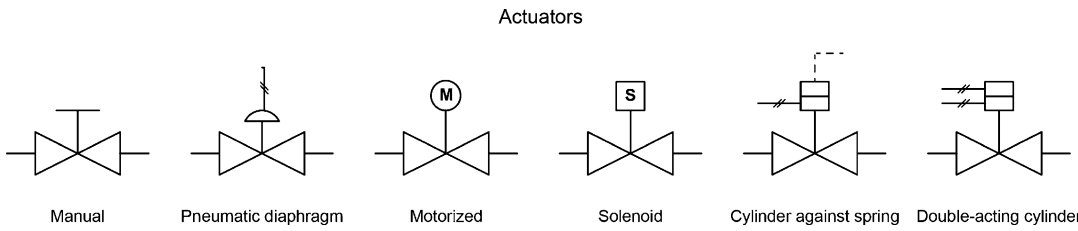


Fig. 3.15 Different types of valve actuators

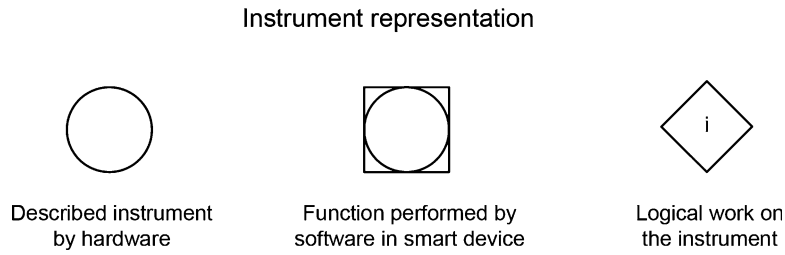


Fig. 3.16 Representation of sensors and controllers

Table 3.6 Most commonly used letters to describe sensors and controllers

	Position		
Letter	1°	2°	3°
A	Analyzer		Alarm
C	Conductivity		Controller
D	Density	Difference	
F	Flow		
H	Manual		High
I	Current	Indicator	
L	Level		Low
P	Pressure		
S	Speed	Safety	Switch
T	Temperature		Transmitter

3.5 Proposed Questions

1. What is the set point of a controller and what should be considered in defining it?
   A: The set point is the value you want as a process variable. It must be selected properly to ensure an efficient operating point.
2. What is the most important information in a P&ID and what type of user does it address?
   A: A P&ID should contain all elements that explain how a given process is controlled and will depend on the instrument that is being made. It is particularly useful for maintenance engineers, development engineers, and process engineers seeking to diagnose the causes of a process failure or determine why a process is out of control or just looking to improve a process.
3. Suppose that the tank in Fig. 3.17 has a level controller. Then, suddenly the input flow increases twice, and after a while, the tank reaches a new steady-state condition. What will the level be at the new steady state?
   A: Because the controller regulates the level and the set point has not been changed, the level will return to the set point, stabilizing the process at the same level prior to the disturbance. In addition, to satisfy the material balance, the input flow must equalize the output flow; therefore, the relief valve will open more as the output flow should be double that of the input flow to maintain the system under steady state.
4. Regarding sensors, what are the precision and accuracy of measurements and why are they important for control?
   A: Precision is a measure of the dispersion of values obtained from making the same measurement repeatedly. Accuracy indicates how close those values are, on average, to the actual value they are meant to reflect. Both concepts are important because the control will act based on the state of the process. That information is provided by the sensors. If the sensors are not able to convey accurate information about the process, the control will act incorrectly.

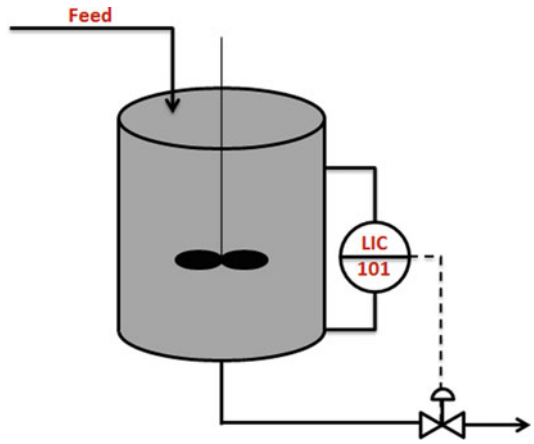


Fig. 3.17 Control system of a tank

5. Why is it important to use a standard as a guide for identifying the elements of a PFD and P&ID? According to the ANSI/ISA and what is an element identified as TAHL-102?
   A: It is essential to use standards because, in the face of possible contingencies, process diagrams should be easily understood by anyone consulting them. If standards are not followed, someone might misread the diagrams and not understand in time the steps that need to be taken or the logic behind the process, making decisions difficult. Regarding the identification of the component, the ANSI/ISA norm indicates that the first letter (T) specifies the nature of the measurement, in this case temperature. The second letter indicates that the instrument is an alarm (A) that is activated whenever the temperature exceeds an upper limit (H) or a lower limit (L). The number is the unique identifier of the element.

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How Capacitive Liquid Level Sensors Work http://www.youtube.com/watch?v=0du-QU1Q0T4
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4[2] Learning from Nature and Its Potential Applications in Chemical and Bioprocess Engineering

The best remedy for those who are afraid, lonely or unhappy is to go outside, somewhere where they can be quite alone with the heavens, nature and God. Because only then does one feel that all is as it should be and that God wishes to see people happy, amidst the simple beauty of nature. As long as this exists, and it certainly always will, I know that then there will always be comfort for every sorrow, whatever the circumstances may be. And I firmly believe that nature brings solace in all troubles.

Anne Frank, The Diary of a Young Girl

I like this place and could willingly waste my time in it.

William Shakespeare

Adopt the pace of nature: her secret is patience.

Ralph Waldo Emerson

4.1 Introduction

It is possible that most, if not all, of the problems that an engineer will want to solve now and in the near future may already have been solved by nature, although in a different context.

First, what is it that we call nature? According to *oxforddictionaries.com*, nature is defined as "*the phenomena of the physical world collectively, including plants, animals, the landscape, and other features and products of the earth, as opposed to humans or human creations*" (emphasis added). It is thought that living organisms have been around on Earth for approximately 3.8 billion years (Gordon 1976). Yes, 3.8 billion years of evolution, where nature not only has been hosting life in a sustainable manner but also has dynamically changed, utilizing the minimum resources to attain the maximum performance. As Einstein once said: "*Look deep into nature, and then you will understand everything better*."

Since ancient times, the wonders of nature have inspired human beings. For example, Leonardo da Vinci and the Wright brothers based the shapes of their designs on observations of living creatures. More than 3,000 years ago, the Chinese tried to mimic the remarkable strength of the spider web to produce hard synthetic silk. One of the most illustrative recent examples is the invention of Velcro by the Swiss inventor George de Mestral (1907–1990). In the early 1940s, de Mestral was walking in the woods with his dog when he realized that seeds were stuck to his pants and to his dog's fur. What for de Mestral and other dog walkers was a problem was, from the point of view of nature, a way of

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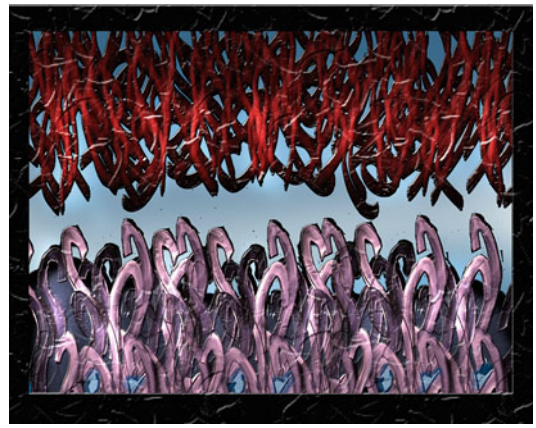

Fig. 4.1 Velcro developed by inventor George de Mestral

propagating new plants to another place. As a talented inventor and electrical engineer, de Mestral asked himself why this happened, which led to an understanding of the mechanism and the realization that this might have industrial and commercial applications and implications. By performing careful microscopic observations, de Mestral understood the mechanism by which seeds adhere so strongly to various objects. After around 8 years of toil, de Mestral developed a commercial product that came to be called Velcro (Fig. 4.1).

Looking, observing, learning, and finding inspiration from nature

First we will discuss, in some detail, certain phenomena presented to us by nature so that we may begin to comprehend its immeasurable beauty and discover how important it is to have a better understanding of it. Appreciation of nature will help us to develop applications in a more efficient and sustainable way. Nature hides a wealth of knowledge (it has a lot of "patents") that we can use to improve our lives, but it is very important that we learn from nature and that we do so and develop applications in a respectful and sustainable way, like nature naturally does. In addition, a strong message to us as engineers will be to recognize the importance of complementing our basic studies with biology. Perhaps 50 years ago, process engineer had to have a strong background in three subjects: mathematics, physics, and chemistry. Nowadays, biology must be included in our basic preparation to be a chemical and bioprocess engineer in the twenty-first century.

Honeybees and honeycombs

Honeybees are around three-quarters of an inch long, and most of them are workers (females). The males are called drones and are slightly larger than the females. Drones have a unique and specific function in this community: to mate with the queen. Females are responsible for all the work inside and outside the hive. Among other activities, this work involves (indirectly) pollination and (directly) honeycomb construction. As outlined below, pollination is very important and crucial for the survival of humanity; in addition, understanding the logic behind the construction of honeycombs can help us improve and optimize the design and construction of various pieces of equipment and structures.

According to Allsopp et al. (2008), pollination by insects is a critical function for ecosystems and is important for many crop systems. Furthermore, most commercial crops are pollinated by insects. It is estimated that insects are responsible for pollinating 80–85 % of total commercial acreage, such as fruits, vegetables, oilseeds, legumes, and fodder (Allsopp et al. 2008). Most of this pollination is carried out by honeybees. Although estimating the value of honeybees as pollinators is a complex undertaking, a rough estimate is on the order of US$15 billion per year in the USA alone (Morse and

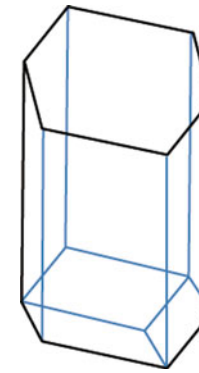

Fig. 4.2 At the *bottom* are depicted the three rhombi that close the hexagonal prism at an angle of 72°32′

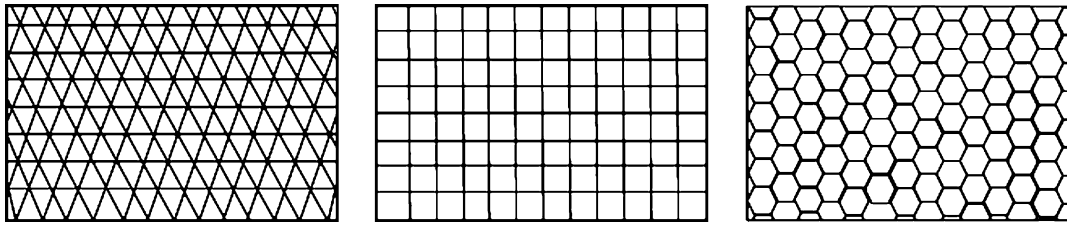

Fig. 4.3 Potential prism cross section, equilateral triangles, squares, hexagons, etc. The area enclosed in a perimeter of 1 unit is 0.048 units square (triangle), 0.063 units square (square), and 0.075 units square (hexagon), respectively, where the hexagon is the most efficient

Calderone 2000). As stated on the Web site http://www.gpnc.org/honeybee.htm, honeybees are the state insect in several U.S. states.

Although bees have not been "rewarded" or recognized for honeycomb design and construction, it is indeed a marvelous accomplishment. In a simple way, we can describe a honeycomb as being constructed by a series of hexagonal prisms of wax and closed at the bottom by three rhombi at an angle of 72°32′, as was carefully measured by Giacomo Maraldi (1665–1729) in the early eighteenth century (Fig. 4.2). This design is unique because it not only provides the required mechanical strength given by the hexagonal prism configuration but also minimizes the amount of wax needed for the honeycomb construction. The amount of wax used is vital because honeybees need approximately 4–6 g of honey to produce 1 g of wax. In terms of design, the prism cross section can have different forms, such as, for example, equilateral triangles and squares (Fig. 4.3), and so the question is: why do honeybees choose the hexagonal prism and that particular way of closing the ends with three rhombi? Approximately four centuries ago, mathematicians started analyzing and trying to understand the logic behind this particular and highly efficient design. From an evolutionary point of view, it is logical to think that honeybees would try to minimize the effort they would have to expend and the amount of wax needed for their honeycomb construction, in other words, they optimized the disign of honeycombs while keeping it simple.

First, the mathematician Johann Sammuel Köning (1712–1757), on the request of his French friend René Antoine Ferchault de Réaumur (1683–1757), mathematically proved that the optimum angle for closing a hexagonal prism with three rhombi was 72°34', contradicting the results obtained by Giacomo Maraldi and, more importantly, contradicting nature. Then, in 1743, Colin MacLaurin (1698–1746), a British mathematician, established mathematically that Giacomo Maraldi was correct in confirming that the angle was 72°32′ and, furthermore, that honeybees were right; therefore, nature was right. Later on, the well-known mathematician László Fejes Tóth (1915–2005) studied and

analyzed other possibilities in reference to the closing bottom of the hexagonal prism. Without going into detail, given the complexity of the math, Dr. Tóth concludes that it is possible to design a hexagonal prism closure using less wax than in a honeycomb design. One alternative is to close the bottom of the hexagonal prism using two hexagons and two rhombi instead of three rhombi, although Tóth admits, "We must admit that all this has no practical consequence . . . by building such cells the honeybees would save less than 0.35 % of the area of an opening and a much smaller percentage of the surface of a cell." In conclusion, although the use of three rhombi involves slightly more wax, it would require much less effort for honeybees to construct the honeycomb. Consider that humanity, with its great mathematicians, has had a hard time understanding and determining that the design of a honeycomb is optimal in all respects. It is highly probable that before the eighteenth century, the mathematics required to adequately address this complex problem had not yet been developed.

Bacteria are better than us at Fixing Nitrogen

The air we breathe contains approximately 79 % nitrogen, with the balance comprised of mainly oxygen. Nitrogen is essential to life, but it must be transformed to be used. Nitrogen fixation is the process in nature by which nitrogen is reduced to form ammonia (NH_3).

NH_3 could be considered the compound of the twentieth century. On the one hand, it has been essential to the development of agriculture, but it also has played a major role in explosives (e.g., TNT). Just as it has been essential to the welfare of billions of people, it is also estimated to have caused the deaths of around 150–200 million people. There is a long list of processes and equipment that have been made possible by the industrial production of ammonia, such as, for example, nylon, cattle feed, cleaning equipment, and air conditioning.

In 1908, Professor Fritz Haber filed for a patent on the "synthesis of ammonia from its elements." Ten years later, Professor Haber was awarded the Nobel Prize in Chemistry. According to Jan Willem Erisman (Energy Research Centre of the Netherlands): "The increasing demand for food and biofuels makes efficient use of nitrogen fertilizer and more sustainable energy a challenge for many. Haber–Bosch is perhaps the most significant invention of the twentieth century, yet it has many side effects. Now we need a new invention that changes the world just as much, but without the environmental impact." In the commemoration of the 100-year anniversary of Professor Haber's patent, all scientists agreed that "the patent filed by Professor Haber changed the world." The chemical industry saw tremendous growth in the twentieth century thanks to the Haber–Bosch process. According to Mark Sutton (UK's Centre for Ecology & Hydrology), "It is remarkable how a century of Haber–Bosch nitrogen has transformed all our lives. Without it, half of us might not be alive today. At the same time, the environmental impacts of nitrogen cut across all global change issues. To reduce these effects, we must improve nitrogen use efficiency, especially in food production."

Briefly, the Haber–Bosch process is a reaction that is reversible and exothermic:

$$N_{2(g)} + 3H_{2(g)} \leftrightarrow 2NH_{3(g)} \qquad \Delta H = -92 \text{ kJ/mol of } N_2.$$

In the Haber–Bosch process, nitrogen is obtained from air and hydrogen from natural gas. The final product is liquid ammonia. The industrial operating conditions for the reaction are usually at a pressure in the range of 200 atm and temperatures between 400 and 500 °C. To increase the efficiency and speed of the reaction, a catalyst is normally added (iron catalyst).

As mentioned earlier, this reaction also takes place in nature as nitrogen fixation. One big difference between the Haber–Bosch process and nature's pathway are the conditions in which the reaction occurs. Whereas in the Haber–Bosch process it is necessary to employ high pressure (in the range of 200 atm) and high temperatures (400–500 °C), bacteria fix nitrogen at *ambient* pressure and temperature. Again, this is another example of how nature has better "patents" for processes.

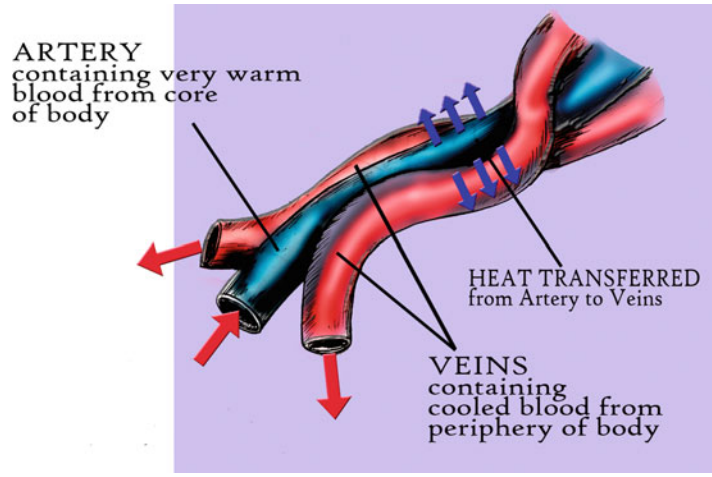

Fig. 4.4 An example of a counter-current heat transfer in dolphins

Can bottlenose dolphins (order cetacea) teach us heat transfer?

Cetaceans include all whales, dolphins, and porpoises.

One of the things that surprises us about dolphins is their intelligence and how they always seem to be smiling and appear very happy. For most of us, dolphins are probably among our favorite animals. Those of us who have had the opportunity to see them in aquariums are always surprised by their extreme agility as they move in and leap out of the water; in addition, they are perhaps one of the best animals at following the instructions of their coach. In the ocean, it is common to see them in groups. To communicate with each other, they scream, whistle, and even use body language. Dolphins are wonderful animals: we can even learn heat transfer from them.

Dolphins live in tropical waters (10–32 °C) and their size is approximately in the range of 2–4 m and they weigh 150–200 kg. Dolphins are highly adaptable in regulating temperature. For example, in flukes, flippers, and dorsal fins, arteries are surrounded by veins to increase heat transfer and maintain body temperature. In addition, the heat transfer is very efficient because it is countercurrent (Fig. 4.4).

The dorsal fin might help stability, but it also has an important role in body's thermoregulation. What can we learn from this? Dolphins have different mechanisms and strategies to regulate temperature:

(a) The surface-area-to-volume ratio can vary. A dolphin's shape (fusiform) decreases the surface-area-to-volume ratio (Chap. 10), thus minimizing heat transfer (Chap. 6). Therefore, depending on the environment (cooler waters), dolphins have a relatively large body and small flippers.
(b) Dolphins accumulate more fat near the surface of their bodies, and thus they control the flow of heat. As we will see in Chap. 6, fat is a poor conductor of heat.
(c) As mentioned previously, in flukes, flippers, and dorsal fins, dolphins have a "heat exchanger" (the artery is surrounded by veins) that operates in countercurrent mode.
(d) Dolphins have a higher metabolic rate compared with animals of the same size.
(e) Dolphins breathe less than other animals and so lose less heat.

Why do gastric acids not dissolve your stomach?

The stomach secretes extremely strong gastric acids (hydrochloric acid and pepsin) to dissolve foods. But if gastric acids can dissolve metals and kill all living cells, then why do they not dissolve the stomach itself? The stomach is a mixture of water, electrolytes (sodium, potassium, and calcium), and pepsin enzymes that break down proteins. When a person eats food, the body discharges a hormone,

gastrin, into the bloodstream whose function it is to stimulate the cells that produce stomach gastric acids. The key is the mucus that covers the internal surface of the stomach. This mucus acts as a lubricant for the food bolus to facilitate its circulation in the digestive tract, and in addition it forms a thick revetment for the internal wall of the stomach to prevent the stomach from being digested or dissolved by its own gastric acids. Although gastric acids incessantly attack the mucus wall, the stomach wall continuously regenerates mucus. Furthermore, the stomach revetment produces its own antacid by the secretion of bicarbonate to neutralize it.

Woodpecker skulls

Felipe Massa (Ferrari) is a Formula 1 driver who started racing at an early age (9 or 10 years old). With the passage of time, he traveled to compete in the Italian Formula Renault, until, finally, in 2002, his dream came true and he raced Formula 1. During the qualifying rounds for the Hungarian Grand Prix (July 2009), a spring approximately 5 cm in size and with a mass of 1 kg suddenly came from the car of Rubens Barrichello (Brawn-Mercedes) at a speed of 230 km/h and violently struck the driver, Felipe Massa, on his helmet. According to the news, despite the violent impact, Felipe Massa survived and the accident did not cause serious injury. It is clear that the main factor that saved Felipe Massa's life was the excellent design—the strength and deformable structure—of his helmet.

To significantly improve the mechanical strength of helmets (for different uses), scientists have turned their gaze toward nature. They have been carefully observing how the skulls and brains of woodpeckers can function without any brain injury after 12,000 strikes to a tree per day. In addition, scientists are analyzing and coming to understand how walnut shells are composed and structured.

Recently, an interesting article was published under the leadership of Dr. Yubo Fan (Wang et al. 2011). The title is very suggestive: *Why do woodpeckers resist head impact injury: A biomechanical investigation*. As mentioned in the article, head injuries are a common cause of death and disability around the world. According to the Centers for Disease Control and Prevention, the five most common causes of head injury or TBI (traumatic brain injury) are car accidents (passenger and pedestrian), bicycle/motorcycle accidents, falls (especially children and the elderly), sports, and acts of violence/assault.

Dr. Wang and his coworkers set out to investigate the role of the 3D kinematics and macro/micro structures of beak and cranial bones in avoiding impact injury to a woodpecker's head. They used high-speed video cameras and scanned the birds' skulls to obtain 3D simulations. The idea was to quantify the effect of pecking forces on the birds' heads. After 3 years of research, they discovered why the brains of woodpeckers are not affected by the constant blows.

Dr. Wang and his group think that it will be possible to design even more impact-resistant helmets than the one Felipe Massa was wearing. In addition, one can think of several applications and implications of this research, such as improving the design of helmets used by (American) football players.

4.2 Lessons and Challenges from Nature

Although human beings have marveled at nature for centuries, it is only relatively recently that scientists, and later on engineers, as a community have intentionally focused methodically on nature to improve the design of materials, devices, structures, and processes. Moreover, it is now easier to follow Einstein's advice when he suggests looking deep into nature because today we have microscopes and tools that allow us to better observe nature and that have improved our understanding of the micro and nano worlds.

What is Biomimetics? (Although less common, other expressions are biomimicry and bionics.) Biomimetics refers to the art of adapting, for our own benefit, processes, substances, devices, or systems that resemble what nature has devised. Learn from, understand, be inspired by, and adapt from nature. Perhaps the main motivation will be the economic benefit, but more importantly, we can study nature and learn how to develop a sustainable way of life as nature "naturally" does.

It is beyond the scope of this chapter to describe the entire body of knowledge related to biomimetics that has been accumulated and studied by the scientific community. An excellent compilation is the *Biomimetics,* edited by Dr. Yosep Bar-Cohen (Bar-Cohen 2006). In addition, because of the complexity of nature and related applications, it is not plausible for a first-year student to comprehend all the details. Therefore, we will limit ourselves to a few examples, but with the intention, or at least hope, that students will understand, appreciate, and become passionate about this novel practice and value its importance for their future careers and professions, reiterating and rediscovering the relevance of biology in our basic courses.

A quick and random search for the word *biomimetic* in some engineering and science journals revealed the following results:

Journal of Chemical Engineering	21
Biochemical Engineering Journal	33
Journal of Environmental Engineering	13
Journal of Food Engineering	9
Journal of Food Science	10
Nature	170
Science	243

First, to understand these numbers, we did a quick search for the words *reactor* and *heat* in the *Journal of Chemical Engineering*. We found 4,008 results for heat and 5,279 for reactor. This shows indirectly that relatively few research efforts are undertaken in the field of biomimetics. As expected, science journals have many more articles related to biomimetics compared with process engineering journals. However, journals in material engineering embrace the majority of the research related to biomimetics.

Why Biomimetics?

Although biomimetics is a highly research-intensive and multidisciplinary field, meaning that it requires the support of both government and industry and seeks input from a wide range of disciplines, it is expected to bear fruit over the medium and long term, not only from an economic point of view but also by improving our way of life and promoting sustainability. In what follows, we will develop, in some detail, examples to show the potential and reality of biomimetics when observing and learning from Mother Nature.

The lotus effect

For oriental religions, the lotus plant (*Nelumbo nucifera*) represents purity and is a sacred plant in Hindu mythology. The leaf of this plant is unique in that it is highly hydrophobic. When a water droplet is deposited on its surface, it practically forms a sphere, with very small contact angle between the water and the leaf (Fig. 4.5). This super-hydrophobic surface makes it so that the water droplets roll off the leaf, taking with it dirt particles and then performing self-cleaning. As recently as 1997, two botanists, Professors Wilhelm Barthlott and Christoph Neinhuis from the University of Bonn, discovered why the lotus plant is so repellent to water droplets. Using a scanning electron microscope (SEM) with a resolution as small as 1–20 nm, they discovered that the lotus leaf surface was not as smooth as is normally thought but has a micro- and nanostructure with a rough surface. The lotus effect is explained

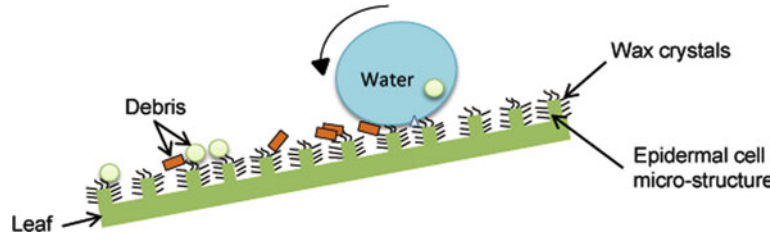

Fig. 4.5 Contact angle between the water and the lotus leaf

by a consideration of two physical characteristics: possession of water-repelling microstructures and a nanostructure situated above the microstructures and composed of by a waxy material that repels water. Moreover, most leaves of plants in the forest have water-repellent characteristics. Hydrophobicity helps leaves to significantly improve gas transfer and minimize the probability of infection since spores and microorganisms need air and water to propagate (Maheshwari and Maheshwari 2008). As mentioned by these authors, water-repellent characteristics go beyond plant leaves; for example, they help some insects to self-clean their large wings and facilitate flying.

Many potential applications and a variety of commercial products that might benefit from these key features of lotus leaves come to mind, for example, waterproof jackets, sails for boats, and paints for walls.

Engineering challenges and some solutions

Today, society faces complex situations. Increasingly, global warming is discussed with increasing frequency in the news. Although a few years ago this topic generated some controversy, it is becoming increasingly clear that global warming is real and requires urgent action and adequate policies. Recently, scientists have argued that global warming is irreversible. According to Will Steffen, executive director of the Australian National University's Climate Change Institute, this decade is critical. One of the most important challenges is to significantly increase the efficiency of energy and water utilization. As was consistently expressed in previous paragraphs, one option in this regard is to look at, appreciate, and understand how nature manages critical resources. As has been well expressed by Professor Marc-Oliver Coppens (Coppens 2003), chemical and biological processes must be inspired by nature and achieve a sustainable development that is in harmony with the environment. As an example of how nature manages chemical reactions, let us look at enzymes. Enzymes are biological molecules (proteins) that are characterized as being excellent catalysts. That is, enzymes significantly accelerate the rate of a chemical reaction by reducing the activation energy (strictly speaking, a lower activation energy because it uses a different path). Virtually all reactions occurring within cells are catalyzed by enzymes. An essential feature of such reactions is that within the cell they occur at moderate temperatures (0–50 °C) and typically around atmospheric pressure. It has been consistently documented that these reactions would take a long time, even years, if they were not catalyzed by enzymes. By understanding and mimicking enzyme shape and function, it will be possible to design more efficient catalysts.

Edible films

Another preoccupation of humanity is food preservation, distribution, and shelf life. For a couple of decades, food science researchers have been studying and exploring the possibility of using edible films. The main motivation is to extend the shelf life and improve the quality of food. Generally, an edible film is defined as a thin layer made of edible biopolymers that can be applied preformed (film) or directly on food (cover) to control water loss, the rate of gas transfer (CO_2/O_2), and the addition or loss of flavor and to avoid damage during handling and transport. Other functions, such as the transport of active substances, such as antioxidants, probiotics, and antifungal and antimicrobial agents, have also been studied. Overall, an edible film or coating is made of two major ingredients: (1) biopolymers, which are

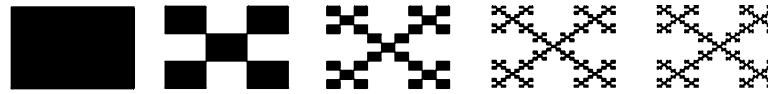

Fig. 4.6 Self-similar patterns that repeat from small to large scales

used as a structural matrix, which may be carbohydrates, proteins, or lipids, and can be incorporated individually or by forming mixtures; and (2) plasticizers, which are used to improve the mechanical and resistance properties of films or edible coatings. In recent years, the use of edible films for the transport of active substances in foods has attracted much attention because it is one of the main attractions for use in the food industry due to their holding capacity or to the controlled release of active compounds. Nature offers a great variety of edible coatings/peels. Most fruits and horticulture products incorporate edible coatings/peels, for instance, apples, tomatoes, grapes, and peaches. Interesting examples that merit further study include banana and avocado peels. The latter can be "stored" for several months on the tree without undergoing much change. In the case of the banana, the peel is edible, though unpleasant; more than that, as a requirement for edible films, banana peels not only extend shelf life but also control gas transfer, retard maturation, show antibacterial activity, and provide UV protection. As a corollary, we can state that the skins and peels of foods can be a source of inspiration for the design and structure of new edible films, but they should also be studied for their composition, where we might extract many compounds of interest. Then we can think about processing in an integral way and designing future processes as nature does, transforming foods without waste. Furthermore, another lesson from nature is that there the package and the product are not separate entities; it is all just one harmonious thing.

Fractals

Fractals are self-similar patterns that repeat from small to large scales (Fig. 4.6). The term *fractal* was coined in 1975 by mathematician Benoit Mandelbrot. Examples of fractal geometry are abundant in nature: rivers, mountains, coastlines, DNA, animal coloration patterns, and many others. In trees, the branches, crown, and roots display a fractal arrangement. The size of the branches and leaves is not related to the overall size of the tree: only the number of branches and generations of the tree increases as the tree grows. As mentioned by Coppens (2003), even architects like Antonini Gaudi have taken inspiration from these desirable mechanical properties of trees. As engineers, we can see a tree as a large and efficient bioreactor for converting carbon dioxide (CO_2) and water into biomass and oxygen. In this bioreactor, one of the critical and interesting characteristics is the extremely high surface area per unit volume due to the extremely large number of leaves. Much better equipment design and operation can be achieved in scaling if we look at how nature has resolved these kinds of issues. For example, a laboratory bioreactor should have a capacity of 1–5 L, whereas an industrial bioreactor might have a capacity of 100,000 L or more.

A potent and striking message from nature is that chemical reactions and processes in living organisms occur mostly at moderate temperatures, close to atmospheric pressures, and at a high rate. Additionally, in processes designed by engineers, it is common to deal with high temperatures and pressures. Another way to solve problems using nature as a guide is by utilizing what we call fractal geometry (for scaling). As an example, in general, branches and leaves maintain their size independently of the size of the tree.

4.3 Biomimetics and Product Development

One question that we can formulate that is not easily tackled by industry is this: how can we accelerate or ultimately realize the process of learning from nature to invent products and processes that are useful to society? Another is: how can we make the same products and processes that we are currently producing and utilizing but in a more friendly and sustainable way?

We outline two approaches:

(a) We have a well-defined engineering problem and want to see the most efficient way to solve it. The research and development group (which includes biologists) led by an engineer looks for similar problems in nature and thinks about how nature has resolved those problems.
(b) A research and development group (which includes engineers) led by a biologist is constantly looking at and learning about how nature has solved problems that are similar to those faced by the group's company.

Although not defined in exactly the same terms as expressed here, these processes are called top-down (a) and bottom-up (b) (Global Watch Mission Report 2007).

4.4 Project Homework

As stated at the beginning of the chapter, an important activity for familiarizing students with the novel and interesting topic of biomimetics will be to assign homework to teams. Although the subject may not be easy without the requisite knowledge in science and basic background, we believe that students working in teams in a specific subject can become reasonably familiar with the whole subject. What we suggest is subdividing the class into several groups and assigning each group a specific topic. Homework could be developed in 2–3 weeks and then shared with all students in 10- to 15-min presentations. Thus, without much effort, the entire class will acquire a broad perspective and will discover potential applications of biomimetics in process and bioprocess engineering. To facilitate the work of the professor/instructor, we have included a long list of potential topics and several references to guide and lead the effort.

The goal is for each group to familiarize itself with a specific topic and then demonstrate to the class the potential applications in commercial products or new products under development.

4.4.1 Potential Topics

General

1. Analyze the composition and study the function of different coatings/peels in fruits and horticultural products.
2. Understand and analyze the lotus effect.
3. Understand honeycomb construction and perform some mathematical calculations (geometry) and compare different alternatives for constructing honeycombs.
4. Show graphically and schematically how woodpeckers resist head impacts.
5. Describe and analyze different types of camouflage used by different animals.
6. Describe the composition, strength, and shape of spider webs.
7. Develop impact-sensitive material to mimic bruised skin.
8. Show how shark skin can be mimicked for efficient swimming.
9. Show how birds can be used as inspiration for the design of flying objects.
10. Discuss how geckos might inspire a solution to sticking surfaces.
11. Show how diffusion processes could be mimicked.

Advanced topics

12. Synthetic life: engineer bacteria to develop drugs that can be extracted from rare plants but at very high cost.
13. Artificial organs. What organs can be replaced?

14. Artificial vision. Which kind of progress has been made in recent years?
15. Use of decoys. Describe different types of decoys found in nature
16. Colors without pigments. How to use optics to block certain colors and show others?
17. Fractals. Potential applications in chemical engineering

4.5 Corollary

"Until the next asteroid hits Earth, it is humanity – more than any other force – that will dictate the future course of all known life. Collectively, we have the power to wipe out most macroscopic species, and to change dramatically the future course of evolution of the rest. We exercise this power with little regard for the implications for human well-being, let alone that of our living companions. In the current 100-year period – "on our watch" – we are projected to drive about half of the world's plant and animal species to extinction.

The realization is dawning that traditional approaches to conservation are doomed to fail. Nature reserves are – and are likely to remain – too small, too few, too isolated, and too subject to change to sustain more than a tiny fraction of Earth's biodiversity (maybe just 5 % of species) and life-support services to society, over the long run. For conservation to have enduring success, it must become both economically and culturally attractive and common-place in the sea of human activity."

Gretchen C. Daily

Professor, Stanford University, USA

The MIDORI Prize for Biodiversity 2010 Prize Winner

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Mathematics of the Honeycomb (Part 1 of 2) http://www.youtube.com/watch?v=A9E7mPhdVp4
Why Woodpeckers Don't Get Brain Injuries http://www.brainline.org/content/multimedia.php?id=6316
Do woodpeckers get headaches? http://www.youtube.com/watch?v=o3AG-BO0Euw
Lotus Effect http://www.youtube.com/watch?v=VHcd_4ftsNY
Biomimetics: its practice and theory http://rsif.royalsocietypublishing.org/content/3/9/471.full
Biomimetics—using nature to inspire human Innovation http://biomimetic.pbworks.com/f/Biomimetics%25E2%2580%2594using%2520nature%2520to%2520inspire%2520humanBar-Cohen.pdf
Nature designs http://www.ted.com/talks/janine_benyus_shares_nature_s_designs.html
The magnificence of spider silk http://www.ted.com/talks/cheryl_hayashi_the_magnificence_of_spider_silk.html
What is cell culture? http://www.ted.com/talks/nina_tandon_caring_for_cells.html

5[5] Challenging Students and Solving Problems with Basic Tools, Testing Students' Attitudes

Continuous effort—not strength or intelligence—is the key to unlocking our potential.

Winston Churchill

Satisfaction lies in the effort, not in the attainment, full effort is full victory.

Mahatma Gandhi

You do not develop courage by being happy in your relationships every day. You develop it by surviving difficult times and challenging adversity.

Epicurus

Opportunities to find deeper powers within ourselves come when life seems most challenging.

Joseph Campbell

5.1 Introduction

Why include this chapter? Why these kinds of problems? We, the authors, consider of utmost importance that engineering students be challenged in their abilities. We understand that at first they will feel unable to handle many of these problems, especially for their level of difficulty. They may feel frustrated, but that is not our intention, and in the end, we hope and expect that they will feel excited and eager to learn. We want to show them that they have the capacity to deal with difficult problems. We hope to motivate them to understand, to learn, and, especially, to have a solid attitude.

As will quickly be appreciated, the skills required to solve the problems presented in this chapter are mainly the basic subjects of mathematics, chemistry, biology, and physics. They do not go beyond the subjects learned (or, rather, what should have been learned) in high school. Some of these problems are very basic, but they require the wit, ingenuity, and, especially, methodology that a good student of chemical or bioprocess engineering must have. The key to solving these problems is first the attitude and then the method. Many problems might seem like brainteasers, which are tricky and not easy to solve, but each of them can be solved to the extent that the student applies a method and then expresses the statement in terms of mathematical equations.

In this chapter, the ultimate goal is to teach students to express in a mathematically sound way, various everyday situations that, more than likely, they have faced and will likely encounter even more when they start their professional careers. With patience, effort, and discipline in following the advice proffered here, the student will develop expertise in problem solving. It is critical and

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fundamental that the student be able to interpret and "translate" statements and express them in a mathematical model in order to solve them. As stated previously, this mathematical representation will be simple to solve because the main goal, at this stage, is to familiarize students with the method and mathematical formulation of problems. In parallel and subsequent courses on chemical or bioprocess engineering, the student will be enrolled in varied and advanced courses in mathematics, chemistry, biology, and physics, and then he or she will be capable not only of formulating intricate problems, but also solving them. For now we are specifically concerned with empowering students with the ability to formulate problems mathematically.

In the near future, freshmen will be challenged in higher-level courses, and during their professional careers, they will appreciate, with greater objectivity, the significance and importance of being able to easily set up and resolve the various problems they will face.

In this chapter, we include different types of exercises that will familiarize students with engineering problems. More importantly, however, the problem sets will prepare students to formulate mathematically more complex problems. The type of problems presented here may seem, somehow, disconnected from chemical and bioprocess engineering, but their importance lies primarily in preparing students to raise and formulate problems in mathematical terms.

5.2 Strategy and Method for Solving Problems

The most important aspect of problem solving is having a strategy and a method, i.e., a set of rules that guarantees the optimal decision and procedure. In this case, these will be mathematical rules and procedures for finding the true solution. But remember, at this stage, the most important aspect is not the final solution, but the correct mathematical formulation. In this chapter, we will show how problems that might seem very difficult are easily and simply solved. To this end, it is vital to follow, in an almost dogmatic (strict) attitude, the strategy and method that will be explained subsequently in detail.

Figure 5.1 outlines a general strategy and approach to address these problems. As you will see, Fig. 5.1 may seem very detailed and often unnecessary for certain problems, and undeniably it is. However, as will be seen throughout the chapter, it is vital in solving most problems and also ensures high efficiency in solving all kinds of problems. A similar strategy will be used to solve most of the problems presented in the book and will possibly be of help in most problems in advanced courses in engineering and other subjects that might be encountered during your professional career.

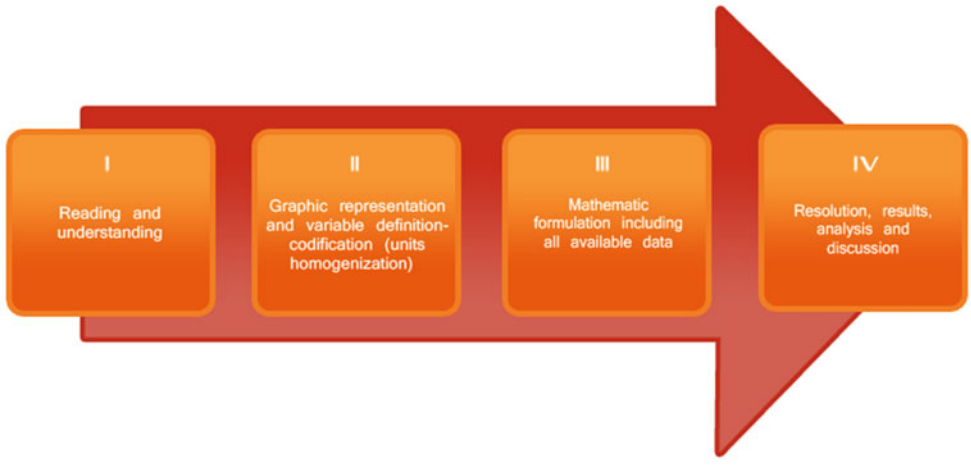

Fig. 5.1 A general strategy and approach to solve problems

Depending on the problem (referring, mainly, to this chapter), some steps can be omitted, e.g., the graphical representation.

Although some of the proposed steps are self-explanatory, an explanation of each step is included as follows.

Step I
Reading and understanding
This first step is critical. Sometimes with long statements you get confused. Although you understand the question, it might be hard to formulate the equations. As long as you have identified your unknowns, and coded accordingly, a good idea is to go phrase by phrase translating them into mathematical expressions.

Step II
Graphical representation and variable definition and codification.
Normally a good scheme or graphical representation lets us have a clear picture of the problem. In addition, we need to identify all the unknowns and then assign each one an adequate code to define each variable. For example, if in a given problem an airplane's speed is unknown, an appropriate coding to define the speed of the airplane could be V_a.

Step III
Mathematical formulation including all available data.
Once all variables have been identified and defined, it is necessary to determine which equations can be formulated. To pose the problem, the number of variables should be equal to the number of equations that can be formulated. In Chap. 7, when we deal with problems related to material balance, we will analyze and extend the concept of degrees of freedom (DF). For now DF is defined as follows:
DF = Number of variables − Number of equations.
If DF > 0, then the problem is underspecified.
If DF = 0, then the problem is correctly specified, and you can proceed to solve it.
If DF < 0, then the problem is overspecified.

In this chapter, we will be dealing with problems that are correctly specified. As presented in Chap. 7, we will further analyze and detail the concept of degrees of freedom in material balance problems.

Step IV
Resolution, results, analysis, and discussion.
After solving a problem, it is highly advisable to test the result in order to understand whether it is an expected and reasonable result and, ideally, to verify whether it is the true solution to the problem.

Before moving on to Sect. 5.3, we cordially invite you to solve the following warm-up examples.

Warm-Up Example 1
Classical problem (2 or 10^+?). Rodrigo has a shoe store, and his friend Andrew, in the same shopping center, has a sports shop. One day, early in the morning, a well-known fraudster decides to buy a pair of shoes. The shoes have a retail value of $180, and the scammer, making mischief, paid with a counterfeit $200 (two $100 bills). At that time in the morning, Rodrigo has no change and so goes to his friend Andrew for change. Andrew calmly changes his $200 and gives Rodrigo 20 $10 bills. Rodrigo returns to his store to give the customer the pair of shoes and his change, $20. The next day, his friend Andrew discovers the scam and goes to Rodrigo's shoe store to get his $200 back. Rodrigo was deceived and gives Andrew his $200 back and in return receives the counterfeit money.

What was Rodrigo's loss from this scam? The answer is at the end of the chapter.

Please stop and attempt to solve the problem!
Did you look at the answer? Surprised? We would like to stress that the problem can be easily solved by fourth and fifth grade elementary school students, but the shocking reality is that when university students, not only freshmen, are given the problem, normally most, if not all, are unable to. Why? As we stressed in the preface of the book, questioning and understanding are very important.

The problem looks so easy, and it actually is, and so it is too tempting to solve it mentally. Perhaps no one got out pen and paper to solve it. This was your big mistake; you clearly underestimated the problems level of difficulty and did not follow any method or strategy to solve it. Do not feel embarrassed. Almost every adult, including engineers, master's degree holders, and even Ph.D.s, fail to solve this problem, but please learn the lesson!

All problems deserve our attention and reflection before giving our final answer, so always follow a method.

Warm-Up Example 2
Monty Hall problem [10^{+}]. Marilyn vos Savant is a well-known American columnist who became famous for being recognized by the *Guinness Book of Records* as the most intelligent person with an intelligence quotient (IQ) of 228. Since 1986, she has been publishing a Sunday column in *Parade* magazine in which her readers ask questions on very diverse topics. As stated by the magazine, her column has elicited responses from the elementary school to the postgraduate level (Ph.D.).

In 1990, Marilyn vos Savant received a letter from a Craig F. Whitaker (Columbia, MD) with the following problem:

Suppose you are on a game show and you are given the choice of three doors. Behind one door is a car, behind the other two, goats. You pick a door, say number **1**, and the host, who knows what is behind each door, opens one of the doors with a goat, say number **3**, and then says to you, "Would you like to pick door number **2**?" Is it to your advantage to switch your choice of doors? What would you do?

This problem is well known (in different versions) and very interesting. We invite you to think and then compare your answer with the one that follows. Because we consider it very important, please take your time and try to solve this problem. We insist that before continuing, you should attempt to solve it and compare your answer with the answer given below. Why? Because most students will not be able to solve the problem, and there is a reason for that. Then, when you read the answer, we will show you why this happens and, most importantly, start to prepare you with a method to efficiently approach a variety of problems.

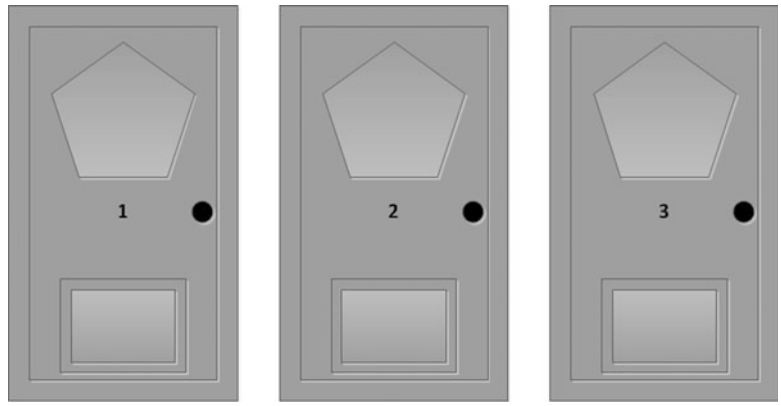

Fig. 5.2 Three doors of the Monty Hall problem

Please stop and attempt to solve the problem!

Solution of Warm-Up Example 2

First, we will give you the correct answer, and then discuss why this problem caused and continues to cause so much controversy.

Yes, it is better to change doors. If you do, your probability of winning will be 2/3, but if you do not change it, that probability will be 1/3. Confused?

As shown and illustrated below, this problem generated much controversy. *Parade* magazine received over 10,000 letters saying, often in rude terms, that Marilyn vos Savant was wrong. Believe it or not, among the detractors were several respected professional mathematicians.

Being faithful to what we have preached in this chapter, to solve this problem we will follow a method. First, make a sketch of the situation (Fig. 5.2).

Second, to address the problem, it is important to first remember what we understand by probability:

$$\text{Probability} = \text{Number of hits/total number of events.}$$

So, what we will do is investigate both cases: if I change doors and if I do not change doors. Considering all the possibilities (total number of events), we will calculate both probabilities and see which one is better. In the table below, we will assume that the car is behind door number 1 and the goats are behind doors 2 and 3 (obviously, we do not know). Then we will analyze all the possibilities, i.e., if we choose door number 1, door number 2, or door number 3, and see what happens in each case when we change doors (or do not change doors). The first column indicates our choice, while columns 2–4 show our choice in boldface.

Our Guess	Door 1	Door 2	Door 3	The Show-Man opens door[a]	Switching to	Switching door	Non switching doors
Door 1	**CAR**	Goat	Goat	3	From 1 to 2	Lose	Win
Door 2	Car	**GOAT**	Goat	3	From 2 to 1	Win	Lose
Door 3	Car	Goat	**GOAT**	2	From 3 to 1	Win	Lose
Probability of wining						**2/3**	**1/3**

[a]The Show host always chooses a door with a goat, e.g. when you chose door 3 his only chance is door 2

As depicted in the table, analyzing all the possibilities, it is clear that it is better to change doors after the host has opened a door with a goat. Strictly speaking, it is not a difficult problem because the number of ways to analyze it is rather small, just three.

In our view, the difficulty is that when faced with a problem like warm-up example 1, most people are wrongly tempted to analyze and solve it mentally and not take the problem "seriously." The strong and potent message here is that one should always consider problems as a worthwhile challenge and solve it using some method. As we have shown, if you analyze all the possibilities (just three), it is easy to see that it is better to change doors!

You can visit the following Web site for more details on the controversy surrounding the correct answer given by Marilyn vos Savant:
http://marilynvossavant.com/game-show-problem/.

What follows are some of the letters sent to *Parade* magazine:

Since you seem to enjoy coming straight to the point, I'll do the same. You blew it! Let me explain. If one door is shown to be a loser, that information changes the probability of either remaining choice, neither of which has any reason to be more likely, to 1/2. As a professional mathematician, I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error and in the future being more careful.

Robert Sachs, Ph.D., George Mason University

You blew it, and you blew it big! Since you seem to have difficulty grasping the basic principle at work here, I'll explain. After the host reveals a goat, you now have a one-in-two chance of being correct. Whether you change your selection or not, the odds are the same. There is enough mathematical illiteracy in this country, and we don't need the world's highest IQ propagating more. Shame!

Scott Smith, Ph.D., University of Florida

Your answer to the question is in error. But if it is any consolation, many of my academic colleagues have also been stumped by this problem.

Barry Pasternack, Ph.D., California Faculty Association

You're in error, but Albert Einstein earned a dearer place in the hearts of people after he admitted his errors.

Frank Rose, Ph.D., University of Michigan

I have been a faithful reader of your column, and I have not, until now, had any reason to doubt you. However, in this matter (for which I do have expertise), your answer is clearly at odds with the truth.

James Rauff, Ph.D., Millikin University

May I suggest that you obtain and refer to a standard textbook on probability before you try to answer a question of this type again?

Charles Reid, Ph.D., University of Florida

THE SHOE AND THE MARILYN VOS SAVANT PROBLEMS HAVE BEEN INCLUDED TO SHOW THE REAL IMPORTANCE OF APPLYING A METHOD WHEN SOLVING PROBLEMS. THEREFORE, WE SUGGEST THAT YOU TAKE A MINUTE AND READ THE FOLLOWING MESSAGE.

ATTENTION

Before turning to the solved problems section, we urge students to observe the following recommendation. Read the statement of each problem solved and try to solve it without examining the proposed solution. You will find the problems hard, but follow step by step the solution given. Following this procedure, you will learn to approach all the problems with a resolute attitude. And never forget the methodology.

5.3 Solved Problems

5.3.1 Rate

Although it might seem strange, in this section we have grouped problems that appear radically different from each other. For example, what do velocity, watches, work, candles, and fluid flow problems have in common? They can all be solved following a common conceptualization. Here the common conceptualization is the rate, in one case that of fluid flow (flow), in others it is the rate at which some work is done, and so on.

From a methodological point of view, we will first solve some problems from each class in order of increasing level of difficulty. The idea is to demonstrate that with an appropriate method, even the most difficult problems can be solved without much trouble. To be consistent with our predicament, for both solved and proposed problems, we have considered a variety of problems, from very basic to very difficult. We hope that after completing this section, you will agree with us that, yes, you can! You will also discover that the key to success is the strategy and method. Initially, perhaps always, it will be critical to work hard, but you will be rewarded. Here, our goal is to get you to the point where you can finally say to yourself, "Yes I can! and so can everyone else!"

As was mentioned in To Our Students, Colleagues and Tutors, each problem will be marked by level of difficulty. For example, the first solved problem has a difficulty of 7 out of 10; therefore, we added [7] to the name of the problem.

VELOCITY

1. **Travel to New York [7]**. Two cars are traveling from Boston to New York. The first car departs ***a*** hours before and arrives ***a*** hours later than the second car going to New York. If the first car takes ***b*** hours for the journey at a speed of V_1 km/h, what is the speed of the second car in kilometers per hour?

Solution

Step I

Reading and Understanding. Although the problem is simple, it involves some challenges because many students are not used to solving such problems with nonnumeric information.

The question involves expressing the speed of car 2 as a function of known data, such as the speed of the first car (V_1 km/h) and the travel time of the first car (***b*** h), and considering the time that it departs before and comes after car 2 (***a*** hours). If we designate the second cars speed V_2, we arrive at an expression such as this:

With V_2 a function of V_1, ***a*** and ***b*** can be expressed mathematically as: $V_2 = f(V_1,a,b)$

Step II

Graphical representation and variable definition and codification (Fig. 5.3)

V_1: Car 1 s speed.

V_2: Car 2 s speed.

Step III

Mathematical formulation including all available data. In this problem, it is possible to formulate two equations, one for the speed of each car. In this case, we do not have degrees of freedom (DF = 0), and the problem can be solved.

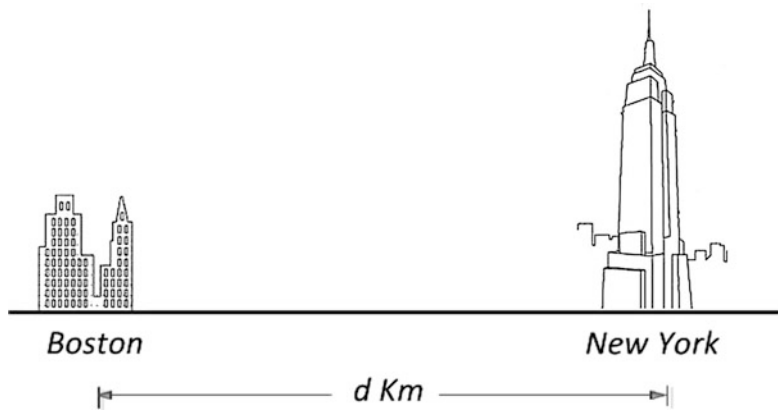

Fig. 5.3 Graphic representation of cars travelling from Boston to New York

Car 1
Given that the distance traveled by the car is d km and that distance is covered in $\boldsymbol{b}$ hours, then

$$V_1 = \frac{d}{b}; \quad V_1 b = d \tag{5.1}$$

Car 2
Given that the distance traveled by the car is $\boldsymbol{d}$ km and that distance is covered in $\boldsymbol{b - 2a}$ hours (departed $\boldsymbol{a}$ hours after and arrived $\boldsymbol{a}$ hours earlier), then

$$V_2 = \frac{d}{b - 2a}; V_2(b - 2a) = d \tag{5.2}$$

Step IV
Resolution, results, analysis, and discussion. From equations (5.1) and (5.2) we have

$$V_2(b - 2a) = V_1 b; \text{then,} \quad V_2 = \frac{V_1 b}{b - 2a} \tag{5.3}$$

meaning: $V_2 = f(V_1,a,b)$.

An important part of solving a problem is analyzing the results. Since car 2 takes less time than car 1 to complete its trip, V_2 should be expected to be greater than V_1. Looking at (5.3) we see that $V_2 > V_1$; this is because the factor $(\boldsymbol{b - 2(a)})$ is less than $\boldsymbol{b}$. Furthermore, for example, when the value of $\boldsymbol{a}$ tends to zero, i.e., when both cars depart and arrive together, in (5.3) it is observed, correctly, that the value V_2 is equal to V_1. This analysis shows us that the result might be correct.

2. Usain Bolt [3⁺]. As a humble amateur, you are capable of running the 100 m dash in a respectable 11.3 s. In your dreams, you are pitted against Usain Bolt, owner of the world record in the 100 m dash—an astonishing 9.58 s. How many meters from the finish line would you be at the moment Usain Bolt completes his sprint in the 100 m dash?

Solution
Step I
Reading and Understanding. Reading carefully, the question is closely related to how many meters you are able to run in 9.58 s. Then to calculate the distance that you are running in 9.58, we first need to calculate your speed.

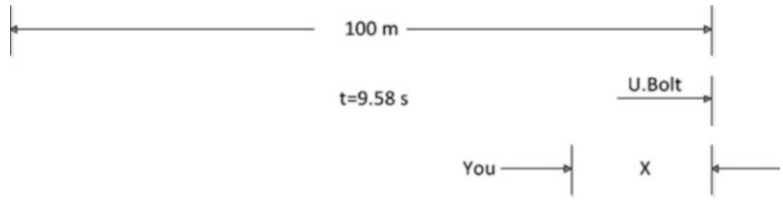

Fig. 5.4 Graphic representation of the running of your dreams

Step II

Graphical representation and variable definition and codification. We can make a simple sketch to represent what is happening in your dreams (Fig. 5.4).

v: Your speed in the 100 m dash (we assume that you run at a constant speed for the whole race. Do you think this is a reasonable assumption? If not, why not?)

d: Distance that you will cover in 9.58 s

Steps III and IV

Mathematical formulation including all available data and resolution, results, analysis, and discussion. We have the data to calculate your speed and then the distance that you will cover in 9.58 s (DF = 0).

First, we calculate your speed; remember that you run 100 m in 11.3 s; therefore,

$$v = \frac{100 \text{ m}}{11.3 \text{ s}} \cong 8.85 \text{ m/s}$$

Then in 9.58 s you will cover

$$d = v \times t = 8.85 \text{ m/s} \times 9.58 \text{ s} \cong 84.8 \text{ m; therefore, } x = 100 - 84.8 = 15.2 \text{ m.}$$

At the moment when Usain Bolt is crossing the finish line, you have covered approximately 84.8 m. Then you will be slightly more than 15 m from the finish line (15.2 m). Not too bad for an amateur!

3. Shoplifter [10$^+$]. A shoplifter is about to escape after robbing a grocery store on some street corner when an employee sets off the alarm.

At that time, a policeman who is four blocks away hears the alarm and runs toward the supermarket (assume that one block is 125 m and the police hears the alarm instantly, i.e., do not consider the time it takes for the sound to travel from the store to the policeman four blocks away).

When the police arrives at the grocery store, the employees tell him that the shoplifter is already two blocks away and getting into a taxi. The policeman increases the speed of his car by 20 % to overtake the shoplifter.

The shoplifter goes several blocks in the taxi when he has to stop at a red light. He estimates that the taxi will be stopped long enough for the police to catch up to him, so he gets out and takes off on foot. His speed this time is 20 % faster than when he was making his escape from the supermarket, but the policeman catches up with him 10 s after he gets out of the taxi.

As the shoplifter is getting out of the taxi, the distance between him and the police is one block. It is known that if the taxi had not stopped, the policeman would have caught up to the shoplifter in 10 min (from the time he got in the taxi).

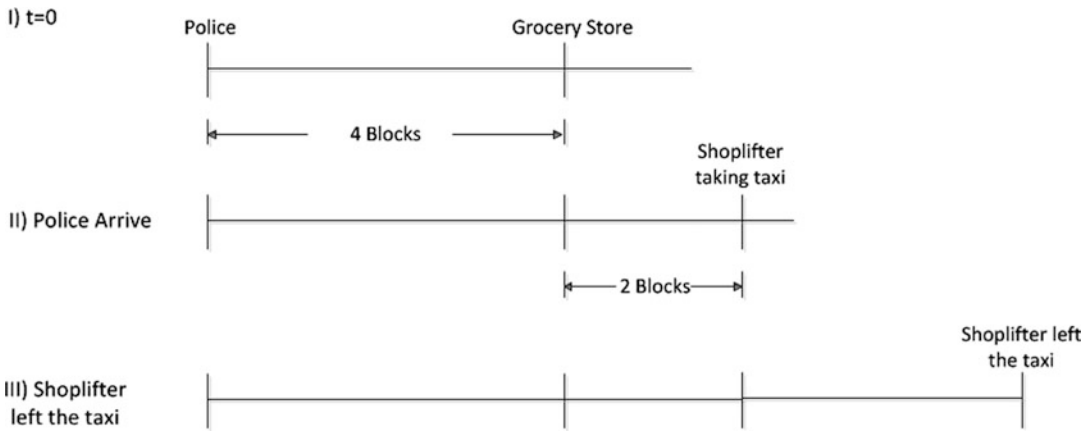

Fig. 5.5 Graphic representation of the shoplifter escape

What is the speed of the shoplifter, the policeman, and the taxi?

Solution

Step I

Reading and understanding. This is a classic example where the hard part of the problem is formulating it in mathematical terms. We need to translate step by step this intricate statement into equations. Here, a careful reading with the help of a good sketch will help us formulate the required equations. In addition, you must always be very tidy!

Step II

Graphical representation and variable definition and codification.

We have three variables, and these are and coded as follows:
V_S: speed of shoplifter m/min
V_P: speed of policeman m/min
V_T: speed of taxi m/min

Step III

Mathematical formulation including all available data. We have three unknowns, and with the information provided we can formulate three equations, then DF = 0.

Looking at the sketch (Fig. 5.5) and reading the problem statement, when the policeman arrives at the supermarket, the shoplifter is two blocks away. Meanwhile, the policeman runs four blocks to get to the supermarket, while at the same time the shoplifter runs two blocks away. Therefore, the policeman's speed is double the speed of the shoplifter. Mathematically, we express this as

$$V_P = 2 \times V_S. \tag{5.4}$$

Then the shoplifter takes a taxi and the policeman increases his speed by 20 %, so now his speed is $1.2 \times V_P$. After several blocks, the taxi has to stop at a red light, so the shoplifter gets out and makes a run for it, but this time at a speed that is 20 % faster than before. Now his speed is $1.2 \times V_S$.

In addition, when the shoplifter gets out of the taxi, the policeman is just one block away. The policeman takes 10 s to reach the shoplifter, and the equation for this situation should look like (Fig. 5.5) this:

Distance run by the police = Distance run by the shoplifter+1 block (125 [m])

First, distance is the product of speed and time. If we write an equation, we must remember that both speeds, that of the policeman and that of the shoplifter, were increased by 20 %, thus

$$1.2 \times V_P \times \frac{10}{60} = 1.2 \times V_S \times \frac{10}{60} + 125. \tag{5.5}$$

Given that the speed is in meters per minute, the time should be in minutes! This is why we put 10/60 min (10 s) in (5.5).

Finally, if the taxi had not stopped at the red light, the policeman would have caught the shoplifter in 10 min from the time the shoplifter got into the taxi. At that point, the policeman was two blocks away. The equation for this situation should look like this (Fig. 5.5):

Distance run by the policeman = Distance run by the shoplifter in the taxi+2 blocks (250 [m])

$$1.2 \times V_P \times 10 = V_T \times 10 + 250. \tag{5.6}$$

In this case, the time is in minutes.

Step IV

Resolution, results, analysis, and discussion. As shown in the mathematical formulation, we have three equations and three unknowns. From (5.4) and (5.5), we obtain

$$V_P = 1,250 \ [\mathrm{m/min}]$$

$$V_S = 625 \ [\mathrm{m/min}]$$

And replacing V_P in (5.6) we obtain

$$V_T = 1,475 \ [\mathrm{m/min}]$$

Although it should be evident, it is important to notice that the speed of the policeman must be greater than the speed of the shoplifter. If not, why would the shoplifter take a taxi? In addition, although subtle, it is logical to expect that the speed of the taxi was greater than the speed of the policeman. Remember, when the policeman becomes aware that the shoplifter is taking a taxi, he speeds up by 20 %.

This specific problem shows again the power of following a method for solving problems. Read and understand, represent things graphically, and so on.

WATCHES

4. Travel to San Francisco [6]. You set out by car from Portland, Oregon, to San Francisco, California, at 10 a.m. Your friend Peter leaves a little later (but before 11 a.m.), when the minute and hour hands overlap. If both arrive at the same time in San Francisco, how much less time did Peter take for the trip?

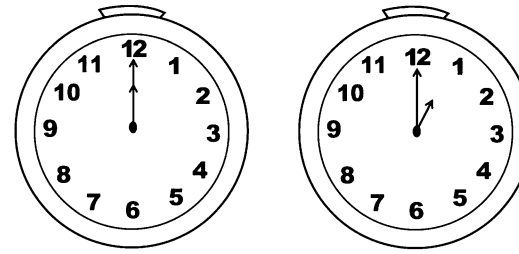

Fig. 5.6 Clocks at 12:00 AM and another clock at 1:00 PM

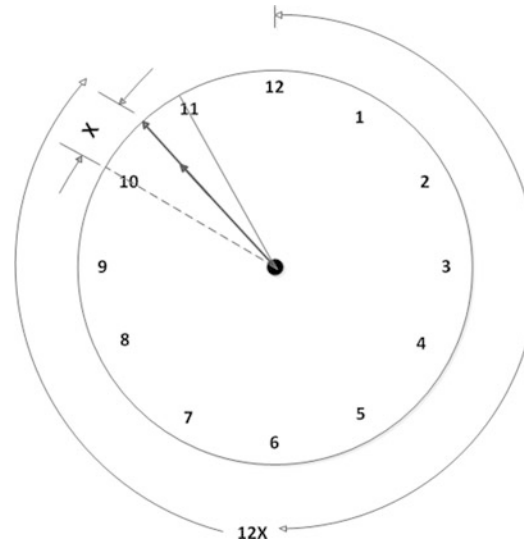

Fig. 5.7 The clock indicates Peter's departure, after 10:00 AM but before 11:00 AM

Solution

Step I

Reading and understanding. Unlike the first problem (Travel to New York), in this case, the data are numeric, but the difficulty is that we have to relate the speed ratio between the hour hand speed and the minute hand speed. A simple observation (Fig. 5.6) shows a clock at 12 noon and another clock at 1 p.m. We can infer that within 1 h the minute hand will have made a full turn (60 min) and the hour hand will have advanced from 12 to 1 p.m., that is, 5 min. From this we can deduce that the minute hand moves 12 times faster than the hour hand (60/5).

Step II

Graphical representation and variable definition and codification. A clock is shown schematically at 10 a.m. and another that indicates Peter's departure, after 10 a.m. but before 11 a.m., and with the minute and hour hands overlapping.

$12x$: minutes advanced by the minute hand

x: "minutes" advanced by the hour hand

Step III

Mathematical formulation of problem including all available data. As shown in Fig. 5.7, we have called x the "minutes" advanced by the hour hand and $12x$ the minutes moved by the minute hand

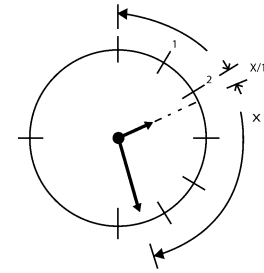

Fig. 5.8 Starting time of the exam of Introduction to Chemical and Bioprocess Engineering

(remember that the minute hand moves 12 times faster than the hour hand). Therefore, if we consider that from 12 to 10 there are 50 min, then we can formulate the following equation:

$$50 + \mathbf{x} = 12\mathbf{x}.\text{Solving for } x, \text{we have } \mathbf{x} = 50/11; \text{approximately } 4 \text{ min } 33 \text{ s}.$$

So Peter set out at 10 h 54 min 33 s. Given that they arrived at the same time, he used 54 min 33 less time than you to make the trip from Portland to San Francisco.

Step IV

Results, analysis, and discussion. The key to solving this problem, as in many engineering problems, is to build an adequate graphical representation of the situation. In this particular case, the graphical representation facilitates our understanding of why the speed of the minute hand is 12 times faster than the hour hand speed. With this background and understanding, most problems involving time will be easy to solve.

5. How long was the exam? [7]. An exam in your Introduction to Chemical and Bioprocess Engineering class starts when the hour hand of the clock is between 2 and 3 p.m. and the minute hand is between 5 and 6. The exam ends when the positions of the hour and minute hands are reversed. How much time do you have for the Introduction to Chemical and Bioprocess Engineering exam?

Solution

Step I

Reading and understanding. As was deduced in the previous problem (Travel to San Francisco), the minute hand moves 12 times faster than the hour hand. With the information provided in the problem statement, we can calculate when the exam started and when it finished. Then, it is straightforward to calculate the amount of time for the exam.

Step II

Graphical representation and variable definition and codification. Figures 5.8 and 5.9 show schematically and graphically when the exam starts and finishes.

As depicted in Fig. 5.8 X represents the minutes after 2 p.m. (when starting the exam), and in Fig. 5.9 Y represents the minutes after 5 p.m. (when finishing the exam).

X: minutes after 2 p.m.

Y: minutes after 5 p.m.

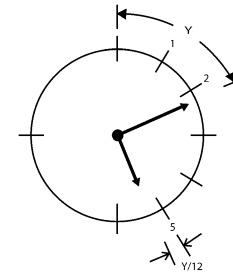

Fig. 5.9 Finishing time of the exam of Introduction to Chemical and Bioprocess Engineering

Step III

Mathematical formulation of problem. We have two unknowns, and we can formulate two equations, then DF = 0. Observing Figs. 5.8 and 5.9 simultaneously, and considering that the hour and minute hands are reversed, we can write the following equations:

$$\frac{Y}{12} + 25 = x, \tag{5.7}$$

$$\frac{X}{12} + 10 = y. \tag{5.8}$$

Step IV

Results, analysis, and discussion. Solving (5.7) and (5.8) we obtain

$X = 26$

$Y = 12$

So the exam started at 2 h 26 min and finished at 5 h 12 min. Therefore, the exam lasted 2 h 46 min.

WORK

6. **Overtime [6]**. A worker is able to complete a job in two shifts of 8 h. Due to an emergency, his boss wants him to complete the work as quickly as possible, and therefore the worker has to work without a break. In this case, during the first 10 h, the worker works like he normally does, but after that, because he tired, he works at 75 % of his normal rate. How many hours will it take him to finish the job?

Solution

Step I

Reading and understanding. For this type of problem it is necessary to define the concept of worker speed. It will be defined as follows:

$v = \frac{T}{t}$, where v is the work speed, T is the work done (completed work), and t is the time to complete the entire job.

Step II

Graphical representation and variable definition and codification. Although in most cases it is useful to have a graphical representation, in this case it is unnecessary.

v: work speed

T: completed work

t: time

Step III

Mathematical formulation including all available data. There is one unknown variable, and so one equation can be formulated (DF = 0). We are told that the worker can complete the job in two shifts of 8 h, i.e., 16 h; therefore, the work speed can be represented by

$$v = \frac{T}{16}. \tag{5.9}$$

Working continuously, the employee works the first 10 h at this rate and the remaining time at 75 % of his normal rate. Therefore, part of the work will be completed in the first 10 h ($10v$) and the rest of the work ($t \times 0.75v$) will be fully completed (T); then

$$10v + 0.75vt = T. \tag{5.10}$$

Step IV

Resolution, results, analysis, and discussion. Substituting (5.9) into (5.10) we obtain

$$10\frac{T}{16} + 0.75\frac{T}{16}t = T.$$

Simplifying T, we obtain $t = 8$ h, and so the work done by the worker without a break is completed in 18 h (10 + 8).

The result is logical, given that part of the work was performed at a slower speed. In the first case, the work was finished in 16 h in two shifts, and in the second case in 18 continuous hours. Problems like this could perhaps be solved in a simpler way, but it is vital to stress that the method is fundamental, especially for more complex situations. Recall the lesson of the shoe problem. In many of the problems proposed in this chapter, it is necessary, perhaps essential, to continue using the proposed method. We emphasize that, however basic a problem may be, the method for dealing with it is of utmost importance.

7. Apprentices and masters [10$^+$]. A group of apprentices and masters is building a large wall made of 50,000 bricks. They complete the job in 25 days. The apprentices work 2 h more than the masters per day. The boss pays per brick, and at the end of the day an apprentice earns three-quarters of what a master earns. If the masters work alone, but with the working hours per day of an apprentice, they will complete the big wall in 50 days. The boss pays $12,000 for the completed job, and a master earns $96 a day. Calculate (a) the number of masters, (b) the number of apprentices, (c) the working speed of a master, (d) the working speed of an apprentice, (e) the daily working hours of masters, and (f) the daily working hours of apprentices.

Solution

Step I

Reading and understanding. As in the previous problem (Overtime), the key is to correctly define work speed, in this case number of bricks per hour, then formulate all the equations from the problem statement.

Step II

Graphical representation and variable definition and codification. In this case it is not necessary to have a graphical or schematic representation. To code and define the unknowns, we will follow the order of the questions:

N_M: number of masters
N_A: number of apprentices
V_M: working speed of a master bricks/h
V_A: working speed of an apprentice bricks/h
t_M: daily working hours of a master h/day
t_A: daily working hours of an apprentice h/day

Step III

Mathematical formulation including all available data. We have six unknowns, and after a careful reading of the problem statement, we can write five equations. At first glance, the problem seems to have no solution because we need six equations. But if we assume that the problem has been correctly stated, then we need to further analyze the situation in step IV: Resolution, results, analysis, and discussion.

Taking into account that for the time being, the problem cannot be solved, we will write all the equations that can be inferred from the problem statement.

From "*A group of apprentices and masters are building a large wall made of 50,000 bricks and are able to complete the job in 25 days*" we can write

$$25 \times N_M \times V_M \times t_M + 25 \times N_A \times V_A \times t_A = 50,000. \tag{5.11}$$

To understand (5.11), remember that V_M is the work speed [bricks/h] per master, which, multiplied by t_M [h/day], gives us the number of bricks per day for masters. Then multiplying this result by the number of masters and the number of days that masters work, we obtain the total number of bricks laid by the masters. In the same way, the second term of the equation calculates the total number of bricks laid by the apprentices. Adding these two numbers, we arrive at the completed job: 50,000 bricks.

Given that the apprentices work 2 h more per day than the masters, we obtain

$$t_M + 2 = t_A. \tag{5.12}$$

From *"The boss pays per brick and at the end of the day an apprentice gains three-quarters of what a master earns"* we can write the following:

First, the boss is paying \$12,000 for the completed job (50,000 bricks) and \$0.24 per brick; therefore,

$$0.24 \times V_A \times t_A = \frac{3}{4} \times 0.24 \times V_M \times t_M. \tag{5.13}$$

Masters working alone, but with the daily working hours of an apprentice, would complete the wall in 50 days; thus,

$$50 \times N_M \times V_M \times t_A = 50,000. \tag{5.14}$$

Finally, a master earns \$96 a day, so

$$0.24 \times V_M \times t_M = 96. \tag{5.15}$$

Step IV

Resolution, results, analysis, and discussion. Assuming that the problem has a solution, we need to think about it. Clearly, from the problem statement we can formulate five equations (5.11–5.15). We need to be confident that we have already written all the possible equations from the problem statement, so we need to try to understand and look for some other, hidden, information*. For example, it sounds logical that N_M and N_A should be integers. With this in mind, we will do some algebra to get an equation to relate N_M and N_A.

From (5.15) we get that $V_M \times t_M = 400$, and substituting it into (5.13) we obtain $V_A \times t_A = 300$. Substituting both results into (5.11) and simplifying we get

$$4 \times N_M + 3 \times N_A = 20. \tag{5.16}$$

Because N_M and N_A should be integers and positive numbers, we will analyze the potential valid results from (5.16).

N_M	N_A
5	0
4	1.33
3	2.66
2	**4**
1	5.33
0	6.66

From the table we can infer that the only possible result is $N_M = 2$ masters and $N_A = 4$ apprentices. Then substituting N_M in (5.14) and dividing by (5.15) we obtain

$$t_A = \frac{5}{4} t_M.$$

That relates t_M to t_A, and solving together with (5.12) we obtain

$$t_A = 10 \text{ h and } t_M = 8 \text{ h}.$$

Finally, it is easy to obtain that $V_M = 50$ bricks/h and $V_M = 30$ bricks/h.

The lesson here is that sometimes certain equations, information, or constraints are somewhat hidden.

*Diophantine equations are a type of equation where variables are integers. Usually, they have fewer equations than variables, but the fact that the variables are integers makes it possible to find a single solution. The name of these equations comes from the Greek mathematician Diophantus of Alexandria, third century BC.

CANDLES

8. **Two candles [7].** A candle with height L_a burns in $\boldsymbol{a}$ hours; another candle with height L_b burns in $\boldsymbol{b}$ hours. Given that $L_a > L_b$ and $\boldsymbol{b} > \boldsymbol{a}$, if the candles are lit simultaneously, after how much time will they be the same height?

Solution

Step I

Reading and understanding. This problem combines two degrees of difficulty. First, it is very important to outline graphically the situation. Second, no numerical information is given, and

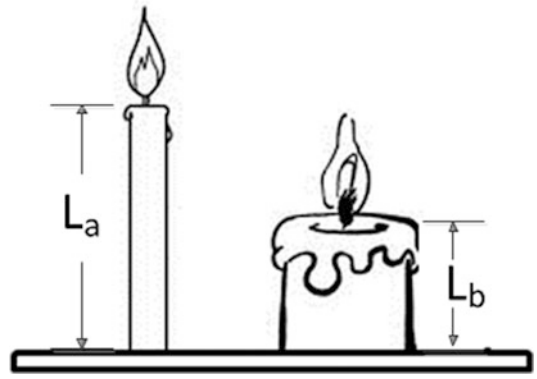

Fig. 5.10 Candles of height L_a and L_b

therefore the mathematical formulation is somewhat more difficult. As in the previous problems, it is important to conceptualize the rate of consumption of the candle. In this case, it could be generalized as

$v = \frac{L}{t}$, where v is the rate of consumption of the candle, L is the candle height, and t is the time it takes for a candle to be consumed completely.

Step II

Graphic representation and variable definition and codification (Fig. 5.10).

X: distance consumed by candle b
t: time for both candles to reach the same height

Step III

Mathematical formulation including all available data. It is possible to formulate two equations, one for each candle. We have two unknowns, so DF = 0.

According to the definition of speed, we may define

$$v_a = \frac{L_a}{a} \text{ and } v_b = \frac{L_b}{b}.$$

Once both candles reach the same height (Fig. 5.11), after a time t, we can write

$v_a = \frac{L_a - L_b + X}{t}$ and $v_b = \frac{X}{t}$, and replacing v_a and v_b we have

$\frac{L_a}{a} = \frac{L_a - L_b + X}{t}$ and $\frac{L_b}{b} = \frac{X}{t}$, and solving for X in both equations we obtain

$X = \frac{L_a}{a}t - L_a + L_b$ and $X = \frac{L_b}{b}t$, respectively, and matching both equations and solving for t we obtain

$$t = \frac{ab(L_a - L_b)}{bL_a - aL_b}. \tag{5.17}$$

Step IV

Resolution, results, analysis, and discussion. First, we can determine whether for all values of L_a, L_b, $\boldsymbol{a}$, and $\boldsymbol{b}$ the result obtained for time t is positive. Since $\boldsymbol{a}$ and $\boldsymbol{b}$ are positive, and the problem states that $L_a > L_b$, then the numerator in (5.17) is positive. In the case of the denominator, $\boldsymbol{b}L_a > \boldsymbol{a}L_b$ because $L_a > L_b$ and $\boldsymbol{b} > \boldsymbol{a}$; therefore, it is also positive. In conclusion, t is always positive. Another

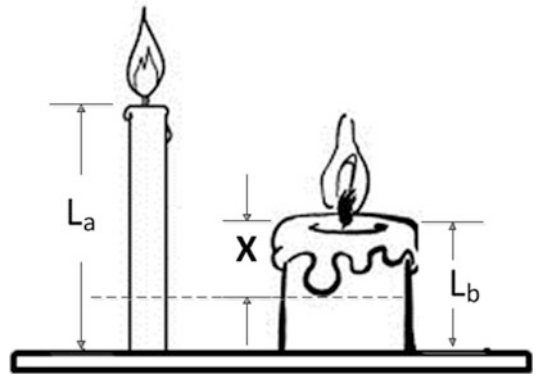

Fig. 5.11 Time t where both candles reach the same height

condition to be met is that t should be less than $\boldsymbol{a}$ before the candle with height L_a is consumed, and therefore rewriting (5.17), we obtain

$t = \frac{ab(L_a - L_b)}{b\left(L_a - \frac{a}{b}L_b\right)}$, simplifying and rewriting $t = a\frac{(L_a - L_b)}{\left(L_a - \frac{a}{b}L_b\right)}$, the multiplier for $\boldsymbol{a}$ is less than 1. Remember that $\boldsymbol{a} < \boldsymbol{b}$, and therefore $L_a - L_b < \left(L_a - \frac{a}{b}L_b\right)$, then $\boldsymbol{t} < \boldsymbol{a}$.

Flow

9. **Two valves [8]**. A tank is filled through valves A and B. Each separate valve fills the tank in $\boldsymbol{a}$ and $\boldsymbol{b}$ hours, respectively ($\boldsymbol{a} > \boldsymbol{b}$). After valve A is opened, when should valve B be opened so that the tank is filled in $\boldsymbol{b}$ hours?

Solution

Step I

Reading and understanding. As in the previous problems, first it is necessary to conceptualize the flow rate. In generic terms, the volumetric flow rate will be defined as

$C = \frac{V}{t}$, where C is the volumetric flow, and V is the volume to be filled in time t.

Step II

Graphical representation and variable definition and codification. Although in most cases it is useful to have a graphical representation, in this case it is unnecessary because a graphical representation would not help us in writing the equations.

V: volume of tank

C_a: volumetric flow rate of valve A

C_b: volumetric flow rate of valve B

Step III

Mathematical formulation including all available data. There is one unknown variable, and one equation can be formulated (DF = 0). Since valve A fills the tank in $\boldsymbol{a}$ hours and valve B fills it in $\boldsymbol{b}$ hours, the flow rate of each of the valves can be expressed as

$$C_a = \frac{V}{a}, \tag{5.18}$$

$$C_b = \frac{V}{b}. \tag{5.19}$$

The tank should be filled in $\boldsymbol{b}$ hours. The filling process is started by valve A, and after t hours valve B starts working. Therefore, the volume filled by valve A for b hours, plus the volume filled by valve B in $\boldsymbol{b} - \boldsymbol{t}$ hours must equal the volume of the tank, V:

$$C_a b + C_b(b - t) = V. \tag{5.20}$$

Step IV

Resolution, results, analysis, and discussion. Substituting C_a and C_b from (5.18) and (5.19) into (5.20) we obtain

$\frac{V}{a}b + \frac{V}{b}(b - t) = V$, solving for t,

$$t = \frac{b^2}{a}. \tag{5.21}$$

A condition that should be fulfilled is that t must be less than $\boldsymbol{b}$. If we rewrite (5.21), we obtain $t = \frac{b}{a}b$, and since $\boldsymbol{b} < \boldsymbol{a}$, then $\boldsymbol{b}$/a (multiplier of $\boldsymbol{b}$) is less than one. Therefore, $t < \boldsymbol{b}$.

10. Wine [3⁺]. A vat of wine is filled by three valves, A, B, and C. To completely fill the vat of wine, each valve, operating alone, takes a, b, and c hours, respectively. How quickly will the vat of wine get filled if valves A and B operate simultaneously? How quickly will the vat get filled if valves A, B, and C operate simultaneously?

Solution

Step I

Reading and understanding. First, it is very important to define mathematically the flow rate:

$C = \frac{V}{t}$, where C is the volumetric flow rate, and V is the volume to be filled in a time t.

Step II

Graphical representation and variable definition and codification. In this case, a graphical or schematic representation is unnecessary.

V_{W}: volume of vat of wine (l)

C_{A}: flow rate of valve A L/h

C_{B}: flow rate of valve B L/h

C_{C}: flow rate of valve C L/h

Step III

Mathematical formulation including all available data. According to the hour variable codification, we will express mathematically the statement "*To completely fill the vat of wine, each valve, operating alone, takes a, b, and c hours, respectively.*"

When valve A operates alone, it fills the vat of wine completely in a hours; thus,

$$C_{\mathrm{A}} = \frac{V_{\mathrm{W}}}{a}. \tag{5.22}$$

Analogously, for valves B and C we have

$$C_{\mathrm{B}} = \frac{V_{\mathrm{W}}}{b}, \tag{5.23}$$

$$C_{\mathrm{C}} = \frac{V_{\mathrm{W}}}{c}. \tag{5.24}$$

Now, if valves A and B operate together, the volume that valve A contributes plus the volume contribution of valve B in time t should completely fill the vat of wine. Therefore,

$$C_{\mathrm{A}} \times t + C_{\mathrm{B}} \times t = V_{\mathrm{W}}. \tag{5.25}$$

Step IV
Resolution, results, analysis, and discussion.

Substituting C_{A} and C_{B} in (5.25) we obtain

$$\frac{V_{\mathrm{W}}}{a} \times t + \frac{V_{\mathrm{W}}}{b} \times t = V_{\mathrm{W}}. \tag{5.26}$$

Simplifying V_{W} and rearranging we obtain

$$t = \frac{1}{\frac{1}{a} + \frac{1}{b}} = \frac{ab}{a+b}. \tag{5.27}$$

If the three valves operate simultaneously we get

$$C_{\mathrm{A}} \times t + C_{\mathrm{B}} \times t + C_{\mathrm{C}} \times t = V_{\mathrm{W}}. \tag{5.28}$$

As before, substituting C_{A}, C_{B} , and C_{C} into (5.28) we obtain

$$\frac{V_{\mathrm{W}}}{a} \times t + \frac{V_{\mathrm{W}}}{b} \times t + \frac{V_{\mathrm{W}}}{c} \times t = V_{\mathrm{W}}. \tag{5.29}$$

Simplifying V_{W} and rearranging (5.29) we obtain

$$t = \frac{1}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}. \tag{5.30}$$

If valves A and B had equal flow rates, say C_{A}, then we would expect that both valves operating simultaneously will reduce the time to fill the vat of wine completely by half. In (5.27), if b equals a, then

$$t = \frac{1}{\frac{1}{a} + \frac{1}{b}} = \frac{ab}{a+b} = \frac{a^2}{2 \times a} = \frac{a}{2}.$$

In the same way, if the three valves had equal flow rates, then if the three valves operated simultaneously, it would take $a/3$ h to fill the vat of wine. If b and c are equal to a, then in (5.30) we obtain

$$t = \frac{1}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}} = \frac{1}{\frac{1}{a}+\frac{1}{a}+\frac{1}{a}} = \frac{a}{3}.$$

These results show that our solution is apparently correct.

11. *N* valves [8]. A tank is filled by a valve in 10 h. Another tank is filled by an unknown number of valves. Each valve of the second tank provides the same flow rate in liters per hour, and in addition, each valve of the second tank separately fills it in 100 h. First, an operator opens the valve of the first tank, and 1 h later the operator opens the set of valves on the second tank. Two hours later, the operator closes the set of valves on the second tank, and at this point, the volume of the second tank is 50 L greater than that of the first tank. Five hours later, the first tank has double the volume of the second tank and the valve of the first tank is closed. Finally, the operator pumps water from the first tank to the second until it is completely filled. The operator then checks that the remaining volume in the first tank is 200 L.

(a) What is the volume of the first tank?
(b) What is the volume of the second tank?
(c) What is the flow rate of the valves (first and second tank)?
(d) How many valves does the second tank have?

Solution

Step I

Reading and understanding. As was shown in the previous problem (Wine), the flow rate was defined as follows:

$C = \frac{V}{t}$, where C is the volumetric flow, and V is the volume to be filled in time t.

Step II

Graphical representation and variable definition and codification. In this case, it is not necessary to have a graphical or schematic representation. As explained earlier, a schematic representation would not help us to develop the necessary equations.

V_1: volume of first tank L
V_2: volume of second tank L
C_1: flow rate of valve on the first tank L/h
C_2: flow rate of each valve on second tank L/h
N: number of valves that feed the second tank

Step III

Mathematical formulation including all available data. We have five unknowns. From the problem statement we should be able to formulate five equations. If so, DF = 0, and we can solve them.

As we did in previous problems, we will be constructing the equations by translating the verbal expression into a mathematical expression.

If the first tank is filled in 10 h, then

$$10 \times C_1 = V_1. \tag{5.31}$$

If each valve of the second tank fills it in 100 h, then

$$100 \times C_2 = V_2. \tag{5.32}$$

The following statement indicates that the operator first opens the valve on the first tank, and 1 h later the valves on the second tank are opened. Then, 2 h after those valves are opened, the volume of the second tank is 50 L more than that of the first tank. Therefore,

$$3 \times C_1 + 50 = 2 \times N \times C_2. \tag{5.33}$$

The first tank took 3 h to fill and the second tank 2 h, but by N valves.

Our next statement says: "*Five hours later the first tank has double the volume of the second tank*"; mathematically this is

$$8 \times C_1 = 2(2 \times N \times C_2). \tag{5.34}$$

Finally, water is pumped in to fill the second tank completely, and 200 L remains in the first tank. Therefore, the amount pumped from the first tank, $8 \times C_1 - 200$ L, plus the amount already in the second tank, $2 \times N \times C_2$, should be equal to the volume of the second tank, V_2. Mathematically:

$$(8 \times C_1 - 200) + (2 \times N \times C_2) = V_2. \tag{5.35}$$

Thus, we have formulated five equations (5.31–5.35), and so we can solve the problem.

Step IV

Resolution, results, analysis, and discussion. From (5.33) to (5.34) we obtain

$$C_1 = 50 \text{ [L/h]}, \text{and from } (5.31)\ V_1 = 500 \text{ [L]}.$$

We will leave to you the remaining algebra to obtain the following results:

$$C_2 = 4 \text{ [L/h]}, V_2 = 400 \text{ [L]}, \text{and } N = 25 \text{ valves}.$$

5.3.2 Concentrations

12. **Wine barrel [10⁺]**. A drunk man is standing next to a large wine barrel (a mixture of ethanol and water) of volume V. To conceal his actions and deceive the barrel owners, the drunk man does the following: he takes a cup of volume v ($v <<<< V$) from the barrel and, in return, pours a cup of water into the barrel, so that the barrel's volume is held constant.*If, initially, the barrel had pure wine, show that after the drunk man removes wine and pours in water n times the final ethanol concentration in the barrel will be

$$C_n = C_0 \left(\frac{V - v}{V} \right)^n, \tag{5.36}$$

where C_0 is the initial concentration (unmixed wine) [liters of alcohol/liter solution], and C_n is the concentration after the nth operation (1, 2, 3, …, n) [liters of alcohol/liter solution].

*Suppose the volumes can be considered additive. If two liquids are mixed (for example, if you mix ethanol and water), then the volume of the mixture is not necessarily the sum of the volumes of the liquids since one of the liquids could occupy the empty spaces between the molecules of the other

liquid. In this particular case, the wine has an alcohol concentration (ethanol) of about 12 %, and it is known that ethanol/water mixtures are not perfectly additive.

Solution

Step I

Reading and understanding. Initially, the alcohol volume is C_0V, and when the drunk man takes the first glass of wine and replaces it with water, the amount of alcohol left in the barrel is then $C_0(V - v)$, which is equivalent to C_1V.

Step II

Graphical representation and variable definition and codification. Although in most cases it is useful to have a graphical representation, in this case is unnecessary. In addition, the variables were already defined in the problem statement.

Step III

Mathematical formulation of the problem

First operation

$$C_0(V - v) = C_1V, \text{then} \quad C_1 = C_0\left(\frac{V - v}{V}\right) \tag{5.37}$$

Second operation

$C_1(V - v) = C_2V$, replacing C_1, then $C_2 = C_0\left(\frac{V-v}{V}\right)^2$

Third operation

$C_2(V - v) = C_3V$, replacing C_2, then $C_3 = C_0\left(\frac{V-v}{V}\right)^3$

By induction, we can express that

$$C_n = C_0\left(\frac{V - v}{V}\right)^n. \tag{5.38}$$

Step IV

Resolution, results, analysis, and discussion. First, we observe in (5.38) that if the drunk man's cup volume tends to zero ($v = 0$), then the concentration in the barrel (C_n) will not change (virtually). On the other hand, if the drunk man's cup volume was similar to the volume concentration of the barrel (v tends to V), then this would approach zero quickly.

13. Two streams [9]. Two salt-free streams of unknown flow are mixed to form one stream. It is known that the mixed stream has a flow of ***a*** kg/h. Adding a soluble salt to the first process stream produces a solution of 4.76 % (w/w) of salt, and the salt content in the mixed stream is 0.62 % (w/w). What is the ratio of the two streams?

Solution

Step I

Reading and understanding. First, when the salt is added to the first stream, the resulting salt concentration is 4.76 % (w/w), but then when both streams are mixed, the salt concentration becomes significantly reduced. If both streams have the same flow rate, then we would expect the salt

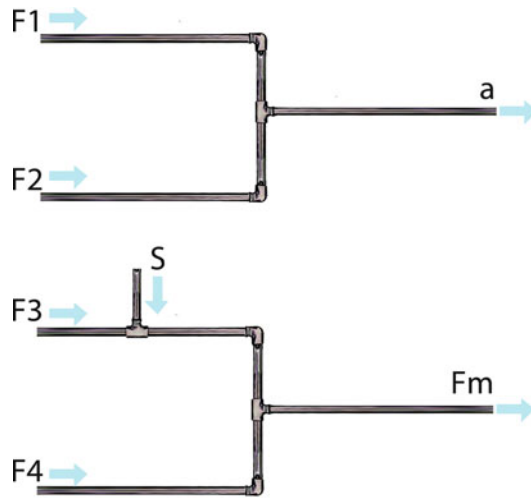

Fig. 5.12 Schematic representation of the mixture of two streams, firstly without adding salt and then with the addition of salt

concentration in the mixed stream to be cut in half. Given that the salt concentration is reduced several times (more than 7), we expect the second stream to have a much higher flow rate than the first one (on the order of 7 times).

Step II

Graphical representation and variable definition and codification.

According to Fig. 5.12, the variables are as follows:

F_1 : flow rate of stream (1 kg/h)

F_2: flow rate of stream (2 kg/h)

S: kilograms per hour of salt added to stream 1

F_M: flow rate of mixed stream with salt added kg/h

We assume that the flow rate of the mixed stream (with salt) is a known value (a kg/h).

Step III

Mathematical formulation including all available data. We have four unknowns (F_1, F_2, S, and F_M), and we can formulate four equations, as shown in the mathematical formulation, thus DF = 0. But do not forget that the problem is not to solve the equations for all unknowns, just to calculate the ratio of the streams 2 to 1, i.e., F_2/F_1.

Before the salt is added (Fig. 5.12), the addition of the flow rate of streams 1 and 2 must be equal to the flow rate of the mixed stream, thus:

$$F_1 + F_2 = a. \tag{5.39}$$

In the same way, when the salt is added (Fig. 5.12b) we can write

$$F_1 + F_2 + S = F_M. \tag{5.40}$$

When salt is added to stream 1, the salt concentration is 4.76 %, and thus

$$4.76 = 100\frac{S}{F_1 + S}. \tag{5.41}$$

And the salt concentration in the mixed stream is 0.62 %; therefore

$$0.62 = 100\frac{S}{F_M}. \tag{5.42}$$

Step IV

Resolution, results, analysis, and discussion. Performing some algebra with (5.39–5.42), we obtain

$$\frac{F_2}{F_1} = 7.$$

One approach is to work with (5.41) to obtain

$$S = \frac{1}{20}F_1.$$

Then with this result, working on (5.42), we get

$$F_M = 8.0645 \times F_1.$$

Substituting these two results into (5.40) we obtain

$$\frac{F_2}{F_1} = 7.$$

As we expected and almost correctly predicted, the flow rate of stream 2 was approximately 7 times the flow rate of stream 1. Normally, this is a really tough problem for students, but with the correct reading and understanding, and with the help of simple but vital graphical representation, the problem solution looks straightforward.

14. Calcium chloride solution [6⁺]. A student must prepare 15 L of calcium chloride with 6.0 % w/v concentration. She has 7.0 L of a 4.0 % w/v solution and 20 L of a 10 % w/v solution. The student carries out some calculations and discovers, to her surprise, that the solution of 4.0 % w/v is limiting. Then she decides to use the entire solution of 4.0 % w/v, an unspecified amount of 10 % w/v solution, and distilled water. (a) How much distilled water was used? (b) How much of the 10 % w/v solution was used?

Solution

Step I

Reading and understanding. First, we need to understand what is really meant by "the 4.0 % w/v solution is limiting." First, we want to know the lowest concentration that we can get in a 15 L solution if we use all of the 4.0 % w/v solution. A quick calculation shows us that if we use the entire solution of 4.0 % w/v (7.0 L) and 8.0 L of the 10 % w/v solution (to complete the required 15 L), then the final concentration will greater than 6.0 % w/v. This is why the student proposes to use some distilled water, to dilute the 10 % w/v solution, and then combine it with the 7.0 L of the 4.0 % w/v solution to get the required 6.0 % w/v.

Step II

Graphical representation and variable definition and codification. In this case, it is not necessary to use a graphical representation.

x: amount (L) of 10 % w/v solution used

y: amount (L) of distilled water used

Step III

Mathematical formulation including all available data.

We have two variables, and we can write two equations, one that takes into account that the final volume should be 15 L and the second one that complies with the required 6.0 % w/v solution. Thus, DF = 0. First, the total amount of solution should be 15 L, and taking into account that all 4.0 % w/v solution will be utilized (7.0 L)

$$x + y + 7 = 15 \tag{5.43}$$

And the final concentration in the 15 L solution should be 6.0 %. The mass of calcium chloride obtained from the 4.0 % w/v solution plus the mass of the calcium chloride obtained from the 10 % w/v solution divided by the total volume should be 6.0 % w/v; therefore

$$\frac{0.04 \times 7 + 0.1 \times x}{15} = 0.06 \tag{5.44a}$$

Step IV

Resolution, results, analysis, and discussion.

Then, from (5.44), $x = 6.2$ L of 10 % w/v solution, and substituting into (5.43) we obtain $y = 1.8$ L of distilled water.

As expected, some distilled water was needed to reach the 6.0 % w/v in the final solution of 15 L. As stated at the beginning, although we used the entire 4.0 % w/v solution, the lowest concentration in the 15 L solution was higher than 6.0 % w/v (7.2 %). Please check this result!

5.3.3 Percentages

15. **Concentrated product [10]**. A company offers two types of varnish, one with 25 % water and another, substantially dryer, with 4 % water. The product with the higher moisture content (25 % water) sells for $10 per 100 kg if you buy it at the company. The "dry" product (4 % water) sells for $12.5 per 100 kg if you buy it at the company. If the transportation cost to your company is $0.80 per 100 kg, which of the two products will it be more cost-effective to buy?

Solution

Step I

Reading and understanding. First, we must understand that we should be willing to pay more for the dry product. For example, if we buy 100 kg of the moist product (25 % water), we are paying for 75 kg of varnish, while in the case of the dry product, we are paying for 96 kg of varnish. In addition, the cost of transportation per kilogram of varnish is lower for the dry product. Therefore, we should be willing to pay more for the dry product, the one with a higher varnish concentration.

Step II

Graphical representation and variable definition and codification. Although in most cases it is useful to have a graphical representation, in this case it is unnecessary.

Y: kilogram of product with 25 % water to obtain 100 kg of varnish

Z: kilogram of product with 4 % water to obtain 100 kg of varnish

X: maximum price to be paid for dry product (4 % water)

Step III

Mathematical formulation of problem. We will calculate the most we should pay for the dry product and compare that to the price of $12.5 per 100 kg of product. To do this, we calculate the cost of 100 kg of pure varnish with 25 % water delivered at your factory and compare it to the cost of 100 kg of pure varnish with 4 % water at a price of $*X* per 100 kg, also delivered at your factory.

Product with 25 % water. If we want to buy 100 kg of the pure varnish, then we must buy *Y* kg of the 25 % water (75 % varnish) and then pay for transportation. Therefore, we can write the following equation:

$Y \times 0.75 = 100$. Then $Y = 133.34$ kg of product with 25 % water will give 100 kg of pure varnish.

The cost of this 133.34 kg with 25 % water is the cost of the product plus the cost of transportation, as seen in the following equation:

$$133.34 \times \frac{10}{100} + 133.34 \times \frac{0.8}{100} = \text{US\$14.40}.$$

Product with 4 % water. First, we estimate how many kilograms of product (*Z* kg) must be purchased to obtain 100 kg of pure varnish and then calculate the total cost, including transportation:

$Z \times 0.96 = 100$. Then $Z = 104.167$ kg of the product with 4 % water will yield 100 kg of pure varnish.

If we are willing to pay $*X* per 100 kg of product with 4 % water, then the cost of this 104.167 kg of product with 4 % water, including transportation, should be equal to $14.40 (and no more!):

$104.167 \times \frac{X}{100} + 104.167 \times \frac{0.8}{100} = \text{US\$14.4}$, then $\boldsymbol{X} = \$13.00$.

Since the dry product price is $12.50, we should buy it. Moreover, we could even pay up to $13.00, and it would still be more cost effective.

Step IV

Analysis and discussion. As we predicted, we can pay more for the product with the higher concentration of varnish. First of all, every kilogram of product has more varnish, and also transportation per kilogram of the dry varnish is cheaper because it carries less water per kilogram of product.

5.3.4 Ages

16. 50 years old [5]. Today my age is triple the age of my oldest son. Ten years ago, my age was seven times his age. Within how many years I will be 50 years old?

Solution

Step I

Reading and understanding. This is a middle school problem, but the idea is to show you how the method works with this easy problem and with intricate ones.

Step II

Graphical representation. Here there is a nice way to solve this kind of problem. The idea is to construct, in general, a table like the following one:

Person	Past	Present	Future
Person 1			
Person 2			
...			
Person N			

Variable definition and codification. Here we have the option to define X as my present age and Y as the present age of my son, or directly assume that at present my age is triple his age ($3X$ and X, respectively). Let us say that X is my present age and Y is his present age. Then the table will look like this:

Person	Past	Present	Future?
Me	$X - 10$	X	50
My oldest son	$Y - 10$	Y	

Step III

Mathematical formulation including all available data. We have two unknowns, and with the provided information, we can formulate two equations, one for the present and a second one for the past. Thus, DF = 0. Looking at the table and the problem statement we can write the following equations.

First, currently my age is triple the age of my son, therefore

$$X = 3 \times Y. \tag{5.44b}$$

Ten years ago, my age was seven times his age:

$$X - 10 = 7(Y - 10). \tag{5.45}$$

Step IV

Resolution, results, analysis, and discussion. Solving (5.44) and (5.45), we obtain $X = 45$ and $Y = 15$; therefore, within 5 years I will be 50 years old.

17. Marilyn's age [10$^+$]. N years ago, Marilyn's age was a fifth of her present age. Within M years her age will be double her present age. If Marilyn's age in the future (M years later) is ten times her age N years ago, what is Marilyn's present age?

Solution

We will skip step I.

Step II

Graphical representation and variable definition and codification.

As proposed in the previous problem (50 years old), we will build a table. In this case:

X: Marilyn's present age

Person	Past	Present	Future
Marilyn	$X - N$	X	$X + M$

Step III

Mathematical formulation of problem. We have three unknowns, and so it seems we can formulate three equations. If so, then DF = 0.

First, N years ago Marilyn's age was one-fifth of her present age:

$$(X - N) = \frac{X}{5}. \tag{5.46}$$

Second, within M years, Marilyn's age will be double her present age:

$$(X + M) = 2 \times X. \tag{5.47}$$

Finally, Marilyn's age in the future (M years later) will be ten times her age N years ago:

$$(X + M) = 10(X - N). \tag{5.48}$$

Step IV

Resolution, results, analysis, and discussion. Dividing (5.47) by (5.46) we obtain

$$\frac{X + M}{X - N} = 10. \tag{5.49}$$

This is identical to (5.48). Unfortunately, there is not enough information to know Marilyn's age. In fact, with the data provided in the problem statement, it is possible to write just two independent equations, not three. Equation (5.48) is not independent of (5.46) and (5.47). Therefore, DF = 1. This means that assigning a value to one unknown we can solve the problem. For example, if $M = 10$, then from (5.47) $X = 10$, and from (5.48), $N = 8$. You can verify these results with the problem statement.

5.3.5 Miscellaneous

18. Palindrome* [5]. The odometer on the family car shows 17,971 miles. The driver, driving on a highway in Columbus, Ohio, realizes that this number is palindromic. Curious, he thought, "It will be a long time before this happens again." But 2 h later, the odometer again shows a palindromic number. How fast did the car go during those 2 h?

*A palindrome is a phrase or number that reads identically from left to right and right to left. There are many famous palindromic phrases, e.g., "A man, a plan, a canal—Panama." In the case of numbers, there are not only palindromic numbers (131) but also palindromic equations, such as $11 \times 11 = 121$.

Solution

The problem is interesting, but because it is a somewhat different problem, we will not follow strictly the detailed method that we have employed in every problem so far.

Step I

Reading and understanding. Given that 2 h later the odometer on the family car showed a palindromic number, the distance that the car advanced will be approximately in the range of 100–130 miles if we assume the car travels at a rate of 50–65 miles/h on the highway. Therefore, we can estimate that the number that we are searching for is greater than 18,071 miles and less than 18,101 miles. In addition, to be palindromic, the last number should be 1 (because the first is 1). With the same reasoning, the preceding number should be 8 (because the second number is 8), and then the number is 18,081. The number cannot be 18,181 (also palindromic) because according to the speed limit, the number on the odometer 2 h later should not be much higher than 18,101.

Step II

Results, analysis, and discussion. If the searched for palindromic number is 18,081, then the speed of the car should be the distance traveled by the car divided by the time it took to go that distance. Therefore,

$$\text{Distance} = 18{,}081 - 17{,}971 = 110 \text{ miles and time} = 2.0 \text{ h}$$
$$\text{Then, } v = \frac{110}{2} = 55 \text{ miles/h}$$

19. The Lucas problem [10⁺]. This problem was invented by Edouard Lucas, a nineteenth-century French mathematician. Every day at noon a ship leaves Le Havre (a port in France) to the port of New York, and at the same time another ship—every day at noon—leaves New York for Le Havre (assume the same time zone for both ports). The trip takes seven days and seven nights. How many ships will the ship that left Le Havre for New York encounter on its way?

Solution

Step I

Reading and understanding.

Again, as in the previous miscellaneous problem, this is an interesting challenge. Experience has shown us that almost everybody gets this one wrong, even bright graduate students and professors. As in the shoe problem in the introduction to this chapter, everybody is tempted to solve it mentally and fails. This problem really needs a graphical representation of the ship's journey. Then the solution is straightforward.

Step II

Graphical representation.

Step III

Results, analysis, and discussion. Looking at the diagram (Fig. 5.13), we see clearly that the ship encountered 15 ships on its travel. Why? For example, when a ship starting out from Le Havre completed 6 days of travel (noon of the sixth day), another ship was setting out from New York for Le Havre at the same time (Fig. 5.13). These two ships meet at midnight (x), from which you can infer that this ship encounters other ships not only at noon everyday but also at midnight. In addition, it will meet a ship from New York when the journey starts and also when it is ending its journey. In Fig. 5.13, you can count the encounters, which total 15. Surprised?

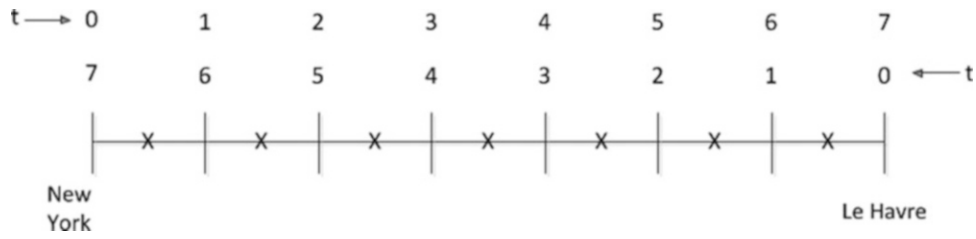

Fig. 5.13 Schematic representation of ships leaving Le Havre (a port in France) to the port of New York and simultaneously ships leaving New York to the port of Le Havre

As commented in the "Reading and Understanding" section of this problem, the key here is a graphical representation. For most of us, it is very hard to imagine this situation, and normally people underestimate the number of encounters.

5.4 Proposed Problems

VELOCITY

1. **Increasing the speed [7⁺].** A automobile travels 20 km for a certain amount of time. If the automobile needs to cut the travel time by 1 h, the speed of the automobile should be increased by 10 km/h. What is the new speed?
   A: V = 20 km/h
2. **Sebastian Coe [6].** In the early 1980s the world record in a 1,500 m race was 3 min 32 s. In his workouts, Sebastian Coe (considered by many to be one of the greatest athletes of the twentieth century) ran against his archrival Steve Ovett. In his last workout with Ovett, Ovett started the race 44.1 m over Coe, but they crossed the finish line together. If Ovett ran at a speed equivalent to 3 min 37.4 s for 1,500 m, what was Coe's time for the 1,500 m?
   A: 3 min 31 s
3. **Travel to Santiago [7].** You are traveling from Viña del Mar to Santiago (120 km) on the same day as your cousin John. Your cousin departs a certain amount of time earlier than you, and that time is equal to 10 % of your travel time on your trip to Santiago. If your average speed is 20 % greater than that of your cousin John and his travel time to Santiago is 1 h and 35 min, (a) what is the speed of each driver? (b) what was your travel time from Viña del Mar to Santiago? and (c) who arrives first in Santiago?
   A: (a) 75.8 km/h (John) and 91.0 km/h (you). (b) 1.319 h or 1 h 19 min and 9 s. (c) As your travel time to Santiago is about 79 min and your cousin John departed just 7 min and 54 s before you, this is not enough time for your cousin to beat you to Santiago (remember his travel time was 1 h and 35 min), so you arrive first.
4. **Two trains [7].** A train leaves New York on a nonstop trip to Washington, DC, at 60 km/h. At the same time, another train leaves Washington, DC, heading for New York at 40 km/h. How far are both trains half-hour before crossing each other?
   A: 50 km (30 + 20)
5. **A restless fly [8].** Two cyclists begin biking simultaneously, each in the direction of the other. One sets out from Columbus, Ohio, the other from Cleveland, Ohio. When the two cyclists are 80 km away (from each other), a fly lands on the shoulder of one of the cyclists, flies to the other cyclist, and then flies back. The restless fly goes back and forth until the cyclists cross paths, then

lands on the nose of one of the cyclists. If the fly is traveling at a speed of 40 km/h and the cyclists are riding at 20 km/h, how many kilometers did the fly?

A: The problem is simpler than it appears. The cyclists' time to meet up was 2 h. Therefore, the fly traveled 80 km in 2 h.

6. **Harvest time [6$^+$]**. A farm must deliver her harvest to a packinghouse. The farm manager, in Greenville, has decided that the trucks must arrive in town at exactly 11 a.m. Traveling at 30 km/h, the trucks would reach the city at 10 a.m., i.e., 1 h early; and if they traveled at 20 km/h, they would arrive at noon, i.e., 1 h late. (a) How far is Greenville from the city? (b) How fast must the trucks travel to arrive exactly at 11 a.m.?

   A: (a) The Greenville farm is 120 km from the city. (b) The truck should travel at a speed of 24 km/h.

7. **Vacations [9]**. Two students are traveling by train from their home city to a summer house on the beach. One of the students says, "I have noticed that every 5 min, we pass an oncoming train. How many trains arrive in the home city in 1 h if trains travel at the same speed in both directions?" "12 trains, obviously," says the other student, because 60 divided by 5 equals 12. The first student disagrees. What do you think? **Hint**: Remember the Lucas problem.

   A: 6 trains per hour (in some ways, this problem is similar to the Lucas problem).

8. **Can John save time? [7]**. John is going home from Valparaiso, Indiana. He ride halfway. John rides 15 times faster than he walks. The second half of his trip John traveled by oxcart. He can walk twice as fast as that. (a) Would he have saved time if he had walked the entire way? (b) How much time would he have saved?

   A: (a) Yes, he would have saved time. (b) Assuming that the walking time is t, riding halfway and in the oxcart the other half would have a travel time of $(31/30) \times t$.

9. **Average speed [8$^+$]**. A horse runs half of its route with no load at **A** km/h, and the other half it does with a heavy load at **B** km/h (**A** $>$ **B**). What is the horse's average speed in kilometers per hour?

   A: $2AB/(A + B)$

10. **More or less? [9$^+$]**. A rower travels 5 miles with the current and 5 miles against the current. On another occasion, the rower travels 10 miles in a lake where there is no wind. (a) Which one is longer? (b) Explain your answer in (a).

    A: (a) It takes longer in the first situation. (b) The rower requires less time rowing with the current. A good way to visualize this situation is to analyze an extreme case, such that the current is very strong. It The first leg will take a short time, but the second leg will take a very long time.

11. **A swimmer [10]**. A boat travels with the current. A swimmer jumps out and swims against the current for a certain amount of time then swims back to the boat. Did the swimmer swim for a longer time against the current or when he was going back to the boat? (Assume that the swimmer's strength does not depend one whether he is swimming with or against the current.)

    A: Both times were the same. The current carries the man and the boat downstream at the same speed. The distance between the swimmer and the boat is unaffected.

12. **How keen are you? [9]**. Two boats are in front of each bank of a river. One boat leaves to meet the other boat, which is leaving the other shore. Both boats are moving at a constant speed. They meet 400 m from the first shore. Each boat moves to the other side and returns to shore without stopping and crosses at 320 m off the second shore. How wide is the river and what is the relationship between the speeds of the two boats?

    A: 1,000 m; the speed ratio is 2:3

13. **Heavy truck [6$^+$]**. A heavily loaded truck sets off on a long trip from point A. When the truck has gone 200 km (now at point B), a motorcycle moving at a speed that is 6 times faster than the

loaded truck's speed begins traveling toward it. How far from point B does the truck get until the motorcycle meets up with it?
A: 40 km

14. **Fast, but with caution [3]**. A bus leaves New York headed for Boston at noon. An hour later, a cyclist sets out from Boston to New York, obviously moving much slower than the bus. Upon crossing, which of the two is farther from New York?
A: Neither! They intersect, so they are the same distance from New York!
15. **Traveling to Cincinnati [9$^+$]**. Rodrigo and Elizabeth are traveling by car from Chicago to Cincinnati, and during the trip, Rodrigo notes that the telephone poles on the road were equally spaced. Elizabeth, a little bored with the long trip, times 1 min and tells Rodrigo the number of telephone poles she counted during that time. Rodrigo tells his traveling companion that if that number is multiplied by 6, it gives the car's speed in kilometers per hour. Considering that when Elizabeth started timing they were exactly between two telephone poles, and in the same way, when she finished counting the telephone poles, they were exactly between two of them, how far apart are the telephone poles?
A: 100 m
16. **Train encounter [7$^+$]**. Two freight trains of the same length travel at the same speed, 90 km/h. Upon their encounter, they cross each other completely in 12 s. How long is each train?
A: 300 m
17. **Changing tires [5]**. As always, I kept a spare tire in my car. During the year, I drove **A** km and I was continuously rotating the tires (including the spare), so that at the end of the year, each of the tires had traveled the same number of kilometers. How many kilometers did each tire travel?
A: 4/5*A* km
18. **Going to San Francisco [8$^+$]**. An airplane is flying from Portland, Oregon, to San Francisco, California, and then back to Portland, maintaining constant speed on the outbound and return flights (no wind for or against). The next day, it makes the same flight at the same speed, but this time, there is a slight tailwind from Portland to San Francisco. That is, the plane flies downwind from Portland to San Francisco but then against the wind from San Francisco to Portland. On what day is the flight shorter, the first or the second? Explain!
A: The first (similar analysis to problem 10 Sect. 5.4).
19. **Boat trip [6]**. A man takes his motorboat and travels downstream at a constant speed. Because he is going with the current, it takes only 6 min to go 3 km. When he returns, against the current, it takes 12 min to cover 3 km. How long would it have taken to travel 3 miles in still water?
A: 8 min
20. **Heavy traffic [4$^+$]**. On a normal working day, I travel from home to office at an average speed of 60 km/h (round trip). Today, as usual, I drove at a speed of 60 km/h to work. Unfortunately, on the return trip, I got stuck in a traffic jam, and so my average speed was only 40 km/h. What was my average speed on the round trip?
A: 48 km/h
21. **Long trains [7$^+$]**. Two trains of equal length travel by two parallel rails in the same direction. From the moment the fastest train, at a speed of 100 km/h, overtakes the slower train (60 km/h), it takes 3 min to overtake completely. (a) How long is each train? (b) If they travel in opposite directions, how much time will it take them to completely pass each other?
A: (a) Each train is 1 km long. (b) 0.75 min
22. **Long journey [4$^+$]**. Let's take a long trip and figure out if having four new tires plus a spare is enough. I was told that each tire can be used for up to 35,000 km, and we estimate that our

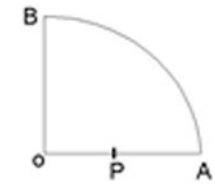

Fig. 5.14 Routes options for an individual who is at point B to get to point P

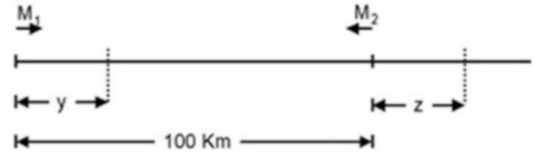

Fig. 5.15 Schematic representation for mobiles M1 and M2

journey, in total, will be 42,000 km. (a) Is it sufficient to have four tires plus the spare? If so, how many kilometers could I travel?

A: (a) Yes, it is sufficient. (b) 43,750 km

23. **Santiago to Talca [5]**. You have decided to take a trip from Santiago to Talca to participate in an important meeting. While you are planning your trip, you see that the distance between these two cities is 300 km. You start early in the morning, but unfortunately, after an hour, your car starts having problems and you must stop right away. This problem delays you for 30 min and forces you to continue the journey at a much slower (40 % slower) speed. If you arrive 1.5 h late to Talca, what was the speed at the beginning of the trip?

    A: $V = 120$ km/h (maximum speed permitted on Chilean highways)

24. **By bus or on foot [5$^+$]**. An individual at point B (Fig. 5.14 a quadrant) has two options to get to point P. The first is to jog from BO to OP. The second is to take a bus from point B to point A and then walk the remaining stretch AP. Knowing that BA is 400 m, the bus speed is 48 km/h, individual walking speed is 6 km/h, individual jogging speed is 18 km/h, and OP:PA = 2:3, what is the best alternative for this person?

    A: The first (1 min 11 s: the second alternative is 2 min 2 s).

25. **Boat trip [10$^+$]**. You travel by boat **a** km upstream and then return to the starting point. You know that you can go **b** km upstream, and in the same amount of time, you can go **c** km downstream. If the round trip takes you **e** hours, what is the speed of the current? **Hint**: start writing two equations for the first situation with both speeds as unknowns (boat and current), and also two times as unknowns, upstream and downstream times, but the total time should be **e** hours. Then you can add two more equations with the supplementary information.

    A: The current speed is $a\frac{(c+b)(c-b)}{bce}$

26. **Two automobiles [10$^+$]**. Two automobiles travel to meet each other, as shown in Fig. 5.15. These automobiles will pass each other at **y** km. With respect to the original position M1, in the case where automobile M1's speed is **n** times higher that of automobile M2. If automobile M2 travels in the same direction as automobile M1, the latter will reach M2 at point **z**. Knowing that the ratio y/z is 1:4: (a) What is the value of **n**? (b) What is the value of **z** and **y** if the distance between the two automobiles is 100 km?.

    Hint: When formulating the equations, you will notice that there are more variables than equations, but be aware that it is not necessary to obtain the value of all variables, just three of them. Of course, if you have too few equations, you cannot calculate all the variables.

    A: (a) $\mathbf{n} = 1.43$, (b) $\mathbf{z} = 235$ km and $\mathbf{y} = 58.8$ km

WATCHES

27. **Overlapping [3⁺]**. At what time exactly after 6 p.m. will the minute and hour hands overlap?
    A: 6 h 32 (8/11) min ~6 h 32 min 44 s.
28. **Right angle [3⁺]**. At what time after 6 p.m. will the minute and hour hands form a right angle (90°)?
    A: 6 h 16 (4/11) min ~6 h 16 min 22 s.
29. **Extended angle [3⁺]**. At what time after 12 am will the minute and hour hands form an extended angle (180°)?
    A: 12 h 32 (8/11) min ~12 h 32 min 44 s.
30. **Laboratory experiments [6]**. The next laboratory on heat transfer begins when the hour hand is between 8 and 9 and the minute hand between 1 and 2. The laboratory will end when the positions have been reversed. How did the laboratory experiments take?
    A: The lab began at 8 h 8.4 min and ended at 13 h 40.7 min, and so the lab experiments took 5 h 32.7 min.
31. **Overlapping [3⁺]**. At what time after 3 p.m. do the minute and hour hands overlap?
    A: 3 h 16 min 22 s
32. **Right angle [3⁺]**. At what time after 12 am do the minute and hour hands form a right angle?
    A: 12 h 16 min 22 s
33. **Watches [9⁺]**. My wristwatch is fast by 1 s/h and Ricardo's wristwatch is 2 s/h slow. Right now, both show the same time, and there is no difference between a.m. and p.m. on either watch. (a) When will both watches show the same time again? (b) After how much time will my watch be a full day ahead and indicate the exact same time? Solve with equations!
    A: (a) After 600 days, (b) 3,600 days (almost 10 years).
34. **Long meeting [6]**. A meeting starts between 10 and 11 a.m. and ends between 1 and 2 p.m. The minute and hour hands have swapped places. When does the meeting begin and end?
    A: The meeting starts at 10 h 9 33/143 min a.m. and ends at 1 h 50 110/143 min p.m.

WORK

35. **The wall [6]**. A worker can build a wall in **a** days and another worker can do it in **b** days. (a) How long will take to complete the job if they work together? (b) How many days after the first worker should the second worker start to complete the job in **c** days?
    A: (a) $\frac{1}{\left(\frac{1}{a}+\frac{1}{b}\right)}$ days; (b) $\frac{c(a+b)-ab}{a}$ days
36. **Apprentice [5]**. A highly skilled worker is able to paint a wall in 2 h, a normal worker in 4 h, and an apprentice in 8 h. (a) How long will it take for the three workers to paint the wall if they work together? (b) What percentage of the wall did each one paint?
    A: (a) 8/7 h (1 h 8 min 34 s) (b) Skilled worker 57.1 %, normal worker 28.6 %, and apprentice 14.3 %
37. **Professional [4⁺]**. A professional worker takes 20 h to build a wall, and an apprentice takes 40 h to complete the same job. Due to the high cost of a professional worker, the foreman decides to start the work with the apprentice; after 10 h he hires the professional to finish the job with him. How long did it take to finish the wall?
    A: 20 h
38. **Builder [6]**. A builder promises to build a set of houses in a certain number of days, which will require 50 workers. From the ninth day, he has a smaller crew and has only three-fifths of his workers; this forced a delay in delivery of 10 days. In how many days did they finish building the houses?
    A: 33 days.

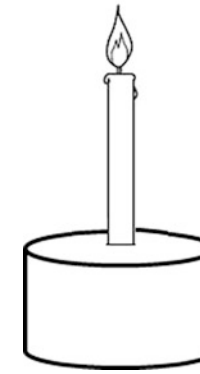

Fig. 5.16 Shape of the candle as proposed by your friend

39. **Harvesters [8]**. A group of harvesters should reap 2 meadows, where the large one is double the size of the smaller one. On the first half-day, the entire staff worked in the large meadow. Then half the group worked in the largest one, and the other half in the smaller one at the end of the day. Finally, in the small meadow, a certain section was not harvested, and the next day, one harvester finishes the harvesting. How many harvesters were in the group?
A: 8
40. **Extra workers [10^+]**. A group of **a** workers is hired to do some work. After **m** days they do the **n**th part of the job. How many workers would have to be hired to complete the work in **b** days?
A: $\frac{a}{b}(mn - m - b)$
41. **Bridge [7^+]**. In the construction of a bridge, an unknown number of workers work for 20 days. To finish the job in 10 days, 10 workers join them. In addition, it is known that the same job can be done by 20 workers in 20 days. How many workers are there?
A: 10 workers
42. **House [8^+]**. 8 workers build a house in 210 days working in 2 shifts of 4 workers each. The first shift works from 8 a.m. to 4 p.m. and the second shift from 4 to 10 p.m. If a worker's performance decreases by 20 % after 10 h, how many workers are required to complete a house in no more than 180 days, with a single shift of 8–20 h?
A: 7 workers (6.1)

CANDLES

43. **Candles of different diameters [9^+]**. Two candles of different diameters have lengths of L and $L/2$, respectively. The taller candle (smaller diameter) is consumed in 3 h, while the smaller candle (larger diameter) is consumed in 4 h. At 9 p.m., your best friend comes over and tells you that the date you have with two other girls is at 11:30 p.m., but unfortunately neither has a watch. You tell your friend, "Don't worry. If we light the candles right now, when both candles are the same height, that will be a good time to go meet them." Are you right?
A: Yes, you're right, the candles have the same height 2.4 h after being lit, which will make it 11:24 p.m.
44. **A single candle [8^+]**. Days later, using the candles from the previous problem (Candles of different diameters), your friend suggests making a candle as depicted in Fig. 5.16, using part of the candle with height L (smaller diameter) and part of the larger diameter candle ($L/2$), so that the time to burn this composite candle is exactly 3.5 h. So your friend asks you, "Why not use half of each candle?" (a) Does that seem like a reasonable suggestion, or would you come up with a simpler solution to get a candle that burned out after 3.5 h?

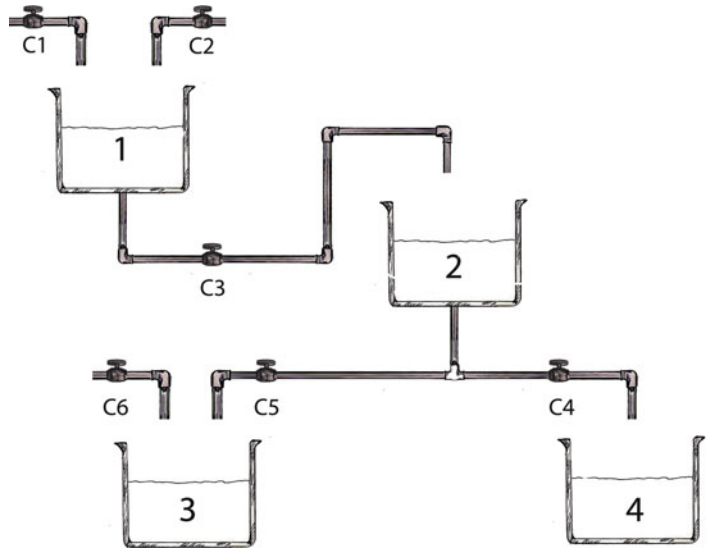

Fig. 5.17 Bioreactor system

A: (a) Yes, but a simpler solution would be to use only the large diameter candle, leaving it with a height of 7/8*L*.

45. **Two candles [8⁺]**. Two candles have different lengths and diameters. The taller one lasts 3.5 h and the shorter one 5 h. After 2 h of being lit, the candles are the same height. What is the ratio of the lengths of the two candles?
A: 7:5

FLOW

46. **Tank [6]**. A tank has two valves. To fill the tank, each valve operating individually takes **a** and **b** hours. How soon will the tank be filled if both valves operate simultaneously?
A: $\frac{1}{\left(\frac{1}{a}+\frac{1}{b}\right)}$ h

47. **Three valves [6]**. A tank has three valves. To fill the tank, each valve operating individually takes **a**, **b**, and **c** hours. How soon will the tank be filled if the three valves operate simultaneously?
A: $\frac{1}{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)}$ h

48. **Bioreactor system [8]**. Figure 5.17 shows a bioreactor system. Initially, all bioreactors are empty and the valves closed. Then, valves 1 and 2 are opened simultaneously. How many minutes later into the operation should valves 3–6 be opened so that all bioreactors are full after 100 min?
Data: The flow per valve and the volume of each bioreactor are as follows**:** $C_1 = 4$ L/min; $C_2 = 6$ L/min; $C_3 = 10$ L/min; $C_4 = 5.5$ L/min; $C_5 = 4$ L/min; $C_6 = 20$ L/min; $V_1 = 100$ L; $V_2 = 220$ L; $V_3 = 1{,}240$ L; $V_4 = 440$ L.
A: Valve 3 opens 10 min later, valve 4 opens 20 min later, valve 5 opens 40 min later, and valve 6 opens 50 min later

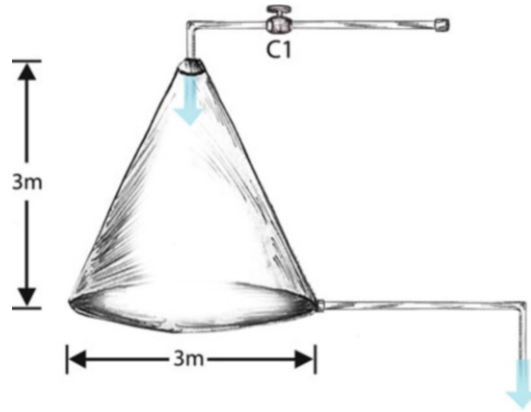

Fig. 5.18 Conical tank

49. **Conical tank [5]**. A conical tank (Fig. 5.18) is fed with a valve that has a flow rate of C_1 L/h. The bottom of the tank has a drain that allows 600 drps/min to flow through it, and 5 drops have a volume of 2 cm^3. If the tank is filled in 50 h: (a) What is the flow rate of the valve? (b) How long after the operation begins will the liquid height be 40 % of the total height?
A: (a) $C_1 = 155.8$ L/h. (b) 39.2 h
50. **Cylindrical tank [10⁺]**. A cylindrical tank with a radius of 5.0 m and height of 1.5 m is fed by a valve whose flow rate is 2.0 m^3/h. If you later add a second valve and it is observed that the linear velocity of the liquid height has increased by 20 %, then: (a) What is the filling time with valve 1? (b) What is the filling time with 2 valves?, (c) What is the flow rate of valve 2?. **Hint:** If the second valve increases the linear speed of the liquid level by 20 %, then valve 2 has one-fifth the flow rate of valve 1.
A: (a) 1.8×10^1 h, (b) 1.5×10^1 h, (c) 0.4 m^3/h
51. **Spherical tank [8]**. A spherical tank ($r = 2$ m), initially empty, is fed by a valve that delivers a flow rate of 5 m^3/h. (a) What is the liquid height after 3.35 h? (b) What is the liquid height after 4 h?
A: (a) 2 m (b) 2.26 m

NUMBERS

52. **Two-digit number [7]**. First, we switch the numbers of a two-digit number. Then we perform subtraction (the first minus the second number) to obtain 9. If we add those numbers, we obtain 55. What is the number? **Hint**: A two-digit number says XY can be defined as $10 \times X + Y$, e.g., 54 can be written as $5 \times 10 + 4 = 54$.
A: 32
53. **How many times? [4]**. Given two numbers, if we subtract half of the smaller number from each number, the result is that the larger number is three times the smaller number. What is the ratio of the original numbers?
A: 2

AGES

54. **John and his cousins [8⁺]**. John has two little cousins. If the age of the oldest cousin squared plus the age of the younger cousin add up to 234, how old are John's cousins? **Hint:** Review solved problem 7 Sect. 5.3.1 (Diophantine equations).
    A: 15 and 9 years old
55. **John and Mary [3]**. John is 5 years older than Mary. If 15 years ago John was twice Mary's age, how old are John and Mary?
    A: John is 25, Mary is 20
56. **Natalie Portman [6]**. A reporter indiscreetly asks Natalie Portman her age. Trying to avoid the question, she answers that her age 2 years ago was half of the sum of her age and the age of her close friend Scarlett Johansson. In addition, 24 years ago, she was twice the age of Scarlett. Undeterred, Natalie tells the reporter: (a) "So, in how many years will I be 40? (b) Also, how old is Scarlett Johansson?"
    A: (a) 8 years and (b) 28 years

PERCENTAGES, PROPORTIONS, AND MIXTURES

57. **Wine [5]**. A jar of undiluted wine is mixed with another jar that has three times the first jar's volume, but the second one is diluted. If the second jar contains two parts water and one part wine, what is the proportion of wine in the final mix?
    A. Half.
58. **Milk [7]**. Milk sold in stores contains **a%** protein, **b%** fat, and **c%** carbohydrates, with the rest being mostly water. In a laboratory experiment, milk is concentrated by evaporating the water, so **d%** of the total mass is evaporated. Due to the high temperatures used in the evaporation process, **e%** of the proteins precipitate (denatured). What is the concentration of protein, fat, and carbohydrates following the evaporation process?
    A: $\%\text{protein} = a\left(\frac{1-\frac{e}{100}}{1-\frac{d}{100}}\right)$; $\%\text{fat} = b\left(\frac{100}{100-d}\right)$; $\%\text{carbohydrates} = c\left(\frac{100}{100-d}\right)$
59. **Big sale [7⁺]**. At a sale, we sell two items for $20 each. On the first article we earn 25 % profit and in the second article, we lose 20 %. (a) Can you tell whether the total sale produces a net loss or profit? (b) How much is earned or lost?
    A: (a) There is a net loss. (b) The first item cost us $16, the second was $25, totaling $41, so we lost $1.
60. **Mixed class [8⁺]**. In a co-ed class, a group of 80 men was selected, and the total number of men from this group who failed the test represented 15 % of the total number of men in the course. The nonselected men represented 80 % of the total women. If 40 % of these women failed and 90 of them passed, while 75 % of nonselected men were approved, then answer the following questions: (a) How many men failed? (b) How many women failed? (c) What percentage of men is in the co-ed class (d) What percentage of the total number of people who failed are women?
    A: (a) 60, (b) 60, (c) 57.14 %, (d) 50 %
61. **Insured vehicles [8]**. Of all city vehicles, **a%** have insurance. Due to a wave of accidents in recent months, the mayor estimated that of the total vehicles, **b%** had accidents in that period. If you also know that all vehicles that are not insured were in accidents (collision), then: (a) What percentage of insured cars were in an accident? (b) What percentage of the cars that were in accidents was insured? (**a** + **b** > 100).
    A: (a) $100\frac{(a+b-100)}{a}$, (b) $100\frac{(a+b-100)}{b}$

62. **Cone [7⁺]**. Increasing the radius of a cone by **a%**, by how much should the height be reduced to maintain the same volume?
A: $100 \times \frac{200a+a^2}{(100+a)^2}\%$
63. **Iron rod [8⁺]**. When an iron rod is heated, the length L is expanded by a%. By what percentage should it be contracted to reach the original length?
A: $\frac{100a}{100+a}\%$
64. **Fuel consumption [9]**. In a small town, there are 30,000 cars and 1,500 trucks. Fuel efficiencies are 8 km/L for cars in the city, increasing by 50 % on the highway, and 7 km/L for trucks on the highway and 10 % less in the city.
The average trip for trucks is 22 times longer than that of cars, and 1/11 of it is urban driving, while cars spend 80 % of their driving time on urban streets. The cost of car fuel is 50 % more expensive than truck fuel. By how much is the total cost of fuel consumption raised, as a percentage, if the cost per liter of fuel rises 10 % for cars and 15 % for trucks?
A: 12.4 %
65. **Alloys [5⁺]**. We have three alloys with 70, 80, and 90 % purity. In a mixture, we use 10 kg of the first alloy, 15 kg of the second alloy, and X kg of the third alloy. If the final purity of the mixture is 10 % greater than the purity of the second alloy, then: (a) How much is used for the third alloy? (b) If we use twice that amount, what is the final purity?
A: (a) 150 kg (b) 88.9 %
66. **Quantity to exchange [6]**. Two tanks with volumes of V_1 and V_2 liters are full. The first tank has an X_1% of solution **A** and the second tank an X_2% of solution **A** (both v/v). How much should be interchanged between tanks so that the concentrations in both tanks are the same? (Assume that the volumes are additive).
A: $\frac{V_1V_2}{V_1+V_2}$
67. **NaOH and NaCl [7]**. A solution containing 10 % (w/w) NaOH and 20 % (w/w) NaCl is concentrated by evaporating 200 kg H_2O. Then an additional 200 kg solution is added containing 60 % NaCl (w/w) and an unknown percentage of NaOH (w/w). If the resulting mixture is 20 % NaOH (w/w) and 50 % (w/w) NaCl, then: (a) What are the NaOH and NaCl concentrations in the first solution after the evaporation of H_2O? (b) What are the NaOH and NaCl concentrations in the second solution?
A: (a) 20 % NaOH and 40 % NaCl, (b) 60 % NaCl and 20 % NaOH
68. **Copper [7⁺]**. If copper having a purity of 70 % is mixed with the same amount of another metal with X% purity, the final mixture will have a purity of **a%**. (a) Obtain an expression for X as a function of **a**. (b) What should the purity of the second metal be if a final purity of 83 % is desired?
A: (a) $x = 2a - 70$, (b) 96 %
69. **Blue and red balls [8⁺]**. A bag contains blue and red balls. If the probability of drawing two red balls (without replacement) in a row is 1/10 and the blue balls exceed the red balls by one unit, how many balls of each color are in the bag?
A: 2 red balls and 3 blue balls
70. **Birthday [9⁺]**. Today, when I went to classes, I noted that more than two of my fellow students had the same birthday. If 40 students are in the class, what is the probability that two classmates have the same birthday?
A: ~0.891 (~89.1 %)

71. **Chemical company [7]**. Two sales representatives at two different chemical manufacturers are talking about sales at their respective companies. The first one says that 4 years ago his sales were only 80.0 % of the sales of his friend (at the second manufacturer). If the second sales rep has increased his sales at a rate of 10.0 % a year and the first salesman has increased his at a rate of 8 % for the first 3 years, then at what rate did the first manufacturer increase its sales in the fourth year if, at present, both manufacturers have the same level of sales?
A: 45.3 %
72. **Survey [9$^+$]**. A survey has yielded the following information about a group of people: 40 % have light eyes and 70 % have dark hair. 5 % of the light-haired people have dark eyes. (a) What percentage of the dark-eyed people have light hair? (b) What percentage of those with dark hair have light eyes?
A: (a) 2.5 % (b) 16.43 %

MISCELLANEOUS

73. **Cows [8]**. If 10 cows eat 10 packages of grass in 10 days, how many packages of grass will 5 cows eat in 5 days?
A: 2.5 packages of grass
74. **Student scholarship [9]**. A university has decided to award scholarships in the amount of $2,000 to every medical student and $1,000 for each engineering student. If the university's total number of engineering and medical students is 1,000, and half of the medical students reject the scholarship, what is the total dollar amount allocated to scholarships?
A: $1 million.
75. **Number of balls [6$^+$]**. In a bag, I have an unknown number of balls. First, I retire half of the balls plus 1, then I retire the remaining half plus 2 and subsequently withdraw the remaining half plus 4. If, finally, there is 1 ball in the bag, how many balls were there at the beginning?
A: 50 balls.
76. **Euler's problem [7]**. A problem attributed to *Leonhard Euler* states that: "In a bed and breakfast are housed 20 people, both men and women. Each man pays 8 coins for his lodgings, and each woman pays 7 coins. If the total bill comes to 144 coins, how many men and women are staying at the bed and breakfast?"
A: 4 men and 16 women.
77. **Valentine's Day [8]**. As Valentine's Day approaches, I rush out to buy roses. Fortunately, this time, for $50 I get 5 more roses than before. Indeed, the price of a dozen roses fell $10. How much was a rose originally worth?
A: $3.34
78. **Notebooks and iPads [7]**. A large purchase was made of televisions, notebooks, and iPads, a total of 100 units. The TVs cost $600, the notebooks $1,000, and the iPads $800. If the total bill came to $90,000, what is the difference between the total number of notebooks and TVs bought?
A: 50
79. **Foot length, mathematics, and logic [5]**. A statistician gives a math test in a town of 60,000 inhabitants and, at the same time, measures the length of all the villagers' feet. Surprisingly, among the inhabitants 5–22 years of age, he finds that mathematical ability in this town is very well correlated with the length of a person's foot. Of course, the same study for people between 22 and 50 years old does not yield a good correlation. Explain this.

A: Clearly foot size is directly related to age, and math skills are also correlated with age. Of course, beyond a certain age, this correlation is lost.

80. **Party [10]**. Rodrigo is excited to go a party this Saturday at the home of Macarena. But first he must get permission from his father. As usual, the father gives him a challenge. Dad tells Rodrigo that he will let him go to Macarena's party, but he has to play three games of chess and win at least two games in a row. Rodrigo must choose one of the following two sequences of play and win two games in a row to get permission to go to the party: play his mother, then his father, then again his mother; or starting with his father, he then plays his mother and finishes by playing his father again. Rodrigo knows that he rarely beats his dad and that most of the time he beats his mom. Which of the two sequences is more likely to end up in Rodrigo's going to Macarena's party?
A: The sequence that improves his chances is: Dad, Mom, Dad.
81. **Preparing for the GRE [4]**. Your father is tired of your lack of preparation for the GRE. Therefore, he offers to pay $8 per solved problem and fine you $5 per unsolved problem. If you submit to him 26 solved problems and neither owes the other any money, how many problems did you solve correctly?
A: 10 correct and 16 incorrect
82. **New Year's Eve party [9$^+$]**. You have a family party to go to on New Year's Eve. When everyone starts giving out hugs, you're curious to know how many hugs are given, so you count them. You come up with 36 hugs. How many people were at the party?
A: Including you, 9
83. **Canning plant [7]**. A canning company has two processing plants, one where seafood is canned and another that cans fruits. In the seafood canning plant there are three production lines, one with 60 % more capacity than each of the other two lines, expressed in cans processed per hour. This plant operates with two shifts of 8 h each. On the other hand, the fruit processing plant has two production lines of equal capacity and operates with three 8-h shifts per day. If both plants process the same amount of cans per day, what is the ratio of production between the first production line of the seafood plant and either of the fruit production lines.
A: First seafood processing line: Fruit processing line = 4:3
84. **Three substances [9]**. If we have 84 kg of **A**, 66 kg of **B**, and 18 kg of **C,** and their densities are in the proportion 1: 2: 3, what is the v/v percentage of each substance?
A: 68.3 %, 26.8 %, and 4.9 %, respectively
85. **Alcohol and water [10$^+$]**. Two vessels whose volumes are in a ratio of **a**:**b** are full of alcohol and water, respectively. Swapping **C** liters equalizes the concentrations in both vessels. What is the total volume of both vessels?
A: $V_T = \frac{C(a+b)^2}{ab}$

Solution to the shoe problem (Sect. 5.2, Warm-Up Example1)
From the scam Rodrigo is out **one pair of shoes and $20**. Surprised? First, if your answer was $200 you were close but not strictly correct. One easy way to solve the problem is to analyze all the transactions made by Rodrigo (Fig. 5.19).

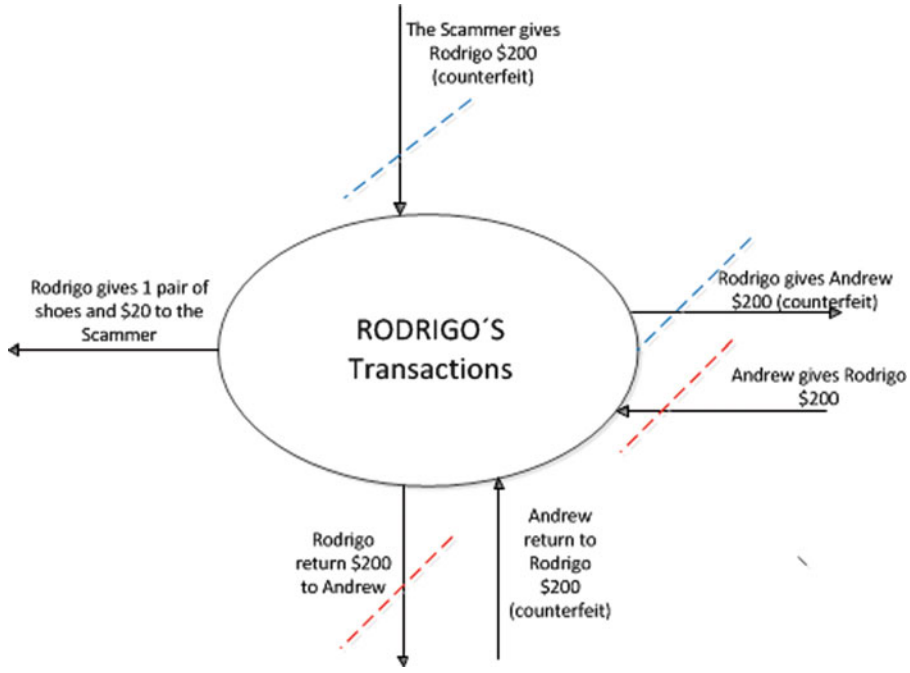

Fig. 5.19 Schematic representation of the transactions made by Rodrigo

From Fig. 5.19 it is clear that Rodrigo is out one pair of shoes and $20. The blue and red dotted lines cancel out, and in the end, Rodrigo is out one pair of shoes and $20 and retains two counterfeit $100 bills ($200).

The problem is easy to solve and has a difficulty level of **2,** but student answers would suggest a level of difficulty of $\mathbf{10^{+}}$.

Additional Web References

Integration of Math and Life http://www.youtube.com/watch?v=ZXq993ZLmg8
Planning a Party by Julia Pollak http://www.youtube.com/watch?v=GHSlVN8N82A

A Glimpse of Thermodynamics and Transport Phenomena

6[2]

Thermodynamics is a funny subject. The first time you go through it, you don't understand it at all. The second time you go through it, you think you understand it, except for one or two small points. The third time you go through it, you know you don't understand it, but by that time you are so used to it, it doesn't bother you anymore.

Arnold Sommerfeld

A theory is the more impressive the greater the simplicity of its premises, the more different kinds of things it relates, and the more extended its area of applicability. Therefore the deep impression that classical thermodynamics made upon me. It is the only physical theory of universal content which I am convinced will never be overthrown, within the framework of applicability of its basic concepts.

Albert Einstein

The second law of thermodynamics is the most metaphysical laws of physics since it points out without interposing symbols, without artificial devices of measurements, the direction in which the world is going.

Henri Bergson

6.1 Introduction

In this chapter, you will become familiarized with thermodynamics and transport phenomena concepts. In addition, you will be able to carry out some simple calculations, but most importantly, you will understand the language of chemical and bioprocess engineering.

As Bloom's categorization for this chapter indicates, the idea is to familiarize students with the language of process and bioprocess engineering. You will be exposed to the most important building blocks of process engineering, a synthetic preview and understanding of the concepts of thermodynamics and transport phenomena. Because an extensive background in mathematics, physics, chemistry, and biology is not a prerequisite for this material, this preview will be mostly qualitative, with the exception of ideal gases. Because you have already been tested by somewhat tricky problems, at this stage, we will present several problems of the same kind, but they will focus on ideal gases and their application to chemical and bioprocess engineering. Ideally, we will impart to you a greater appreciation of the power, scope, and potential of your future profession. One of the beauties of process engineering is its diversity. Some of the fields, knowledge, and applications of chemical and bioprocess engineers are (Fig. 6.1) given below:

R. Simpson and S.K. Sastry, *Chemical and Bioprocess Engineering: Fundamental Concepts for First-Year Students*, DOI 10.1007/978-1-4614-9126-2_6,
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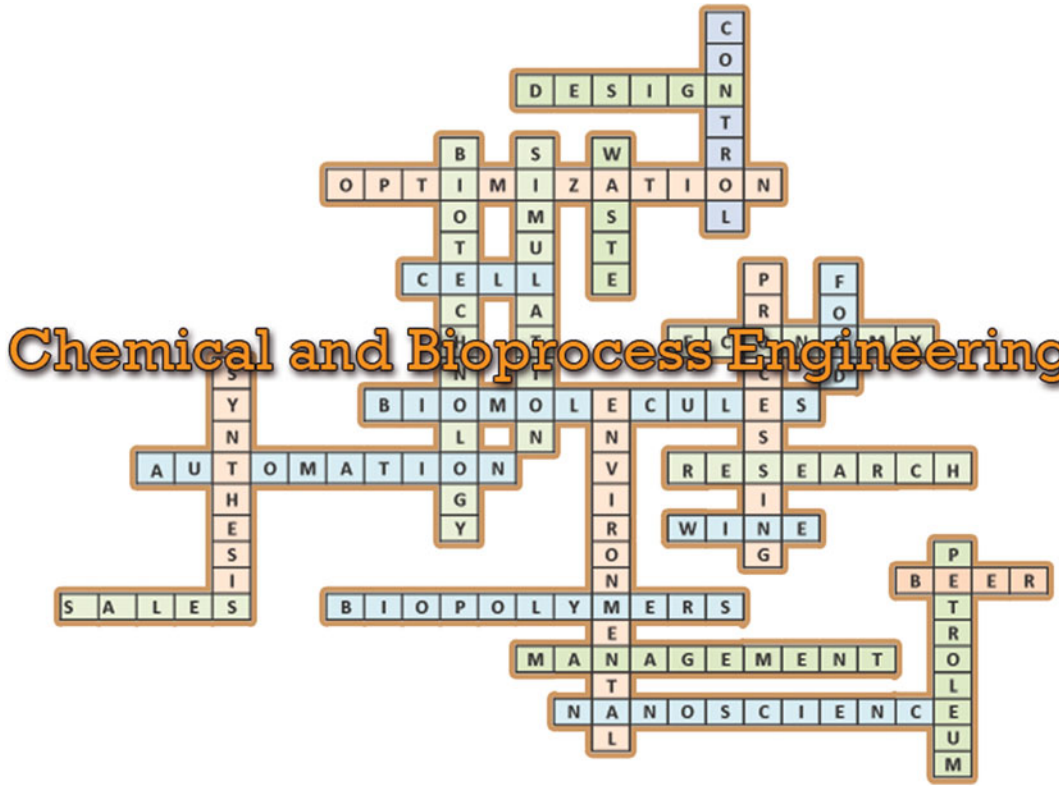

Fig. 6.1 The multiple applications and fields of Chemical and Bioprocess Engineering

6.2 A Glimpse at Thermodynamics

6.2.1 Introduction, Definitions, and Key Concepts

This introduction presents the basic thermodynamic concepts and definitions that arerequired to understand the physical principles that govern the phenomena associated with the transfer of matter, energy, and entropy, i.e., the first and second laws of thermodynamics. These tools provide the framework for the analysis and development of each of the stages constituting an industrial chemical process or bioprocess.

What is thermodynamics? The science that studies energy, in its various manifestations and transformations, and the properties of matter associated with energy.

What is energy? The ability of a system to make changes to itself or to the surrounding environment (operational definition).

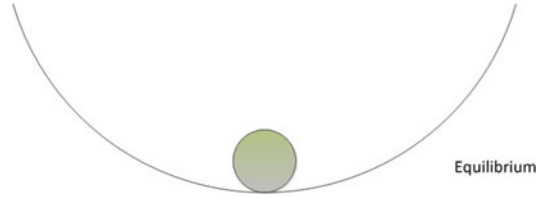

Fig. 6.2 A ball under equilibrium

What forms of energy are there? There are many: energy associated with the structure of matter, energy of motion, energy of position, electromagnetic energy, etc., in addition to so-called energy in transit: heat and thermodynamic work.

What are the fundamentals of thermodynamics? Thermodynamic knowledge is based on experimental observations, which have been encoded through mathematical language, the well-known thermodynamic laws: zeroth, first, second, and third. The zeroth law provides us with a definition of temperature. The first law concerns the exchange of energy, or how much of one type of energy is equivalent to another type of energy (conservation principle). The second law relates to those phenomena that are possible and those that are not efficient. When the first and second laws are combined, they lead to relationships that can determine how much energy can be extracted from a system in a particular environment: energy availability or exergy.

Why study thermodynamics? As engineers, we want to harness energy for purposes that are useful to humanity. The modification of our environment through energy management allows us to attain common benefits.

What is the direct application of thermodynamics in engineering? Thermodynamics, through its laws, participates in the stages of design, construction, and optimization of all processes involving transformations from one form of energy to another.

Three unifying ideas form the core of thermodynamic theory:

(i) **Energy**: the ability of a system to make changes to itself or the surrounding environment.
(ii) **Equilibrium**: a condition of a system in which all driving forces potentially capable of generating a change in the system or surrounding area have been canceled (Fig. 6.2). Here, a sphere comes to rest at the bottom of a curved surface. At this point, the forces acting on it are in equilibrium.
(iii) **State**: physical condition of a system that can be described by specifying a limited number of observable variables (Fig. 6.3).

Definitions. We call a *system* a sector of the universe that is delimited and that you wish to study. The remaining part of the universe is the surroundings. A *closed system* is one that does not exchange matter with the surrounding area (Fig. 6.4a). When there is exchange of matter between a *system* and its surroundings, it is considered an *open system* (Fig. 6.4b). An *isolated system* is one that does not exchange matter or energy with its surroundings.

A process corresponds to a sequence of transformations, quantified by changes in the properties that describe a system, that can get you from one state or initial condition to another state or end condition through one or several stages (Fig. 6.5).

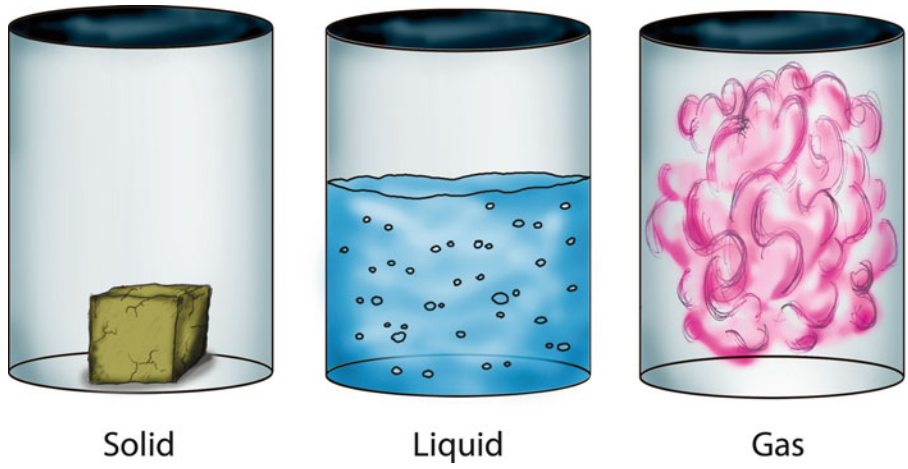

Fig. 6.3 Schematic representation of solid, liquid and gas states

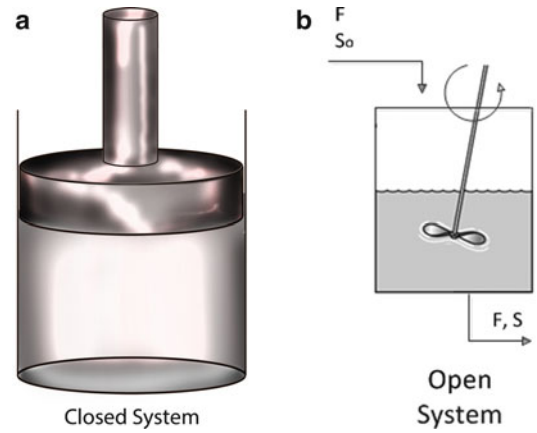

Fig. 6.4 (**a**) A cylinder-piston representing a closed system (**b**) A bioreactor representing an open system

Fundamental concepts

First, let us clarify that heat and temperature are concepts that in colloquial language are normally confused. For example, the expression "It's hot" is commonly used to indicate that the temperature is high, although the word *heat* is used. Heat is a transfer of energy from one part of a body to another, or between different bodies, produced by a temperature difference. Heat is energy in transit, and according to the second law of thermodynamics, the direction of the flow is always from higher to lower temperature.

Temperature (***T***): the property that expresses the energy state or level of inner energy of the molecules of a substance. Intuitively associated with the feeling of hot or cold that is in an object (Fig. 6.6).

The most common method of measuring temperature is with a thermometer that has a fluid inside that expands when heated. In the International System of Units (SI), temperature is measured by an absolute scale. The unit is called Kelvin (K), and the ideal gas is considered a thermometric fluid.

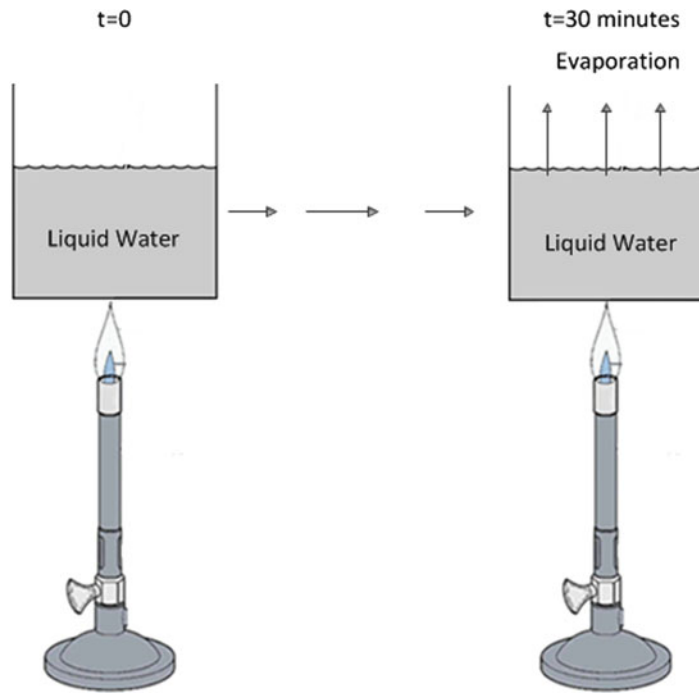

Fig. 6.5 Heating liquid water to get evaporation after some time t

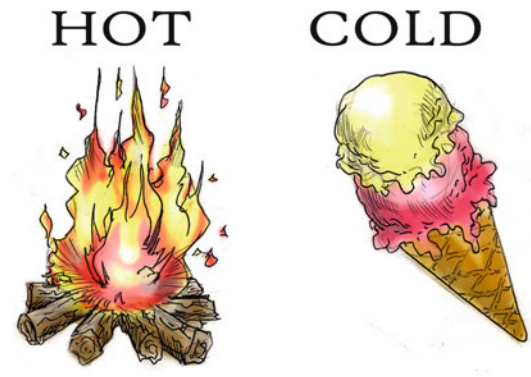

Fig. 6.6 Graphic representation of hot (fire) and cold (ice cream)

A Kelvin corresponds to approximately 1.38×10^{-23} J/particle. Degrees Celsius are related to the Kelvin absolute scale as

$$\mathrm{T(K)} = \mathrm{T}\,(^{\circ}\mathrm{C}) + 273.15. \tag{6.1}$$

In the English system, temperature is measured on the Fahrenheit scale. The unit is called a degree Fahrenheit (°F). The absolute scale of this system corresponds to the Rankine scale (R):

$$\mathrm{T(R)} = \mathrm{T}\,(^{\circ}\mathrm{F}) + 459.67. \tag{6.2}$$

Celsius degrees and Fahrenheit degrees are related by (Chap. 2)

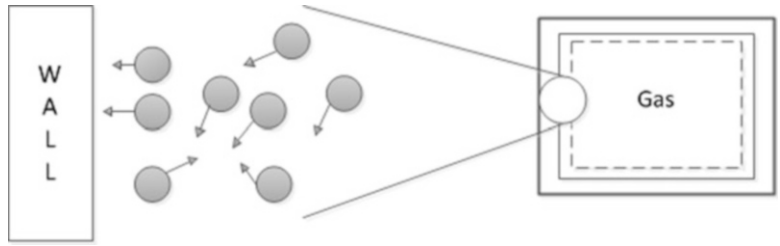

Fig. 6.7 Gas particles exchanging momentum through collisions with the molecules of the container wall

$$^{\circ}\mathrm{C} = (5/9) \times (^{\circ}\mathrm{F} - 32). \tag{6.3}$$

The zeroth law of thermodynamics states that if two systems A and B are in thermal equilibrium with a third system C, then systems A and B are in thermal equilibrium with each other.

Example. (1) In the middle of a closed room at 25 °C is a temperature sensor. What value of temperature will the sensor give? What are the factors that will cause it to detect that temperature?

(2) Now all the air in the room is removed using a vacuum pump. Will the reading of the temperature sensor change? Why or why not?

Pressure (*P*): defines a fluid pressure on a surface as the normal force exerted by the fluid per unit of surface area. Assuming a gas within a cylinder, gas particles exchange momentum through collisions and interactions with the molecules of the container walls (Fig. 6.7).

Momentum exchange is represented by a force exerted by each molecule at the point of collision with the wall. Given a large number of collisions over a large area (A), the macroscopic effect can be represented as force (F) acting on the entire area:

$$P = \frac{F}{A}. \tag{6.4}$$

The basic unit in the SI pressure system is a Pascal (Pa) (Chap. 2), which represents the force exerted by a newton (N) on a square meter (m^2) of surface: N/m^2. The most common units of measure are atmosphere and bar (0.98693 atm $\equiv$ 1 bar). The basic unit of pressure in the English system is psi (pound force/inch^2). More specific units are psia = pound-force/inch^2 (absolute) and psig = pound-force/inch^2 gage (relative):

$$P(\text{psia}) = P(\text{psig}) + 14.7 (1 \text{ atm} = 14.7 \text{ psia; see Chap.2}). \tag{6.5}$$

In the English system, 1 pound-force $\equiv$ 1 pound-mass; thus, $\frac{g}{g_C} = \frac{1 \text{ lb}_f}{1 \text{ lb}_m}$ (g_c reconciles the mass and force units).

Work (*W*). When a body is moved against an external force that opposes its motion, it performs mechanical work (Fig. 6.8).

In the definition of work, the applied force (F) is the one that induces the movement of the body (body weight: mg). Work is energy in action. It is a means of transferring energy. It is energy in transit. Work does not correspond to an inherent property of a system. Work is a form of interaction between the system and its surroundings, meaning that when one gains energy, the other loses energy.

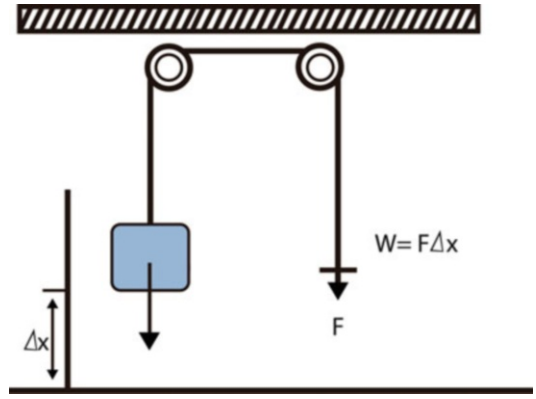

Fig. 6.8 Mechanical work on a body

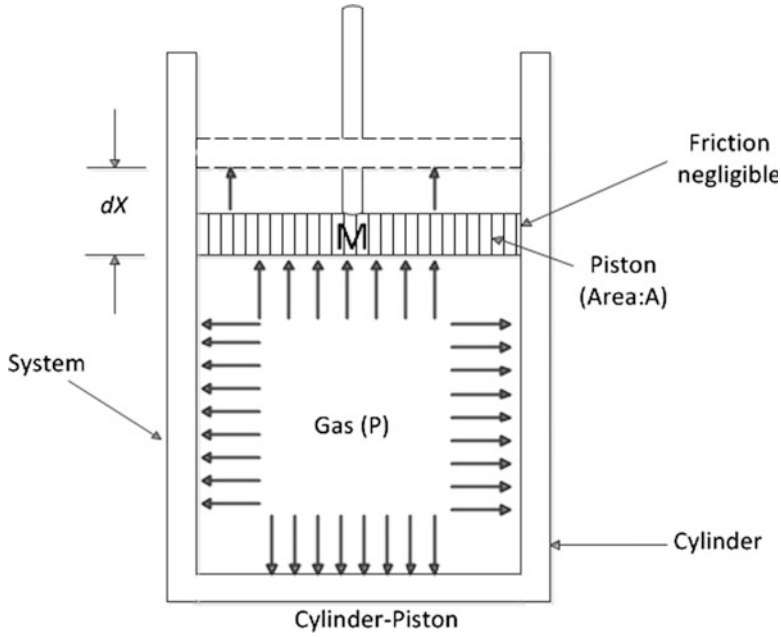

Fig. 6.9 A cylinder-piston system

This type of energy exchange between the system and its surroundings is expressed by the work during displacement.

Consider a closed system formed by a gas confined in a cylinder with a piston of mass M (piston weight: Mg) and negligible friction. The gas displaces the piston up to a length Δx and therefore produces expansion work (Fig. 6.9).

$$W = F\Delta x = (Mg)\Delta x. \tag{6.6}$$

Ideally, $Mg = PA$,

$$W = (PA)\Delta x, \tag{6.7}$$

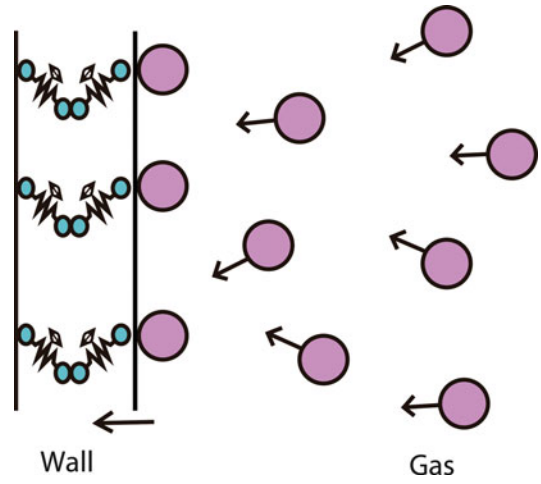

Fig. 6.10 Collision of gas molecules transmitting energy to the molecules of the wall

$W = P\Delta(Ax) = P\Delta V$, $Ax = V$ (volume of gas inside the cylinder).
Then

$$W = P\Delta V(\text{work done by expansion or compression}). \tag{6.8}$$

By convention:	W gained by a system is considered positive. W lost by a system is considered negative.

Heat (Q). What happens with changes in a rigid-wall container? The average (macroscopic) work or energy exchanged between a system (gas) and the surrounding medium (cylinder) because of individual energy exchanges that occur as a result of collisions between molecules of the system and the surrounding environment is called heat (Fig. 6.10). This energy transfer (heat) cannot be expressed as force multiplied by distance (work).

The collision of gas molecules transmits energy to the molecules of the wall, which is stored as vibration. Increased energy inside the solid then makes the temperature rise. Heat is not a new form of energy; it is simply the name given to a transfer of energy in which a large number of particles participate. Macroscopically, it is expressed as an energy transfer due to a temperature difference between particles. Heat does not correspond to an inherent property of a system. Heat is not stored. Like work, it is energy in transit.

By convention:	Q gained by a system is positive. Q lost by a system is negative.

Heat and work are forms of energy transfer and are not stored. It is not correct to speak of heat or work contained in a body or system. Energy is stored in kinetic energy, potential energy, and internal energy, which reside in material objects and exist for the position, configuration, and motion of matter. Energy transformation from one form into another or its transfer from one place to another occurs by heat and work mechanisms. When there is no exchange of energy in the form of heat between two systems in contact, it is said that the system is in thermal equilibrium. The SI unit for energy and work is the joule. In the English system, the unit is the British Thermal Unit (BTU). As mentioned in Chap. 2 (Table 2.3), 1 BTU = 1055.056 J.

6.2.2 Ideal Gases

In this section, we describe in simple terms what an ideal gas is. It includes the concept of an ideal gas and a brief description of the behavior and physicochemical properties that identify it as having ideal fluid characteristics.

An ideal gas is a hypothetical fluid composed of molecules in constant random motion obeying the laws of mechanics. The total number of molecules is high, but the volume of each molecule is a negligible fraction of the total volume occupied by the gas. There are no forces of attraction or repulsion among molecules: each particle travels in a straight line at high speed until it collides with another particle, or the vessel wall that contains it, thus exchanging momentum. On the other hand, a real gas presents characteristics like an ideal gas when the pressure is low because the average spacing between particles is usually several orders of magnitude greater than their diameters, and therefore, the attraction or repulsion energies are small compared to the kinetic energy of each particle. For practical applications, at pressures lower than or equal to 5 [atm], any gas can be treated as an ideal gas.

To predict the behavior of gases, a series of equations has been postulated. One of these equations is discussed in this section and has been denominated as the ideal gas equation. The name comes from the fact that in its theoretical development, as stated earlier, several assumptions are made:

- The volume of the molecules is infinitesimal (negligible in comparison with the container volume).
- It is in thermal equilibrium with its container.
- The gas molecules collide elastically with other molecules and the walls of the vessel.
- The molecules are separated by distances that are large compared to their diameters.
- The net velocity of all the gas molecules must be zero so that, on average, as many molecules are moving in one direction as in another.

For ideal gases, it was experimentally shown (in earlier works by Robert Boyle, Jacques A. C. Charles, and Joseph Gay-Lussac or by theoretical tools like kinetic theory) that the absolute temperature, pressure, and volume (*PVT* behavior) are related according to the equation of state:

$$PV = nRT, \tag{6.9}$$

where P is the absolute pressure of the gas; V is the volume of the container; n is the number of moles of the gas in the container; R is the universal or ideal gas constant (e.g., 1.987 cal/g mol K; 8.314 J/g mol K; and T is the absolute temperature of the gas.

Table 6.1 shows different values of R as a function of the units used (to perform unit conversions, see Chap. 2). A good exercise would be to start with the first value of R and then convert it and obtain those presented in Table 6.1.

Ideal gas mixtures

Dalton's law (additive pressures)

Generally in chemistry it is common to work with gas mixtures. One of the first researchers who studied multicomponent gaseous systems was John Dalton (1766–1884), who went on to enunciate, in 1881, the law that bears his name: "At a constant temperature the pressure of a gas mixture in a defined volume is equal to the sum of the partial pressures of each gas (as long as they do not react)." Mathematically it can be expressed as

$$P_{\mathrm{T}} = \sum_{i=1}^{n} P_i, \tag{6.10}$$

where P is the total pressure, and P_i is the partial pressure of component i.

Table 6.1 Values for R in different units

R	Unit
0.082057	(l atm)/(g mol K)
82.057	(cm^3 atm)/(g mol K)
62.361	(l mmHg)/(g mol K)
10.731	(ft^3 psi)/(lb mol R)
555.000	(ft^3 mmHg)/(lb mol R)
1.314	(ft^3 atm)/(lb mol K)
998.9	(ft^3 mmHg)/(lb mol R)
0.0848	(l kg_F/cm^2)/(g mol K)
1.9872	cal/(g mol K)
1.9869	BTU/(lb mol R)
8.3144	J/(g mol K)

In addition,

$$P_i = \frac{n_i}{n_T} P_T, \tag{6.11}$$

where n_i is the number of moles of component i, and n_T is the total number of moles.

Amagat's law (additive volumes)
According to Amagat's law, in a mixture of gases, each gas occupies its own volume as if the other gases were not present. The specific volume of a given gas in the mixture is called the partial volume. The total volume of the mixture is calculated by simply adding the partial volumes of all the gases that are present in the mixture:

$$V_T = \sum_{i=1}^{n} V_i, \tag{6.12}$$

where V is the total volume, and V_i is the partial volume of component i.

In addition,

$$V_i = \frac{n_i}{n_T} V_T, \tag{6.13}$$

where n_i is the number of moles of component i, and n_T is the total number of moles.

6.2.3 Material and Energy Balance in Open Systems Under Steady-State Conditions

In this section, we carry out a general and conceptual analysis of systems that exchange matter and energy with their surroundings in the absence of chemical reactions. We present, conceptually, the general material balance as well as the overall energy balance for open systems under steady-state

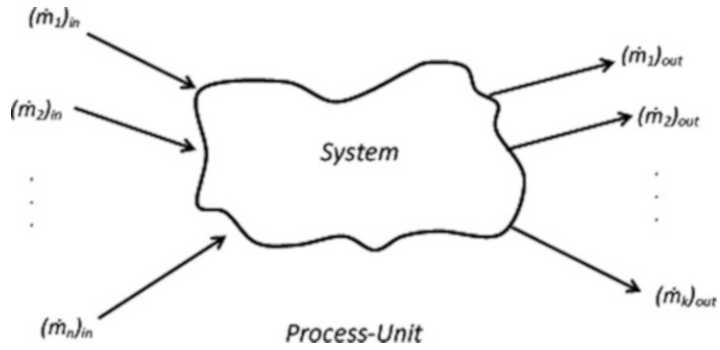

Fig. 6.11 General material balance for an open system

conditions (these concepts are further analyzed in Chap. 7). Both tools are the starting bricks in the foundation for the analysis and development of each of the stages that make up an industrial chemical process and bioprocess. In the next chapter (Chap. 7), we will analyze material balances in great detail with emphasis on quantitative calculations under steady-state conditions.

Material balance, conceptually (quantitative calculations are considered in Chap. 7)
Antoine Laurent Lavoisier (1743–1794) was a scientist who proposed that matter is neither created nor destroyed but conserved. The law of conservation of matter postulates that the amount of material before and after a process is strictly the same. What the law of conservation of matter implies is that, beyond transformations, matter is always constant. In other words, atoms react with each other as substances, but atoms are neither created nor destroyed.

General material balance in open systems for a specified system (Fig. 6.11):

In general,

$$\sum \text{Inputs} - \sum \text{Outputs} = \text{Accumulation(depletion)}. \tag{6.14}$$

If the accumulation (depletion) is 0, then the system is under steady state, and we can express the material balance as

$$\left(\sum_{i=1}^{n} \dot{m}_i\right)_{\text{in}} - \left(\sum_{j=1}^{m} \dot{m}_j\right)_{\text{out}} = 0 \text{ and } \left(\sum_{i=1}^{n} \dot{m}_i\right)_{\text{in}} = \left(\sum_{j=1}^{m} \dot{m}_j\right)_{\text{out}},$$

where $\left(\dot{m}_i\right)_{\text{in}}$ is the mass flow rate of each stream entering the system (mass/time), and $\left(\dot{m}_i\right)_{\text{out}}$ is the mass flow rate of each stream flowing out of the system (mass/time).

In addition, we can write one material balance for each component of the stream. If we have p components, then we can formulate p additional equations. Writing the material balance for some specific component r yields

$$\left(\sum_{i=1}^{n} x_{ri}\dot{m}_i\right)_{\text{in}} - \left(\sum_{j=1}^{m} x_{rj}\dot{m}_j\right)_{\text{out}} = 0 \text{ and } \left(\sum_{i=1}^{n} x_{ri}\dot{m}_i\right)_{\text{in}} = \left(\sum_{j=1}^{m} x_{rj}\dot{m}_j\right)_{\text{out}},$$

where x_{ri} is the mass fraction of component r in stream i, and x_{rj} is the mass fraction of component r in stream j.

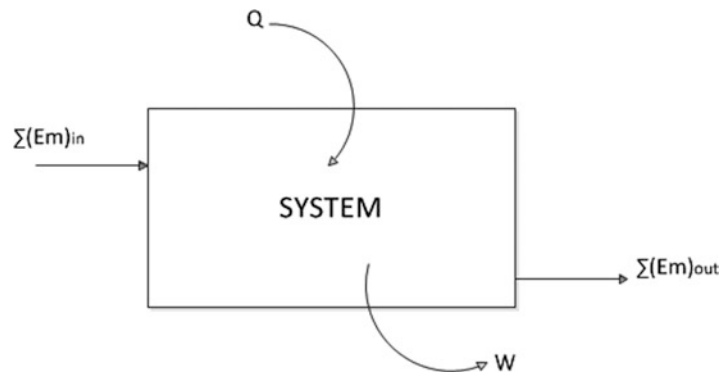

Fig. 6.12 General energy balance for an open system

We can write one total material balance, and, in addition, one material balance per component. Since we have p components, we are able to write, in total, $p + 1$ equations, but ONLY p of them are independent! Why? For example, if we sum up all the equations formulated per component, we will discover that the result is equal to the total material balance. Recall that the number of independent equations is equal to the number of components. Further details of this important concept are analyzed and discussed in Chap. 7.

Energy Balance

The law of conservation of energy states that energy cannot be created or destroyed, only transformed from one form into another. In these transformations, the total energy remains constant, i.e., the total energy is the same before and after a process.

General energy balance in open systems under steady-state conditions (Fig. 6.12):

$$\text{Energy of input flows} - \text{Energy of output flows} + Q + W = 0 \tag{6.15}$$

Taking a close look at the mathematical expression of energy balance, we notice a substantial difference from the material balance expression. In the energy balance formulation two terms appear that are not associated with mass entering or leaving a system. As stated previously, the system can exchange energy with its surroundings due to temperature differences (heat transfer, Q) and if the system is performing or receiving work from its surroundings (e.g., expansion or compression of system boundaries).

6.2.4 Entropy (*S*)

Second law of thermodynamics. R.J.E. Clausius and entropy creation (1854).

The Polish-born theoretical physicist Rudolf Julius Emmanuel Clausius (Koszalin, Poland, 1822–1888), used the results of Nicolas Leonard Sadi Carnot (1796–1832) to establish the concept of entropy (S), which quantifies the degree of dispersion and is sometimes called the disorder of matter and energy that has a system. This parameter is characteristic of one of the most important principles of nature, the second law of thermodynamics. Consider an isolated system, i.e., a system that does not exchange matter or energy with its surroundings. A general approach to the second law is sometimes stated as follows: any isolated system that undergoes a spontaneous process will transit until a final state of equilibrium is achieved in which the maximum degree of dispersion and disorder of matter and energy is reached. In other words, the absolute maximum entropy value is reached.

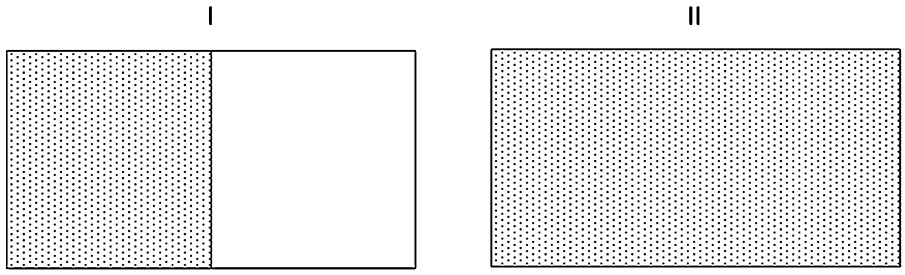

Fig. 6.13 Compartment I with an internal wall and compartment II without the internal wall

The latter ensures that among all possible final states, there will be a preferential path that an isolated system will follow toward that end state that ensures the maximum increase in the value of entropy. There is a tendency to achieve homogeneity in a system, which is the most likely state in which it can exist. In Fig. 6.13, if we remove the wall separating both compartments, then the gas, naturally, will tend to homogenize in the whole container.

Just as the energy balance is different from the material balance because energy balance includes terms that are not related to mass exchange, in the same way, the entropy balance is different from material and energy balances because entropy includes a term that takes into account entropy generation (S_{gen}).

6.3 A Glimpse of Transport Phenomena

The study of transport phenomena follows the study of thermodynamics. Thermodynamics analyzes systems that are in a state of equilibrium. On the other hand, transport phenomena study systems that have shifted from an equilibrium state and attempt to quantify the flow properties of the system (e.g., energy, concentration of species) that arise to guide the system to return to its equilibrium state.

6.3.1 Heat Transfer

When two bodies having different temperatures are put in contact, there is a heat transfer from the higher-temperature body to the lower-temperature body. Heat transfer can be effected by three physical mechanisms: conduction, convection, and radiation.

Heat transfer mechanisms

Conduction. Conduction is a heat transfer mechanism that works through the material by molecular activity and collision among molecules, where molecules with more energy deliver energy to the lower-energy molecules, resulting in a heat flow from the higher temperature to the lower temperature. Notably, no net molecular motion occurs, so the energy transfer is through intermolecular collisions, but the molecules do not change their average position. Conduction can take place in solids, liquids, and gases, but it is characteristic of solids because in gases and fluids convective transfer, or net molecular motion, often occurs simultaneously.

Metals are the best heat conductors. On the other hand, air is a poor conductor of heat, and in general, biological materials (e.g., foods) are poor conductors. Materials that are poor conductors are called insulators (e.g., plastics, asbestos, rubber, cotton).

Table 6.2 Thermal conductivity of some materials in SI system

Material	Thermal conductivity k (W/m K)
Copper	386
Aluminum	204
Stainless steel	15
Ice	2.2
Pyrex glass	1.09
Water	0.611
Food materials (which depend not only on the type of food but also its microstructure)	0.2–0.7
Blood	0.5–0.6
Concrete (medium)	0.4–0.7
Sand (dry)	0.35
Wood (depending on the type of wood and on the direction of the fibers)	0.1–0.3
Fiberglass	0.038
Polystyrene	0.028
Air	0.027

Adapted from: http://web2.clarkson.edu/projects/subramanian/ch330/notes/Introduction%20to%20Heat%20Transfer.pdf

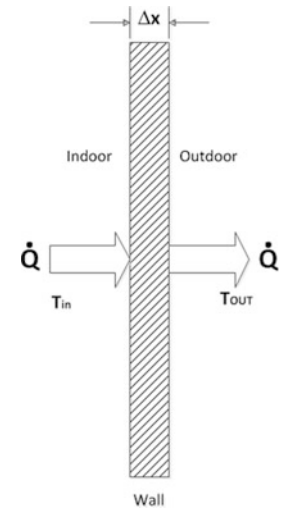

Fig. 6.14 Heat transfer through a wall

Thermal conductivity (k) is a measure of a material's ability to conduct heat (Table 6.2). Its SI units are W/(m K).

Mathematically, heat conduction is modeled and quantified by Fourier's first law (Fig. 6.14):

$$\dot{Q} = -kA\frac{\Delta T}{\Delta x} = -kA\frac{T_{\text{out}} - T_{\text{in}}}{\Delta x}, \tag{6.16}$$

where k is the thermal conductivity of the wall, A is the surface area, and $\Delta T/\Delta x$ is the temperature gradient.

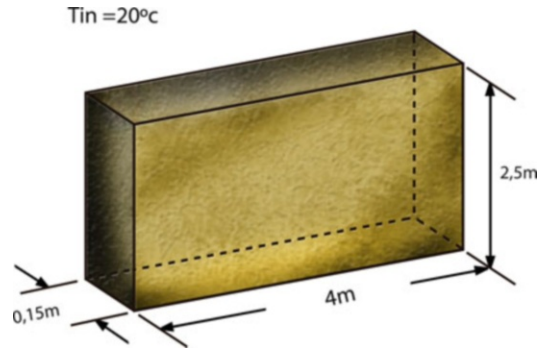

Fig. 6.15 Heat losses from a wall

According to (6.16), we can do a simple calculation and estimate how much heat is lost from a house through one of the walls (Fig. 6.15).

We will assume that the wall is made of concrete ($k = 0.4$–0.7), where the dimensions are 2.5 m height, by 4.0 m width, and 0.15 m thickness. In addition, the temperature inside the house is 20 °C and outside 0 °C. Recall that $\Delta \mathrm{K} = \Delta^{\circ}\mathrm{C}$. Then, assuming the worst-case scenario (the highest thermal conductivity for concrete), from (6.16) we get

$$\dot{Q} = -kA\frac{T_{\text{out}} - T_{\text{in}}}{L} = -0.7\frac{\mathrm{W}}{\mathrm{mK}}(2.5 \times 4)\mathrm{m}^2\frac{(0-20)\mathrm{K}}{0.15\mathrm{m}} = 9.3 \times 10^2\ \mathrm{W}.$$

These losses could be reduced considerably by the addition of an insulator. The thermal conductivity of an insulator is at least ten times lower than that of concrete. As an extreme case, if the wall were made of fiberglass ($k = 0.038$), the losses would be

$$Q = kA\frac{\Delta T}{\Delta X} = 0.038\frac{\mathrm{W}}{\mathrm{mK}}10\,\mathrm{m}^2\frac{20\mathrm{K}}{0.15\mathrm{m}} = 5.1 \times 10^1\ \mathrm{W}.$$

First, we can appreciate that it is possible to significantly reduce heat losses. Second, in a real-life situation, the wall of interest might be composed of concrete and a layer of insulator. Although it is not within the scope of this chapter to do many calculations, if we extend (6.16) to composite walls, we can write

$$\dot{Q} = A\frac{\Delta T}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}. \tag{6.17}$$

Returning to our original problem, let us assume that the concrete wall has 0.05 m of fiberglass. Then

$$\dot{Q} = A\frac{\Delta T}{\frac{L_1}{k_1} + \frac{L_2}{k_2}} = 10\mathrm{m}^2\frac{20\mathrm{K}}{\frac{0.15\mathrm{m}}{0.7\frac{\mathrm{W}}{\mathrm{mK}}} + \frac{0.05\mathrm{m}}{0.038\frac{\mathrm{W}}{\mathrm{mK}}}} \cong 1.3 \times 10^2\ \mathrm{W}.$$

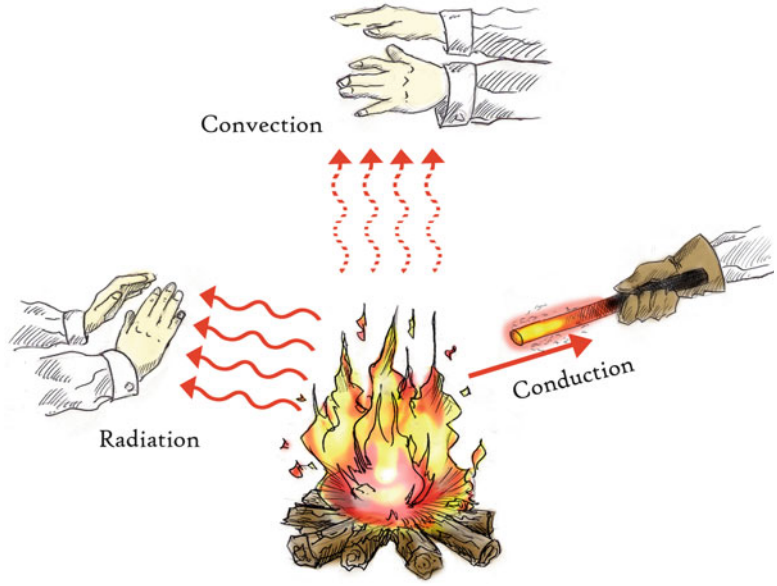

Fig. 6.16 Schematic representation of the three mechanisms of heat transfer

As expected, the addition of a small layer of insulator resulted in a significantly reduced heat loss, from approximately 933 to 131 W. An interesting analysis of the results shows that the heat loss through the composite wall is much closer to the case containing only fiberglass than the case of only concrete.

Convection. Convection is heat transfer with net molecular motion (i.e., by virtue of movement of the substance itself). It can be naturally produced in one of two ways: natural convection, by differences in density, or forced convection, where the mass is made to move from one place to another, such as when using an air fan or water pump. Convection occurs only in liquids and gases where molecules are free to move within the medium.

In natural convection, the fluid movement is due to density differences arising from temperature variation within the fluid.

In forced convection, the fluid is forced to flow by artificial means (e.g., fans, pumps).

Mathematically, convection is modeled and quantified by

$$Q = Ah(T_S - T_f), \tag{6.18}$$

where A is the surface area, h the local heat transfer, T_S the surface temperature, and T_f the fluid temperature.

Radiation. Thermal radiation energy emitted by a material (Fig. 6.16) that is at a given temperature occurs directly from the source outward in all directions. This energy is produced by changes in the electronic configuration of the constituent atoms or molecules and is carried by electromagnetic waves (in a wavelength range of 0.1–100 [μm]) or photons and thus is called electromagnetic radiation.

Unlike conduction and convection, radiation does not require a transmission medium and may occur in a vacuum. The fastest form of heat transfer is by radiation, which occurs at the speed of light and does not suffer attenuation in a vacuum medium.

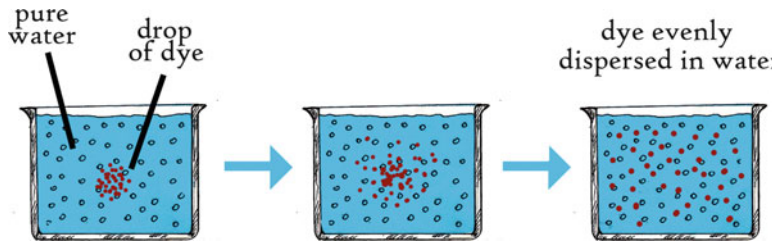

Fig. 6.17 Diffusion of a dye in water

Mathematically, radiation is modeled and quantified by

$$Q = \varepsilon\sigma\left(T_h{}^4 - T_c^4\right)A, \tag{6.19}$$

where Q is the heat transfer per unit time (W), σ is 5.6703 $\times$ 10^{-8} W/m^{2}K^4—*the Stefan–Boltzmann constant*, ε is the emissivity of the object (1 for a black body), T_h is the absolute temperature of the hot body (K), T_c is the absolute temperature of the surroundings (K), and A is the area of the emitting body (m^2).

6.3.2 Mass Transfer

When adding sugar to a cup of coffee, we usually stir it. We know that sugar is soluble in coffee, but still the dissolving process does not occur instantaneously. In fact, it takes time and effort to get the sugar to dissolve. The phenomenon by which substances mix together corresponds to mass transfer. It is of interest to know the parameters that affect mass transfer and the rate of the process. In the case of coffee, we are interested in knowing how long the coffee needs to be stirred; the amount of sugar added, the temperature of the coffee, and even the shape of the stirrer and how fast the coffee is agitated are some variables of interest.

Mass transfer is a main concern in chemical and bioprocess engineering since most industrial processes uses mass transfer operations not only to mix components but also to separate them! The design of these operations is a core part of a chemical and bioprocess engineer curriculum and is based on knowing material balances, energy balances, equilibrium, and mass transfer.

Here we present briefly the mechanism underlying mass transfer. In mass transfer we differentiate the various contributions to it from diffusion, convection, and dispersion.

Diffusion. Diffusion corresponds to mass transfer due to movement of molecules (Fig. 6.17) between the surrounding media. This is the most relevant mechanism for mass transfer in solids. It also occurs in liquids and gases but usually in such cases convection and dispersion become more relevant. Since the rate of diffusion depends on the kinetic energy of the molecules and the degree of cohesion between them, it occurs more easily in gases, then in liquids, and, finally, in solids. For the same reason diffusion tends to occur faster as the temperature increases.

The rate of mass transfer per area (flux ΔC), depends on the concentration gradient $\Delta C/\Delta X$, the surface involved in the process (A) is quantified using the following expression, known as Fick's first law of diffusion:

$$J^{\text{Diffusive}} = D\frac{\Delta C}{\Delta X}, \tag{6.20}$$

where D is the diffusivity or diffusion coefficient and is measured in square liters per unit of time (L^2/t), in SI (m^2/s). Concentration is measured in moles per volume (mol/m^3). The diffusivity

corresponds to a proportionality constant that associates the mass transfer due to diffusion with the concentration gradient. Diffusivity depends on the properties of the molecules that diffuse and the nature of the media where they are diffusing.

If you look closely at this equation and Fourier's law of heat transfer, they look very similar. This similarity is not casual because Fick's observations, based on mass transfer, gave similar results to those observed for heat transfer. This similarity among heat, mass, and momentum transfer are very relevant for chemical and bioprocess engineering and makes it possible to group them into the so-called discipline of transport phenomena.

Convection. Convection is the transport of matter due to the movement of fluid. Returning to the coffee example, the action of stirring helps to bring high sugar concentration from the bottom of the cup to other areas, thus homogenizing the sugar content in the coffee. This is the most relevant mechanism of mass transfer in liquids and gases. Since solid does not flow generally, this is not observed in most cases.

The mathematical description for the convective mass transfer is given by

$$J^{\text{Convective}} = v \cdot C, \tag{6.21}$$

where v is the velocity of the fluid. Since fluids usually do not move at a constant velocity, all the complexities of the flow pattern should be included to evaluate the expression of Eq. 6.21. This convective transport is the most relevant mechanism associated with the streams going in/out of any process.

Dispersion. Most of the flow regimes of interest in chemical and bioprocess engineering are of turbulent flow. This turbulent flow is characterized by the presence of fluctuations, and not only in the direction of the flow. These fluctuations produce an increment on the rate of mass transfer. In most industrial applications, dispersive mass transfer is more relevant than diffusion. For example, this is the most important mechanism in combustion (think flames) and distribution of the pollution (think chimney smoke).

The mathematical description of dispersion mass transfer $J^{\text{Dispersive}}$ is similar to that for diffusion mass transfer (Eq. 6.20), where instead of the diffusion coefficient a dispersion coefficient is considered.

When estimating the total mass transfer in a system, all contributions should be taken in account, diffusion, convection, and dispersion:

$$J^{\text{Total}} = J^{\text{Diffusive}} + J^{\text{Convective}} + J^{\text{Dispersive}}. \tag{6.22}$$

The fact that it is sometimes possible to make useful simplifications and consider only two or one mechanism as being relevant for mass transport permits the estimation of mass transfer rates and, thus, estimation of the process times required (like how long you should stir your coffee!).

Other mechanisms of mass transfer. Several other mechanisms for mass transfer occurs although are less common. As example diffusion can occur or be enhanced by "fields" other than the concentration gradient. As example the migration of ions is enhanced by the electric field (difference in the voltage) in batteries. A slight difference of the "velocity" of diffusion the species under a temperature gradient, i.e., thermophoresis, can be used to separate very similar species.

A case of application in chemical and bioprocess engineering is the use of pressure to promote the permeation of certain species in a membrane. This is the principle behind Reverse Osmosis.

Interfacial mass transfer. Although mass transfer inside the bulk of a solid or fluid phase is of interest and application in chemical and bioprocesses, the mass transfer across interphases is of special interest in the discipline. As example when brewing tea, there is a diffusion process inside of the tea leaves of the molecules responsible for tea flavor and color. Then there is mass transfer (due to diffusion, convection and dispersion) of these molecules to the water in the bulk of the cup.

However is in the interphase of the systems, the boundary between the tea leaves and the water, where more resistance to mass transfer occurs. This interphase mass transfer is usually a combination of different mechanisms. Most of the time this resistance to mass transfer is difficult to calculate but can be estimated using empirical and semi-empirical observations.

Again, similarly to heat transfer, this resistance to mass transfer is modeled using a mass transfer coefficient k_c (analogous to the heat transfer coefficient defined earlier):

$$N = k_c\left(C^{\text{Surface}} - C^{\text{Bulk}}\right), \tag{6.23}$$

where N is the mass transfer (including convective effects) and is proportional to the difference in the concentration between the surface (such as the surface of tea leaves) and the bulk of the fluid. This mass transfer coefficient summarizes all the effects of mass transfer, making it convenient to deal with complex geometries and systems (as occurs in chemical processing equipment). Note the C in the subscript of k_c refers to the use of the concentration (moles per volume) in the evaluation of the expression. Conveniently, other quantities indicating the amount of substance in the media can be used, such as partial pressure or mass fraction.

ATTENTION
As stated at the beginning, all the concepts required in the solved- and proposed-problem sections were learned (or should have been learned) in high school. Assuming that it is necessary to review certain concepts, we will take the opportunity to do an exhaustive review in the solved-problem section. Some of the concepts to be reviewed include, for example, ideal gas mixtures, Dalton's law, and Amagat's law. But we will also keep it simple; the focus will be, as in previous chapters, on tackling problem formulation!

As you will see, most of the quantitative problems in the following sections are concern ideal gases and just a glimpse of material balance. The next chapter is devoted entirely to material balance. Here, the introductory parts on thermodynamics, heat transfer, and mass transfer were included to acquaint you with the chemical and bioprocess engineering world conceptually, but not quantitatively.

6.4 Conceptual and Quantitative Questions

Thermodynamics

1. **Temperature [4]**. (a) At what temperature does a thermometer graduated in degrees Celsius show the same temperature of a second thermometer graduated in degrees Fahrenheit? (b) What temperature in degrees Fahrenheit is double that of degrees Celsius?
   A: (a) If we consider the temperature in Celsius as equal to the temperature in Fahrenheit, then °C = °F therefore substituting °C by °F in °C = (5/9) × (°F − 32) we get that at −40° the temperature is the same in both scales. (b) 320 °F = 160 °C.
2. **Mass balance [6]**. Why is it imperative to always do mass balance and not be tempted to do a volumetric balance when the streams are expressed in volumetric units?
   A: As mentioned earlier, mass is additive. On the other hand, volume is not necessarily additive. For example, if you prepare a mixture of 100 mL of water with 100 mL of ethanol, you will not get 200 mL of mixture. In this case, you get significantly less than 200 mL.

3. **Ideal gases [7]**. Suppose you have a gas in a sealed and rigid container and you add heat (increasing the temperature). What happens to the density of the gas?
   A: It remains the same. Although the temperature is increased, at the same time the pressure increases. More importantly, by definition, density is mass over volume, and neither the mass nor the volume is being changed.
4. **Heat and work [10$^+$]**. As mentioned at the beginning of the chapter, there is a sign convention for heat and work (6.15) in the energy balance (first law of thermodynamics). Can you use a different convention, for example, by calling heat negative when the temperature of the system is lower than the surroundings?
   A: Yes, observing (6.15), we add Q. In this case, if the temperature of the system is lower than the surroundings, then Q is positive (gaining energy). However, if in Eq. 6.15, instead of adding Q, we subtract Q, then Q will be negative. The convention used in (6.15) (adding Q and W) implies that Q is positive when the system temperature is lower than the surroundings and W is positive when work is done on the system.

Heat transfer

5. **Heat transfer mechanisms [2]**. What is the main difference between heat conduction and heat convection compared to heat radiation?
   A: In radiation, heat is transferred by electromagnetic waves. On the other hand, heat conduction and heat convection have in common that the heat is transmitted via the motion of molecules.
6. **Convection [3]**. What is the difference between forced and natural convection?
   A: In forced convection, the fluid is "forced" to flow over the surface by artificial means (e.g., fans and pumps). On the other hand, in natural convection, the fluid movement is due to natural causes (e.g., density differences).
7. **Radiation [3]**. What is a black surface?
   A: A black body has three distinctive characteristics: it absorbs all electromagnetic radiation, is the ideal emitter, and is a diffuse emitter.
8. **Freezer chamber [4]**. A freezing chamber is, an approximately rectangular prism 2.5 m high, 4.0 m wide, and 5.0 m long. To minimize the power consumption of the cooling system, the room is insulated with fiberglass 0.10 m thick. If the ambient temperature is 25 °C and the chamber is maintained at −20 °C, estimate the amount of heat entering the chamber per unit time. **Hint**: First, do not consider the heat exchange with the floor. Second, for calculation and estimation purposes, let us consider that the walls of the freezing chamber are made only of fiberglass, and, in addition, do not consider the intersection of the walls.
   A: The total heat transfer area is 65 m^2. The parallelepiped has two walls of 2.5 × 5.0 m (total 25 m^2), two walls of 2.5 × 4.0 m (total 20 m^2), and one 4.0 × 5.0 m (20 m^2) ceiling. Therefore, substituting into Eq. 6.16 we get

$$Q = 0.038 \frac{\mathrm{W}}{\mathrm{mK}} 65\mathrm{m}^2 \frac{25 - (-20)\mathrm{K}}{0.1\mathrm{m}} = 1111.\mathrm{W} = 3.8 \times 10^3 \frac{\mathrm{BTU}}{\mathrm{h}}.$$

Mass transfer

9. **Sugar [6]**. Powdered sugar takes less time to dissolve than cube sugar when poured in hot tea. Why does this occur? Sugar also dissolves faster in hot water than in cold water. From a mass transfer point of view why do you think this occurs? What are the effects of stirring the cup in this example?
10. **Coffee [8]**. Brewing coffee is a classic example of a separation process, where the coffee is extracted using an agent (water) from ground coffee beans. Examples of this operation are drip

coffee, French pots, and espresso. Although they are similar products (hot coffee), they have their differences. What differences in the process might explain this? Also, coffee beans are ground differently depending on the extraction method. What are the advantages for the extraction process of larger/smaller beans.

11. **Removing soap [7]**. When it comes to removing soap from clothing during the rinse part of the wash cycle, what is easier, using a large amount of water all at once or using half the amount two times in quick succession?

6.4.1 Solved Problems (Ideal Gases)

In this section, we have intentionally omitted mass balance problems. As you will see in Chaps. 7 and 8, this topic is treated in detail and with an emphasis on being quantitative.

1. **Natural gas [5]**. Natural gas has the following v/v composition:

Gas	% v/v
CH_4	90
C_2H_6	5
N_2	5

If gas is transported at 90 °F and an absolute pressure of 55 psi, then: (a) What is the partial pressure of N_2? (b) What is the volume of (pure) N_2 per 100 ft^3 of gas? (c) What is the density of the mixture?

Solution

Step I

Reading and understanding

This problem involves several concepts beyond the ideal gas law. First, the mixture of gases will be treated as an ideal gas. Second, remember that a % v/v composition is equal to a % mol/mol composition. Finally, to answer questions (a) and (c) we will assume that Dalton's and Amagat's laws are valid.

Step II

Graphical representation, variable definition, and codification

There is no need for a graphical representation.

P_{N2}: partial pressure of N_2

V_{N2}: partial volume of N_2

ρ_M: density of mixture

Step III

Mathematical formulation of problem

We have three unknowns, and so we need three equations. The equations are Dalton's equation to estimate the partial pressure of N_2, Amagat's law to estimate the partial volume of N_2, and, finally, the ideal gas law to estimate the density of the mixture.

Dalton's law indicates (6.11) that the partial pressure of component i is equal to the product of the molar fraction of component i and the total pressure. Therefore,

$P_i = \frac{n_i}{n_T} P_T$ and $y_i = \frac{n_i}{n_T}$, where, in this case, $P_{N_2} = y_{N_2} P_T$.

Similarly, Amagat's law indicates (6.13) that the partial volume of component i is equal to the product of the molar fraction of component i and the total volume. Therefore,

$V_i = y_i V_T$, where, in this case, $V_{N_2} = y_{N_2} V_T$.

To calculate the density of the mixture (ρ_M), recall that

$$PV = nRT = \frac{m}{MW_M} RT. \tag{6.25}$$

Rearranging (6.25) we get

$$P \times MW_M = \frac{m}{V} RT = \rho_M RT,$$

$$\rho_M = \frac{P \times MW_M}{RT},$$

where the molecular weight of the mixture (MW_M) is

$$MW_M = y_{N_2} MW_{N_2} + y_{CH_4} MW_{CH_4} + y_{C_2H6} MW_{C_2H6}. \tag{6.26}$$

Step IV

Results, analysis, and discussion

(a) Dalton's law: $P_{N_2} = y_{N_2} P_T = 0.05 \times 50 = 2.5$ psi;

(b) Amagat's law: $V_{N_2} = y_{N_2} V_T = 0.05 \times 100 = 5$ ft^3;

(c) Mean molecular weight and density of the mixture:

$$MW_M = y_{N_2} MW_{N_2} + y_{CH_4} MW_{CH_4} + y_{C_2H6} MW_{C_2H6} = 0.05 \times 28 + 0.9 \times 16 + 0.05 \times 30 = 17 \frac{g}{g\ mol},$$

$$\rho_M = \frac{P \times MW_M}{RT} = \frac{50 \times 17.3}{10.73 \times (90 + 460)} = 0.15 \frac{lb}{ft^3}.$$

2. **Large truck [6$^+$]**. A large truck requires 6,005 g of air for all tires, including spare tires (one front and one rear). Each tire (front or rear) has a volume of 130 L, each front tire should have a pressure of 30 psia, and each rear tire a pressure of 34 psia. How many tires does the truck need? Assume that the ambient temperature is 27 °C and the truck has the same number of front and rear tires.

Solution

Step I

Reading and understanding

First, the pressure of the tires is relatively low (around 2 atm), so we can comfortably use the ideal gas law. Second, given that the gas is air (a mixture mainly composed of oxygen and nitrogen), we need to estimate its mean molecular weight.

Step II

Graphical representation, variable definition, and codification

Again a graphical representation is not needed.

N: total number of tires

M_f: total mass of air in front tires (g)

M_r: total mass of air in rear tires (g)

Step III

Mathematical formulation of problem

We have three unknowns (N, M_f, and M_r), and we can formulate three equations: the ideal gas law for front and rear tires and an equation for the total mass. Before writing the equations we need to estimate the mean molecular weight of air (MW_{air}). According to (6.26) we can write

$$MW_{air} = y_{N_2} MW_{N_2} + y_{O_2} MW_{O_2} = 0.79 \times 28 + 0.21 \times 32 \cong 29.$$

Front tires. The ideal gas law for the front tires is

$$P_f \left[V \frac{N}{2} \right] = nRT = \frac{M_f}{MW_{air}} RT, \tag{6.27}$$

where $V \times N/2$ represents the total volume of the front tires and P_f the pressure of the front tires. From (6.27) we obtain

$$N = 6.395 \times 10^{-3} M_f, \text{with mass expressed in grams.} \tag{6.28}$$

Rear tires. The ideal gas law for the rear tires is

$$P_r \left[V \frac{N}{2} \right] = nRT = \frac{M_r}{MW_{air}} RT, \tag{6.29}$$

where $V \times N/2$ represents the total volume of the front tires and P_r the pressure of the rear tires. From (6.29) we obtain

$$N = 5.6424 \times 10^{-3} M_r, \text{with mass expressed in grams.} \tag{6.30}$$

Total mass

$$6,005 = M_f + M_r \tag{6.31}$$

Step IV

Results, analysis, and discussion

Solving (6.28), (6.30), and (6.31) we obtain $N = 18$ tires (including two spare tires).

3. **Long trip [7]**. A vehicle makes a long trip to a very hot area that has a temperature that is 60 % higher than at the point of origin in degrees Celsius. If at the destination the driver discovers that the

pressure of each tire increased by 2 % and that during the trip each tire lost 2.5 % of its air mass, then what is the weather in the area of origin (cold, warm, hot)? Assume that the volume of each tire remains constant.

Solution

Step I

Reading and understanding

According to the previous problem, the pressure of tires is relatively low (around 2 atm). Therefore, again, we can comfortably use the ideal gas law. In addition, as in the preceding problem, a good estimate for the mean molecular weight of air is 29 g/mol.

Step II

Graphical representation, variable definition, and codification

There is no need for a graphical representation.

P: pressure of each tire at origin (atm)

P^*: pressure of each tire at destination (atm)

V: volume of each tire (liters) (origin and destination)

T: temperature at origin (°C)

T^*: temperature at destination (°C)

m: mass of each tire at origin (g)

m^*: mass of each tire at destination (g)

Step III

Mathematical formulation of problem

We have seven unknowns and can formulate five equations: two equations using the ideal gas law, at the origin and at the destination, and three more relationships for pressure, temperature, and mass. Therefore, we cannot calculate all variables, but we might be able to calculate some of them. Clearly, if we want to determine what the weather in the origin is like, our object should be to calculate the temperature at the origin (T).

Relationships

The temperature at the destination (T^* in degrees Celsius) is 60 % higher than the origin temperature (T):

$$T^* = T + \frac{60}{100}T = 1.6 \times T. \tag{6.32}$$

The tire pressure is 2 % higher at the destination:

$$P^* = P + \frac{2}{100}P = 1.02 \times P. \tag{6.33}$$

The mass of the tires was reduced by 2.5 %:

$$m^* = m - \frac{2.5}{100}m = 0.975 \times m. \tag{6.34}$$

Ideal gas equation at origin:

$$PV = \frac{m}{29}R(273.15 + T). \tag{6.35}$$

Ideal gas at destination

Assume that at the destination the temperature is $1.6T$, the pressure is $1.02P$, and the mass is $0.975m$. Thus,

$$1.02 \times PV = \frac{0.975m}{29}R(273.15 + 1.6 \times T). \tag{6.36}$$

Step IV
Results, analysis, and discussion
Dividing (6.36) by (6.35) we get

$$1.02 = 0.975\frac{(273.15 + 1.6T)}{(273.15 + T)}.$$

$T \sim 22.8\ ^\circ\text{C}$, therefore we can conclude that the weather at the point of origin is warm. On the other hand, at the destination, the temperature was $1.6 \times T$, ~36.5 °C (~97.7 °F) (very hot!).

In this case, we can calculate the temperature at the point of origin and at the destination, but we cannot calculate all unknown variables. As mentioned in step III, we have seven unknowns and only five equations, meaning that we cannot calculate all variables. More details on this topic are discussed in Sect. 7.8.2.1.

4. **Membrane [9⁺]**. Initially, in compartment I there are 20 mol of N_2 and, in compartment II, 984 g of a mixture of N_2, CO_2, and CO with 30 % molar CO_2 and 40 % molar CO. These compartments are separated by a rigid membrane that is permeable only to N_2. The temperature is maintained at 25 °C during the entire operation. The volume of each compartment is 50 L. (a) What is the final pressure in each compartment? (b) How many net moles of N_2 pass through the membrane? (c) What is the mean molecular weight of the gas mixture in compartment II in the initial and final states?

Solution
Step I
Reading and understanding
First, it is important to understand when the system will reach equilibrium and then express this condition mathematically. Given that N_2 is the only gas that can pass through the membrane, equilibrium will be reached when the pressure of N_2 in compartment I (in this case equal to the total pressure of the compartment I) is equal to the partial pressure of N_2 in compartment II. We will assume that the direction of N_2 movement will be from compartment I to compartment II. If we get a minus sign in the amount of N_2 through compartment II, it will imply that the direction is on the other side.

Step II
Graphical representation, variable definition, and codification
Although simple, a graphic is included (Fig. 6.18).
X: moles of N_2 passing through membrane (mol)
P_{IF}: final pressure at compartment I (atm)

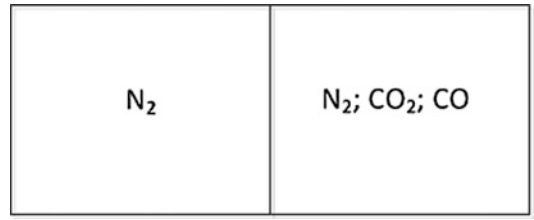

Fig. 6.18 Container separated in two compartments by a selective membrane

P_{IIF}: final pressure at compartment II (atm)
P_{N2II}: partial pressure of N_2 in final state in compartment II (atm).

Step III

Mathematical formulation of problem

We have four unknowns and can formulate four equations: the ideal gas law in the final state in each compartment, the equilibrium condition, and the relationship between the total pressure and the partial pressure in compartment II for nitrogen (Dalton's law).

Final state, compartment I

Given that $V = 50$ L, $n = 20$ mol, and $T = 25$ °C in the initial state, then in the final state

$$P_{IF} \times 50 = (20 - X) \times 0.08206 \times 298.15. \tag{6.37}$$

Final state, compartment II

Given that $V = 50$ L and $T = 25$ °C, then

$$P_{IIF} \times 50 = (n_{II} + X) \times 0.08206 \times 298.15. \tag{6.38}$$

Although we do not know n_{II}, we have the molar composition (30 % CO_2, 40 % CO, and 30 % N_2) and the total mass (984 g). With the molar composition we can calculate the mean molecular weight and then n_{II} as follows:

$$MW_M = 0.3 \times MW_{CO2} + 0.4 \times MW_{CO} + 0.3 \times PM_{N2} = 32.8 \ [g/g \text{ mol}].$$

Therefore, $n_{II} = 984/32.8 = 30$ g mol.

Substituting into Eq. 6.38 yields

$$P_{IIF} \times 50 = (30 + X) \times 0.08206 \times 298.15. \tag{6.39}$$

Equilibrium state

The final pressure of compartment I should be equal to the final partial pressure of nitrogen in compartment II:

$$P_{IF} = y_{NIIF} P_{IIF}. \tag{6.40}$$

Initially, in compartment II we had 30 [mol] in total (n_{II}), and the molar fraction of N_2 was 30 %. Therefore, we initially had 9 [mol] of N_2 in compartment II. Therefore, the final molar fraction of N_2 in compartment II will be the final moles of N_2 in compartment II (9 + X) over the total number of moles in compartment II at the end (30 + X):

$$y_{\mathrm{NIIF}} = \frac{9+X}{30+x}. \tag{6.41}$$

Replacing y_{NIIF} in (6.40) we get

$$P_{\mathrm{IF}} = \frac{9+X}{30+X} P_{\mathrm{IIF}}, \frac{P_{\mathrm{IF}}}{P_{\mathrm{IIF}}} = \frac{9+X}{30+X}. \tag{6.42}$$

Step IV
Results, analysis, and discussion
Dividing (6.37) by (6.39) we get

$$\frac{P_{\mathrm{IF}}}{P_{\mathrm{IIF}}} = \frac{20-X}{30+X}. \tag{6.43}$$

Equalizing (6.42) and (6.43) we obtain

$$\frac{20-X}{30+X} = \frac{9+X}{30+X}. \tag{6.44}$$

Therefore, solving X from (6.44) and obtaining P_{IF} and P_{IIF} from (6.37) and (6.39) we get $X = 5.5$ g mol, $P_{\mathrm{IF}} = 7.1$ atm, and $P_{\mathrm{IIF}} = 17$ atm.

Given that X is positive, the assumption that N_2 will pass from compartment I to compartment II is correct.

6.5 Proposed Problems (Ideal Gases)

1. **Unknown element [2]**. There is an element that, under the conditions of the experiment, can be considered an ideal gas. When 84 g of the element are placed in a cylinder of volume V, the product of $R \times T$ is 1/3 $P \times V$. What is the element?
   A: N_2
2. **Unknown compound [3]**. 98.28 g of a compound occupies a volume of 50 L in normal conditions. What could the compound be?
   A: CO_2
3. **Normal conditions [2]**. One modification that has been proposed for the equation of state of an ideal gas is $PV = nRTZ$, where Z represents a correction factor. This correction factor is mainly affected by the pressure and temperature, i.e., $Z = Z(P,T)$. Check that for an ideal gas, $Z = 1$. Remember, in normal conditions, 1 mol of gas occupies a volume of 22.4 L at 1 atm and 273.15 K.
4. **Piston [4]**. CO_2 is stored in a cylinder-piston system (diameter = 2.000 m, height = 3.000 m) at 25 °C. If the piston has a mass of 100 kg and the system is under equilibrium, what is the pressure of the gas inside the cylinder?

A: 1.003 atm (atmospheric pressure + pressure of piston)

5. **Mean molecular weight [4]**. The mean molecular weight of a gas mixture of CO_2, CO, and N_2 is 34. If the ratio of the molar fraction of N_2 and CO_2 is 0.5, then: (a) What is the molar composition of the mixture? (b) What is the w/w composition of the mixture? (c) What is the volume of 1 lb of the mixture at 2 atm and 580 R?
   A:

(a)		(b)		(c)
CO_2	37.5 %	CO_2	48.53 %	6.226 ft^3
N_2	18.75 %	N_2	15.44 %	
CO	43.75 %	CO	36.03 %	

6. **Air bubble [7]**. An air bubble with a volume of 15 cm^3 is at the bottom of a lake at a depth of 60 m, where the temperature is 4 °C. The bubble rises to the surface, where the temperature is 20 °C. Assuming that the temperature of the bubble is always equal to that of the surrounding water, what is the bubble's volume just before it reaches the surface?
   A: 108 cm^3
7. **Molecules [3⁺]**. Calculate the number of molecules in the gas contained in a volume of 1.0 cm^3 at a pressure of 10^{-3} atm and a temperature of 210 K.
   A: 3.5×10^{16} molecules
8. **Two laboratory bioreactors [4⁺]**. A system of two laboratory-scale reactors is connected by a valve. The first bioreactor has a volume of 5 L and the second has a volume of 10 L. Initially, the valve is closed and the pressure in each bioreactor is 9 and 6 atm, respectively. Then the valve is opened and equilibrium is established. If the temperature was maintained constant during operation, what is the final equilibrium pressure?
   A: 7 atm
9. **Small bulb [7⁺]**. An ideal gas at 650 mmHg occupies a bulb of an unknown volume. Then a certain quantity of gas is removed from the bulb that occupied a volume of 1.52 cm^3 at 1 atm of pressure. After this operation, the pressure in the bulb is 600 mmHg. Assuming that all measurements were made at the same temperature, what is the volume of the bulb?
   A: 23.1 cm^3
10. **Ice cream [8⁺]**. To give low-quality ice cream a smooth consistency, it is customary to add a gas, normally carbon dioxide (CO_2). If the density of the ice cream without gas is 0.96 g/cm^3, and CO_2 is stored at 20 atm and 20 °C, then (a) how many liters of ice cream can be prepared with 20 ft^3 of CO_2? For computation, consider that the ice cream temperature is 0 °C and the density of ice cream with CO_2 cannot be less than 0.81 g/cm^3. (b) If with the same amount of CO_2 we want to make 60,000 L of ice cream, will the density of the ice cream be lower or higher?
    A: (a) ~67,407.25 L (with CO_2) or 56,854 L of original ice cream. (b) Higher (~0.8167)
11. **Natural gas [8]**. A natural gas, containing 11 % v/v of CO_2 and the rest being oxygen, nitrogen, and helium, will be treated to reduce CO_2 and O_2 concentrations. A certain amount of this gas is passed through a CO_2 absorber (where only CO_2 is absorbed). At the absorber outlet, it is verified that the concentration of CO_2 has been reduced to 6.32 %. The gas is then passed through an absorber of O_2. The composition of the exhaust gas of the second absorber is 11.88 % v/v O_2 and 10.58 % v/v helium. If for every 100 mol fed into the absorber system 10 mol of O_2 are retired in the oxygen absorber, a) what is the natural gas composition and b) what is the composition at the absorber system outlet?

A:

(a)		(b)	
CO_2	11.00 %	CO_2	7.40 %
O_2	20.14 %	O_2	11.88 %
N_2	59.85 %	N_2	70.13 %
He	9.03 %	He	10.58 %

12. **Electricity [9⁺]**. In the natural gas plant of a refinery, electricity is generated by expanding high-pressure gas in a turbine. The gas is introduced at a pressure of 200 psia and 30 °C and expanded to a pressure of 25 psia. The gas expansion follows the equation $P_1{V_1}^{1.5} = P_2{V_2}^{1.5}$, and the gas velocity at the inlet and outlet ducts is the same. (a) What is the ratio between the inlet and outlet diameters? (b) What is the gas temperature at the outlet? Although pressures are not necessarily in the range of ideal gases, we will assume, for calculation purposes, that we can utilize $PV = nRT$. **Hint:** The mass flow is constant; thus, $\rho_1 v_1 A_1 = \rho_2 v_2 A_2$.
A: (a) 0.5 and (b) 151.575 K
13. **Chimney [3]**. A chimney gas has the following composition expressed as v/v: CO_2 10 %, CO 0.5 %, O_2 9 %, and N_2 89.5 %. (a) What is the composition of the gas expressed as w/w? (b) What is the volume of 1 lb of gas at 70 °F and a pressure of 30 in of Hg? (c) What is the density of the gas in pounds per cubic feet [conditions of part (b)]?
A:

(a)		(b)	(c)
CO_2	14.7 %	12.88 ft^3	0.077 lb/ft^3
CO	0.47 %		
O_2	9.6 %		
N_2	75.23 %		

14. **Cylinder-piston system [6]**. A cylinder-piston system has a diameter of 3.000 m, and the piston is 4.000 m high (Fig. 6.19). The cylinder contains N_2 at a temperature of 27 °C, and the piston has a mass of 800 kg. When you put rocks on the piston it drops 16 cm. (a) How much do the rocks weigh in kilograms? (b) How much do the rocks weigh when the piston drops 18 cm? Assume that the process is isothermal, that is, the whole process occurs at constant temperature.
A: (a) 3075.4 kg and (b) 3477.5 kg

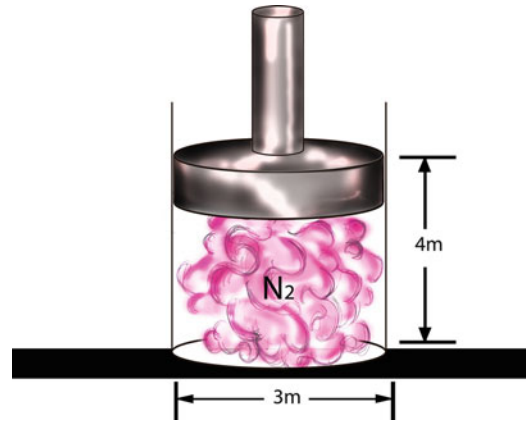

Fig. 6.19 A cylinder-piston system

15. **Polytropic process [9]**. A gas flow of ***F*** (L/h) is compressed until the initial pressure has doubled. If the input temperature is 30 °C and the process can be considered polytropic (PV^a = constant), then: (a) What is the volume flow rate at the output? (b) If $a = 1.4$, what is the temperature at the output?
A: (a) $\frac{F}{\sqrt[a]{2}}$ L/h, (b) ~96.2 °C
16. **Wine fermentation [3]**. During wine fermentation, sugar is converted into ethanol by the following reaction:

$$C_6H_{12}O_6 \rightarrow 2CO_2 + 2C_2H_5OH$$

If for every liter of must (grape juice) there are approximately 450 g of glucose (sugar formula $C_6H_{12}O_6$) per liter, what is the volume of CO_2 produced under normal conditions from 100,000 L of must?
A: 11.207×10^6 L
17. **Gas mixture [10⁺]**. A tank contains 150 g of a mixture of gases (CH_2, O_2, N_2, C_2H_6) with a pressure of 760 mmHg. Then, air containing 20 % CH_4 by volume is added until the pressure reaches 64.7 psia. The mass of gas within the tank is raised 472.9 g, the C_2H_6 partial pressure is 304 mmHg, and the amount of N_2 in the container is 13 mol. If the temperature remains constant throughout the operation, what is the initial composition of gas in the tank?
A:

Gas	Molar %
CH_4	10.1
O_2	19.3
N_2	30.6
C_2H_6	40.0

18. **CO_2 absorption [10⁺]**. A combustion gas containing CO_2, N_2, O_2, and H_2O is used to recover CO_2 by an absorption process with diethanolamine ($C_4H_{11}NO_2$). Before moving the gas to the recovery phase it is cooled to withdraw part of the H_2O (Fig. 6.20). A volume flow rate of 86,900 m^3/h of gas at 300 °C and 1 atm is cooled at constant pressure until the temperature reaches 30 °C. The partial pressure of the water before and after cooling is 142.9 and 32 mmHg, respectively. It is known that after removing all the CO_2 we obtain the following measurements: (1) The gas flow is 35,630 m^3/h (30 °C, 1 atm); (2) moles O_2/moles H_2O = 1.053; (3) 16 lb of diethanolamine are used per pound of CO_2 absorbed.
(a) How many kilograms of water are condensed? (b) What is the composition of the combustion gas? (c) How many kilograms of diethanolamine were utilized?
Hint: Start calculating the molar fraction of water before and after condensation and then do a global mass balance and a water mass balance in the condenser.
A: (a) ~5,065 kg/h
(b)

Gas	Molar (%)
CO_2	7.25
N_2	70.20
O_2	3.75
H_2O	18.80

(c) 94,322 kg/h

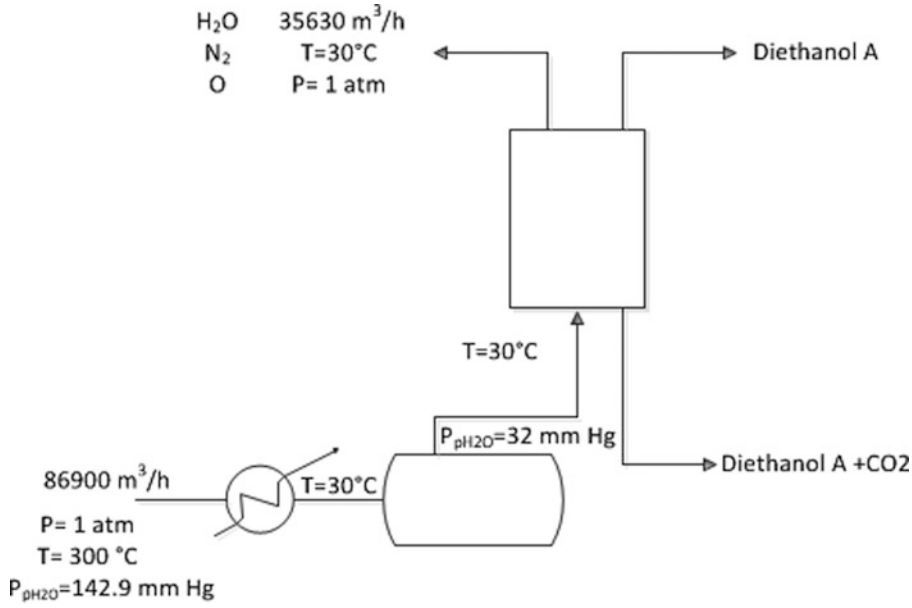

Fig. 6.20 Flow sheet of the process

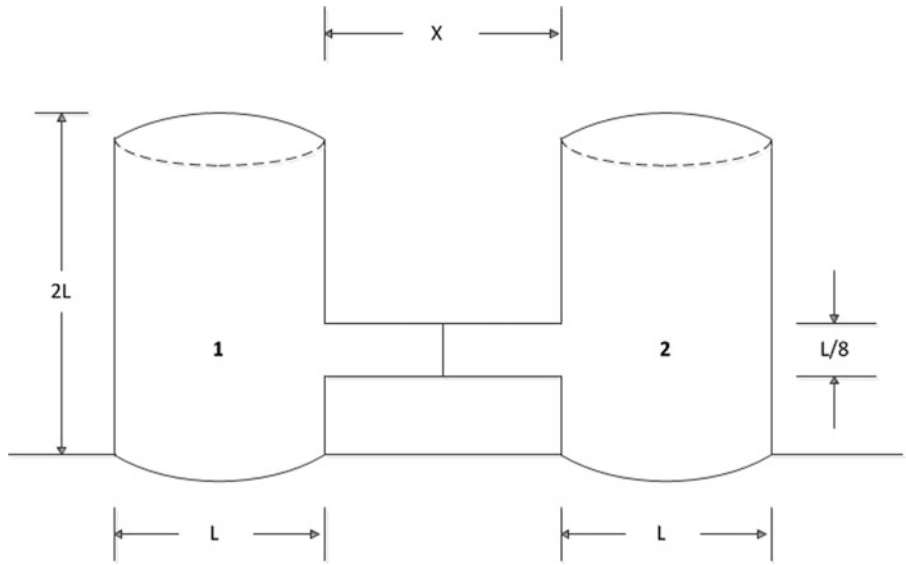

Fig. 6.21 Two tanks that are of the same size and are initially separated by a fixed partition

19. **Fixed partition [8]**. Two tanks are connected with a cylindrical tube as shown in Fig. 6.21. The two tanks are the same size and are initially separated by a fixed partition located at the same distance from each tank. If in the first tank there are 550 g of a mixture of CO_2 and N_2 with 19.26 % molar CO_2 and the second tank has 12 mol N_2 and a negligible amount of CO_2,

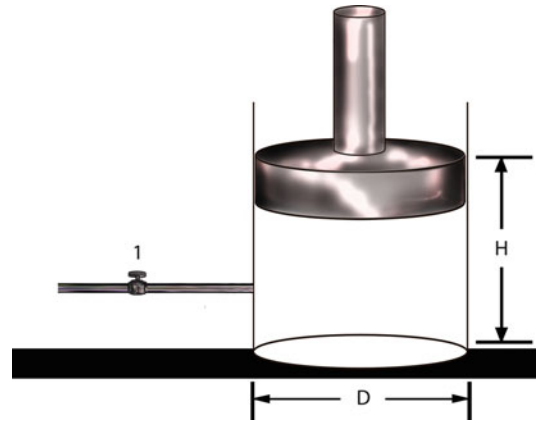

Fig. 6.22 A cylinder-piston system

then (a) what is the length of x, if when the fixed partition is free to move, this moves until reach a negligible distance from tank II, (b) what amount of moles of N_2 are needed in the tank II to equilibrate the fixed partition when freed. (The process can be considered isothermal at 300 K.)
A: (a) 60.06 L m, (b) 17.7 g mol

20. **Cylinder-piston [8⁺]**. Figure 6.22 shows a cylinder-piston system. The piston has a mass of 1,000 kg. Inside the cylinder is CO_2 at a temperature of 25 °C, and the valve is initially closed. When fully open the flow inside the cylinder is 5.0 g mol CO_2/h. **Hint**: The internal pressure is equal to the external pressure and the external pressure is the atmospheric pressure plus the pressure of the piston, where the pressure of the piston is $P = mg/A$.
Data
Cylinder diameter: 2.00 m; initial height: 3.00 m
If P and T are constant, how high will the piston be after 100 h?
A: 6.77 m

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Ideal Gases

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Fundamentals of Material Balance (Nonreactive Systems)

7[5]

Optimists enrich the present, enhance the future, challenge the improbable and attain the impossible.

William Arthur Ward

He that would have the fruit must climb the tree.

Thomas Fuller

Imagination is more important than knowledge.

Albert Einstein

7.1 Chapter Purpose and Strategy

With optimism and effort we will climb together the tree of material balance.

This chapter is about one of the most important and fundamental topics for a process engineer: material balance. The significance and importance of this chapter lies in the fact that we expect to accomplish two objectives simultaneously: delight students with the prospect of a satisfying process engineering career and train and prepare them for one of the most relevant topics: material balance. We will use many examples to teach you, step by step, how to approach, formulate, and solve material balance problems and, at the same time, through diverse examples, reveal the breadth of applications of process and bioprocess engineering. You will be exposed to classical problems of chemical engineering, environmental engineering, food engineering, biochemical engineering, biotechnology, and others.

How difficult is material balance? At this stage, this is a valid question because, as mentioned earlier, and as we will discover, material balance forms the core of this book and is a vital tool for process engineers. Soon enough, certain basic engineering courses will be easier for you because of your knowledge of material balance. In addition, you will discover many applications that are relevant to everyday life.

That is why we strongly suggest that you follow, step by step, all the advice in the first sections of this chapter.

Are material balance problems difficult? The definitive answer is no, and actually, they are easy. Although they might appear difficult, in reality, with adequate training and strategy they can be made easy, fun, and, most important, heartwarming. Some problems, and probably most real-world

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problems, can be nightmarish if not approached following an adequate procedure. Perhaps the key word in this chapter is PROCEDURE!

We base this approach on experience with several generations of freshmen who, after following the appropriate steps, became knowledgeable about material balance concepts and their applications.

We now invite you on this important and pleasurable journey to learn, face, formulate, solve, and apply material balance problems in process and bioprocess engineering.

7.2 What Is a process?

What is a process? A process is a series of operations (normally called unit operations, like dehydration, evaporation, crystallization, or fermentation, for example) accomplished in the manufacture of an intermediate or end product (Fig. 7.1).

What is a system? A system is specific part of a process specifically chosen by the process engineer to carry out a detailed analysis, in this case, a material balance analysis (Fig. 7.1).

The dotted lines in Fig. 7.1 show the different possibilities for choosing a system for analysis. As we will explain later, in order to analyze the whole process, it is advisable to analyze it unit by unit.

How processes are classified? We will identify two classifications: (a) how processes are operated as a function of time and (b) the mode of the process operation.

(a) How processes are operated as a function of time.

A **steady-state process** is one in which the system variables do not change over time. Imagine a system where we are measuring its variables (e.g., temperature, pressure, inflow rates, and outflow rates). We repeat the measurements at various times. If the system is in steady state, every time we take a measurement, all of the variables will have the same value. For example, if an equipment is fed with a stream of 10 kg/h, has a retention mass in the interior of 600 kg, and at the output has two outputs, one with a stream of 8 kg/h and the other with a stream of 2 kg/h, then the question is: is the equipment (system) under steady state? Yes. Why? Because the total mass flow rate at the input is

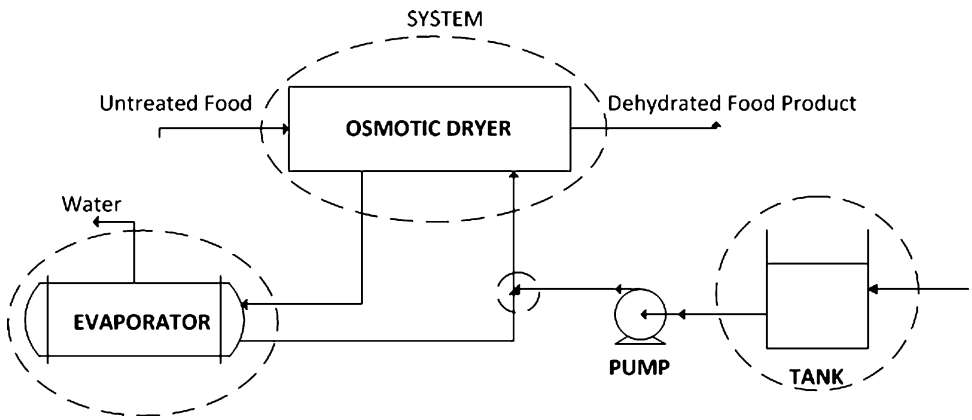

Fig. 7.1 Osmotic dehydration process

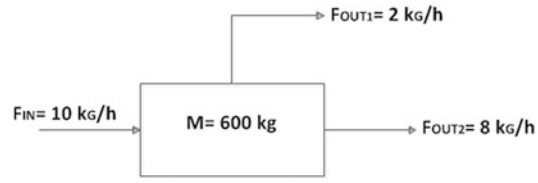

Fig. 7.2 Material balance in a steady-state process

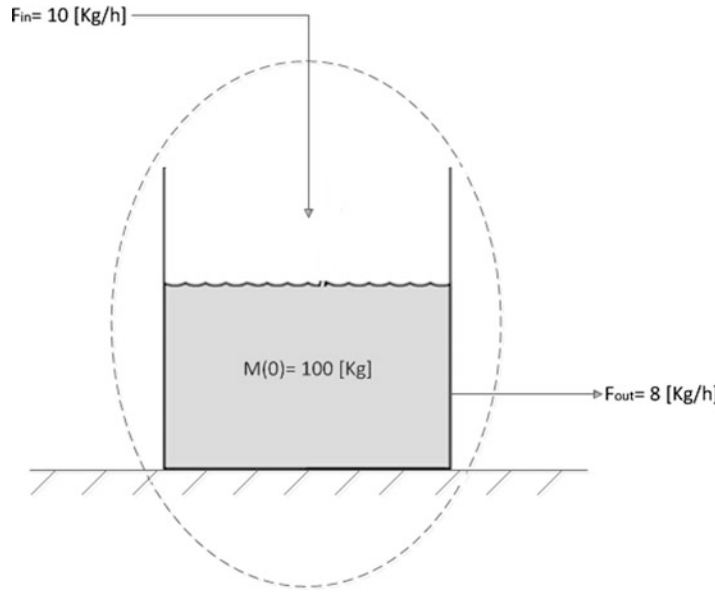

Fig. 7.3 Transient or unsteady-state process

constant and equal to the total mass flow rate at the output; therefore, the retention on the equipment will remain invariable (Fig. 7.2). As expressed in (7.1):

$$\text{Total input mass} - \text{Total output mass} = 0(\text{steady state}). \tag{7.1}$$

The dotted lines in Fig. 7.2 show the chosen system for analysis. Thus,

$$10\ \text{kg/h} - 8\ \text{kg/h} - 2\ \text{kg/h} = 0. \tag{7.2}$$

A **transient or unsteady-state process** is one in which the system variables change over time. Imagine the same system discussed previously under steady-state process. We take measurements of the system variables at some time t. If we return later to take the same measurements and find that the variables have changed, then the system is transient or in an unsteady state. An example would be a system in which the input mass flow rate and output mass flow rate differ. For example, if a tank is fed with a stream of 10 kg/h (Fig. 7.3), has an initial retention mass at the interior of $M(0) = 100$ kg, and at the output has one stream of 8 kg/h, then the question is: is the tank (system) under steady state? No. Why? Because the mass flow rate at the input is greater than the output mass flow rate; therefore, the

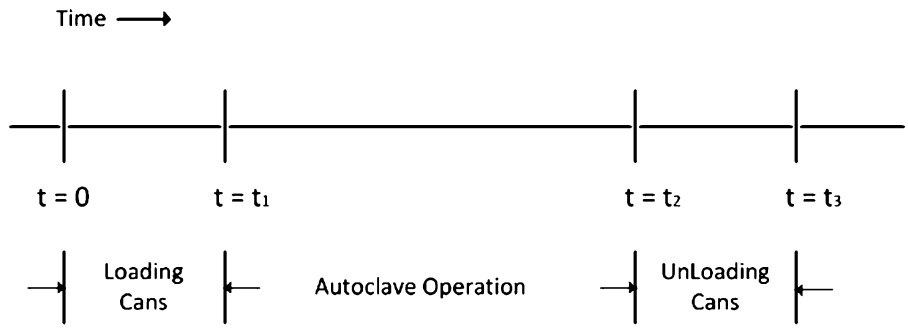

Fig. 7.4 Batch operation of an autoclave in a canning plant

retention on the tank will vary over time [$M(t)$ will accumulate over time]. The accumulation rate will be 2 kg/h. As expressed in (7.3):

As in Fig. 7.3 the dotted lines show the system under study. Thus,

$$10\ \text{kg/h} - 8\ \text{kg/h} = 2\ \text{kg/h}. \tag{7.3}$$

(b) What is the mode of process operation?
A **continuous process** is one in which the input and output streams operate uninterrupted. The product is continuously generated. Some examples are tomato concentration (food process), oil refinery (chemical process), and the brewing of beer (bioprocess).

A **batch process** is one in which the system is fed and closed, and then after a specified amount of time, product is obtained. Examples include soups, jams, specialty chemicals, canned foods, and wine fermentation. Briefly, in the case of canned foods, untreated cans are charged to the autoclave (retort); then the system is closed and operated at approximately 120 °C to sterilize the cans. After a certain amount of time (normally 60–90 min) the system is opened when the cans are commercially sterile and ready for the supermarket.

As shown in Fig. 7.4 the operation is stopped to load cans into the autoclave and then stopped after operation to unload cans.

A **semibatch (also called semicontinuous) operation** is an operation that has some continuous-process features combined with batch processes. Some process units are operated in a batch mode and other process units are operated in a continuous mode. An example of this type of process is also seen in retort processing. Here, individual retorts operate in batch mode, as shown earlier in Fig. 7.4. However, if we imagine a bank of three retorts operating in a plant, one of which is being loaded, another unloaded, and the third being operated (Fig. 7.5), it becomes apparent that from the perspective of the overall system, the process is continuous, even if the individual retorts are batch operated.

7.3 What Is Material Balance?

Material balance involves making an inventory of the quantities going in and out of a system. These quantities could be as simple as the total mass. The principles of material balances can be expanded to the balance of energy, electrical charge, and virtually any quantity that is conserved.

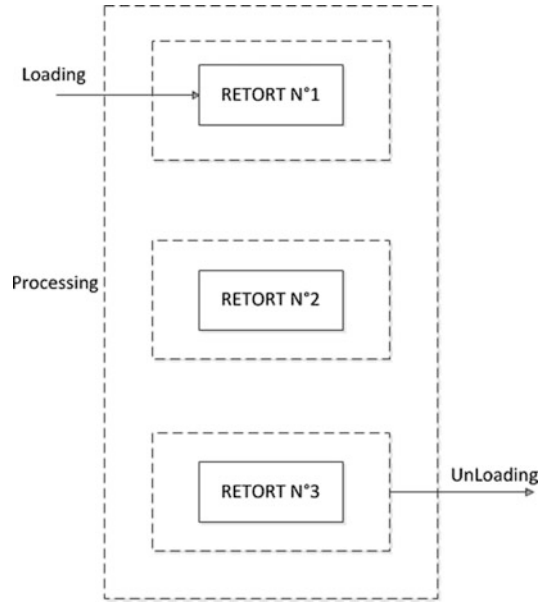

Fig. 7.5 Battery of three batch retorts operated in a continuous mode

Material balance is based on the law of conservation of mass that was established by Antoine Lavoisier (1743–1794) and states that *mass can neither be created nor destroyed, but is conserved (in any ordinary chemical reaction)*.

In general and simple terms (Fig. 7.6) we can express a total mass balance for a given unit or system as follows:

$$\begin{aligned}&\text{Total input mass flow rate} - \text{Total output mass flow rate}\\&\quad = \text{Rate of accumulation [unsteady-state condition]}.\end{aligned} \tag{7.4}$$

If input and output mass flow rates are equal, then we can say that the unit or system is under steady-state conditions because all mass flow rates and the retention mass of the system remain constant (further analysis and more explanations are given in Sect. 7.4). Therefore:

$$\text{Total input mass} - \text{Total output mass} = 0 \qquad [\text{steady-state condition}]. \tag{7.5}$$

For example, Fig. 7.6 shows a tank (system) with two streams, F_{IN} and F_{OUT}. If $F_{IN} = F_{OUT}$, then we have a steady-state condition (7.5), where M is constant, i.e., it does not change with time. On the other hand, if $F_{IN} \neq F_{OUT}$, then the system is in an unsteady-state condition (7.4) and $M(t)$ changes with time, where $M(t)$ will increase versus time if $F_{IN} > F_{OUT}$ or decrease if $F_{IN} < F_{OUT}$.

Numerical examples

Example 1. Steady-state condition. A tank holding 600 kg H_2O (Fig. 7.7) is fed with 1,000 kg/h H_2O and has two streams going out, one with 800 kg/h. What must the mass flow rate of the other stream (F_{OUT2}) be to maintain the tank in steady-state condition?

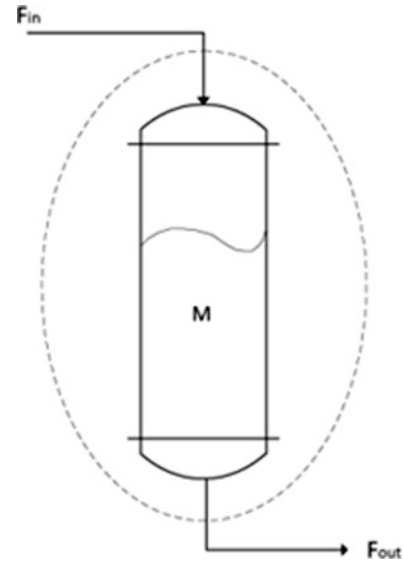

Fig. 7.6 Materials balance in a system

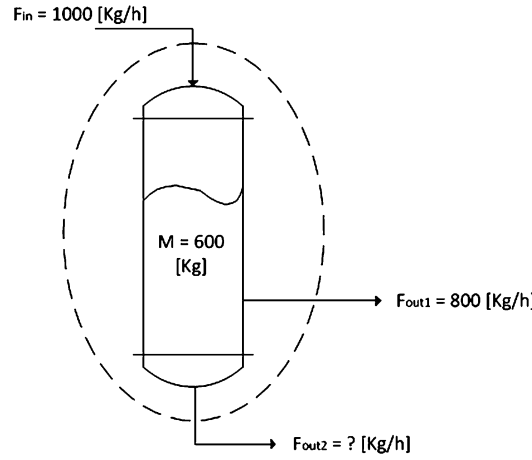

Fig. 7.7 Tank in a steady-state condition

According to (7.1) (steady state), if you sum up all input streams (in this case 1,000 kg/h) and subtract all the streams going out, the result should be 0.

Where F_{OUT2} is the unknown stream, then

$$1,000 - 800 - F_{\mathrm{OUT2}} = 0. \tag{7.6}$$

Therefore, $F_{\mathrm{OUT2}} = 200$ kg/h, where 0 in (7.6) means that the retention mass within the tank (600 kg) does not change with time (steady state); all the mass that enters the tank (1,000 kg/h) leaves (800 kg/h plus 200 kg/h). Although not explicitly mentioned, the tank in this analysis is the system under study.

One of the lessons here is that to carry out a material balance you must always specify the system, then it will be clear how to identify all the streams going in and out of the system; in this case, the

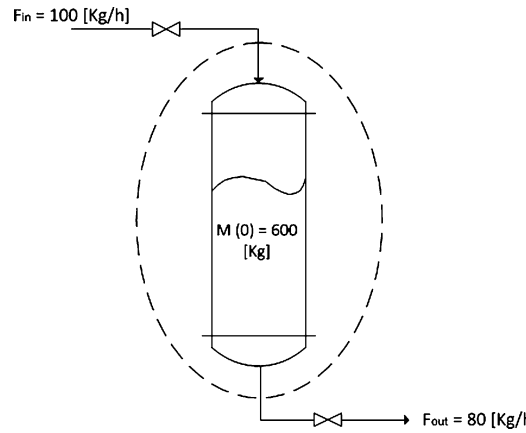

Fig. 7.8 Tank in an unsteady-state condition

system was the tank. In addition, it is very important not only to define variables but also the way they are codified. Real material balance problems have several variables and sometimes tens of variables. Therefore, you need to carefully define each variable and codify them in such a way that it will be easy for you to identify and associate each variable with the flow diagram.

Example 2. Unsteady-state condition. A tank initially contains 600 kg H_2O and is fed with 100 kg/h H_2O and has one stream going out at 80 kg/h. What is the mass balance for this system? Is the system under steady state?

Learning the lesson of the first example, we will first define the system under analysis (dotted lines).

System: tank. Therefore, the mass balance in this case is

$$100 - 80 = 20 = \text{Accumulation}. \tag{7.7}$$

Given that the right-hand-side term in (7.7) is equal to 20 (different from 0), we can state that the system (tank) is in an unsteady-state condition and, in addition, because the term is positive, the system is accumulating water at a ratio of 20 kg/h. As seen in Fig. 7.8, $M(0) = 600$, indicating that the mass in the tank at time 0 is 600 but, as was shown, is increasing at a ratio of 20 kg/h.

Summary

Although simple, some steps are common and necessary in all material balance problems. (a) Graphically represent your process (tank and streams). (b) Define the system under analysis (here, the tank). (c) Define all the variables (input and output streams). (d) Formulate the material balance problem and solve it.

As we progress through the chapter we will be discovering and structuring a general procedure to facilitate the formulation and solution of real material balance problems.

7.4 General Concepts on Material Balance

Mass is conservative. In simple terms, if you feed 1,000 kg of materials to a given process, you will get the same amount at the end; you cannot get 1,500 kg! Although reactions can occur, the total amount of matter will remain the same. Matter can be transformed but neither created nor destroyed. Exceptions are nuclear reactions, but these reactions are beyond the scope of this book.

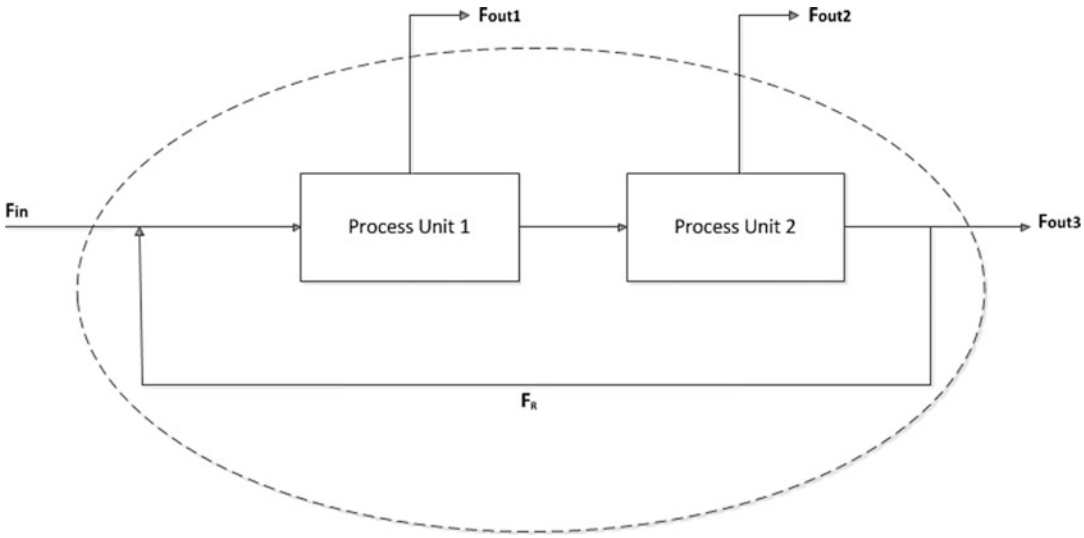

Fig. 7.9 Process with two units and a recycle

Mass is additive. Can balances be done based on the volume rate of the streams in a given process? No, because volume is not always additive. When you add X kg of material A to Y kg of material B, you will always obtain $X + Y$ kg of the A and B mixture. We cannot say the same of volume. For example, 1 L H_2O plus 1 L ethanol (at ambient conditions) will give you less than 2 L of the mixture. On the other hand, 1 kg H_2O plus 1 kg ethanol will always give you 2 kg of the mixture. Students are encouraged to investigate why sometimes (as in the example of H_2O plus ethanol) the volume is not necessarily additive and why the resulting volume, as in this example, is different than expected (see solved problem 12 in Sect. 5.3.2, Chap. 5).

Mass units. Before formulating and carrying out a material balance analysis and calculations (especially for nonreactive systems), it is strongly advised that all streams be expressed in mass units. As we will see in the next chapter, for reactive systems, it might be better to express streams in molar units.

Graphical representation and total mass balance. Figure 7.9 shows a typical scheme or diagram for a mass balance including two units and a recycle stream (F_R). The dotted line shows that the whole system has been selected as a system. Assuming steady state,

$$F_{IN} - F_{OUT1} - F_{OUT2} - F_{OUT3} = 0. \tag{7.8}$$

7.5 Why Material Balance Is Relevant for Chemical and Bioprocess Engineers

Material balance takes an exact account of all materials entering, leaving, accumulating, or being depleted in a given process unit for a specific time interval. The practical use of material balance lies in the fact that in reality it is very difficult for a process engineer to make direct measurements of all the masses of each process stream.

As you will discover in your development and education as an engineer, and later on as a professional, material balance is vital for process engineering. Material balance has broad applications that are even beyond the fields of process and bioprocess engineering (e.g., economics).

Making the inventory of material that enters, leaves, or is generated within a system allows one to know whether the system will be enriched or depleted with the material. In that way it is possible to determine how the system will change, and even the rate of change. This is relevant in equipment sizing, where a decision of how long it will take for the equipment to fill to capacity (e.g., with the tank in a toilet, the time for filling it up is given by the flow of water and the size of the tank; the size of the tank is associated with the flow required to clean it up. At every step in the process, volumes and times are determined by the material balance).

Several applications in chemical and bioprocess engineering work in such a way that the system is designed no to gain or be depleted of materials (or molecules or electrical carriers). This is known as a system operating in steady-state condition. In that case, the material balance changes to one of considering what is required to enter the system (after the generation or transfer of materials) to obtain a desired outflow.

Furthermore, the main focus of many material balance problems is to determine how much the processes operating inside the system must transform materials to obtain a desired product. This is how chemical and bioprocesses like chemical reactors, distillation columns, and fermenters (to mention a few) are designed. It is after this material balance step that the knowledge to design those operations merges (beyond the scope of this book). The sizing and design of this equipment will require knowledge of fluid mechanics, heat transfer, and mass transfer (in Chap. 6 we provided a glimpse of these topics).

In processes consisting of several units (e.g., chemical reactors, equipment for mixing/purification, autoclaves, fermenters) the analysis gets more complex, but the material balance is still a must and, as we will explain later, conducted on each unit, thus enabling the following steps in the design and specification of equipment.

Usually the material balance is the first step in process design and engineering projects. It is present in the day-to-day monitoring and supervision of chemical and bioprocessing plants. It is by far the most relevant tool for chemical and bioprocess engineers.

7.6 Formulating Material Balance Equations (Steady-State and Continuous Operation)

As mentioned at the beginning of Sect. 7.3, Antoine Laurent Lavoisier (1743–1794) was the first scientist to suggest that matter is neither created nor destroyed but conserved. The law of conservation of matter postulates that the amount of material before and after a process is strictly the same. What the law of conservation of matter implies is that, beyond transformations, matter continues to exist. In other words, atoms react with each other as substances, but atoms are neither created nor destroyed.

Formulation of a general mass balance for nonreactive and open systems under steady-state conditions (process unit):

According to (7.5) and Fig. 7.10 we can write a **TOTAL MASS BALANCE** as

$$\text{Total input mass flow rate} - \text{Total output mass flow rate} = 0 \qquad [\text{steady-state condition}]. \quad (7.9)$$

Then

$$\left(\sum_{i=1}^{i=n} \dot{m}_i\right)_{\text{in}} - \left(\sum_{j=1}^{j=k} \dot{m}_j\right)_{\text{out}} = 0, \quad (7.10)$$

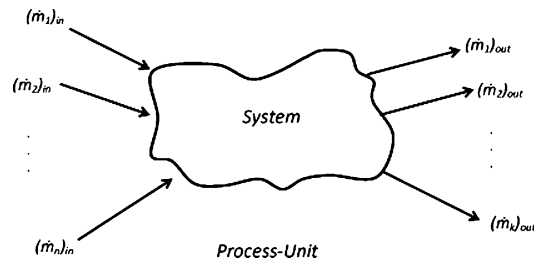

Fig. 7.10 General material balance for an open system

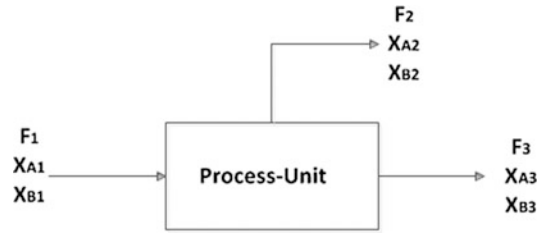

Fig. 7.11 Simple process-unit

where $\left(\dot{m}_i\right)_{\text{in}}$ is the mass flow rate of stream i entering the system (mass/time) and $\left(\dot{m}_j\right)_{\text{out}}$ is the mass flow rate of stream j leaving the system (mass/time).

In addition, given that it is a nonreactive system, the mass of each component is conserved. Then we can write one mass balance for each component. If we have p components, then we can formulate p additional equations. Writing the mass balance for some specific component r,

$$\left(\sum_{i=1}^{n} x_{ri}\dot{m}_i\right)_{\text{in}} - \left(\sum_{j=1}^{m} x_{rj}\dot{m}_j\right)_{\text{out}} = 0, \tag{7.11}$$

where x_{ri} is the mass fraction of component r in stream i, and x_{rj} is the mass fraction of component r in stream j.

The first term in (7.11) represents the addition of masses of component r in all input streams. Thus, the first term represents all masses of component r entering the system. In the same way, the second term represents all masses of component r leaving the system.

Therefore, we can write one total mass balance and, in addition, one mass balance for each component (1, 2, . . ., p). Since we have p components, we are able to write, in total, $p + 1$ equations, but **ONLY *p* of them are independent!** Why? For example, if we sum up all the material balance equations formulated for each component, then we discover that the result is equal to the total mass balance. **Remember, the number of independent equations is equal to the number of components in the process unit (system).**

Example 3. Independent material balance equations. In this example, we will prove quantitatively that the number of independent material balance equations is equal to the number of components. In Fig. 7.11 is depicted a simple process unit with one input stream with components A and B and two output streams, also with components A and B.

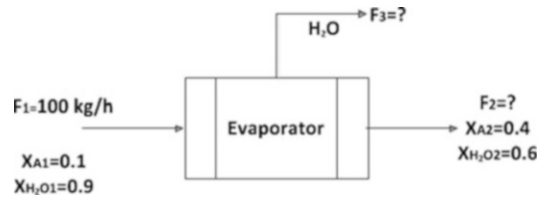

Fig. 7.12 Material balance in a single effect evaporator

Material balance

Total mass balance:

$$F_1 - F_2 - F_3 = 0. \tag{7.12}$$

Mass balance for component A:

$$x_{A1} \times F_1 - x_{A2} \times F_2 - x_{A3} \times F_3 = 0. \tag{7.13}$$

Mass balance for component B:

$$x_{B1} \times F_1 - x_{B2} \times F_2 - x_{B3} \times F_3 = 0. \tag{7.14}$$

As mentioned earlier, in this case we have two components, so we should have only two independent equations. For example, if we sum up (7.13) and (7.14), we obtain

$$(x_{A1} + x_{B1}) \times F_1 - (x_{A2} + x_{B2}) \times F_2 - (x_{A3} + x_{B3}) \times F_3 = 0, \tag{7.15}$$

where $(x_{A1} + x_{B1}) = (x_{A2} + x_{B2}) = (x_{A3} + x_{B3}) = 1$, meaning that (7.15) is exactly the same as (7.12). Therefore, if you use (7.13) and (7.14), then (7.15) is not independent. The same happens if you choose to use, for example, (7.12) and (7.13); then (7.14) will not be independent. The corollary is that if you have n components, then you have n independent material balance equations. However, you have the freedom to select which ones to use.

Example 4. Product concentration. A single-effect evaporator is fed with 100 kg/h of a stream that contains components **A** and H_20. At the output, one stream contains just H_2O (as a vapor) and the other, the stream concentrated in **A**, contains H_2O and component **A**. If the concentration of component **A** is 10 % w/w in the feed stream and 40 % w/w at the output stream, then (a) how much H_2O was removed in the evaporator (evaporated) and (b) what is the mass flow rate of the concentrated product?

According to the earlier examples in this chapter we will follow an intuitive procedure. Thus,

(I) Draw a flow diagram of the problem statement (Fig. 7.12):

(II) Variable definitions and codification

We would like to emphasize the importance of defining all variables and codifying them accordingly. Do not use generic terms like X, Y, and Z as variables because it is much better to codify variables in such a way that you can always associate them with your problem (flow diagram). In addition, it will be easier to have one codification for each variable.

F_1: Feed stream (kg/h) (100 kg/h)

F_2: Stream with concentrated product (component A) (kg/h)

F_3: Stream with evaporated H_2O

x_{A1}: Mass fraction of component A in feed stream (0.1 w/w)

x_{A2}: Mass fraction of component A in output stream (0.4 w/w)
x_{H2O1}: Mass fraction of H_2O in feed stream (0.9 w/w)
x_{H2O2}: Mass fraction of component H_2O in output (0.6 w/w)
Although the mass fraction of H_2O it is not given directly in the problem statement, because there are two components, the addition of mass fraction of component A and mass fraction of H_2O must be 1 in both streams.

(III) Mass balance formulation and solution. In this case we have two components (A and H_2O), so we can formulate two independent equations, as follows:
We will choose, first, the mass balance for component A because it has the advantage that it appears only in two of the three streams.

Mass balance for component A.
Mass of component A entering system − Mass of component A leaving system = 0 (steady state); thus, $0.1 \times 100 - 0.4 \times F_2 = 0$, then $F_2 = 25$ kg/h.

Total mass balance
Total mass entering system − Total mass leaving system = 0 (steady state). Therefore,
$100 - F_2 - F_3 = 0$, then substituting for F_2, we obtain $F_3 = 75$ kg/h. Therefore, (a) 75 kg/h and (b) 25 kg/h.

This easy example helps in understanding some basic concepts of material balance and demonstrates a procedure to be considered when solving complex material balance problems. In the following sections, we will further analyze the specifics of material balance and then design and develop a general procedure for solving material balance problems, whether basic or complex.

7.7 Material Balance Basics

As described here, it is important to consider the process of approaching, formulating, and solving material balance problems.

1. If the flow diagram is not provided with the problem statement, then draw a complete one on a whole page. Clarity at this point is critical and will help you get a feel for the problem. If a flow diagram is provided, sometimes it will be better to redo and complete it with all the available information.
2. Define all variables (known or unknown). Some variables will be specified, but it is advisable to have a definition for each variable and codify it in such a way that it will be easy for you to match each variable with your flow diagram. As was just mentioned in example 3, in general, avoid using generic variable names such as *X*, *Y*, and *Z*. It is much clearer to codify the variable according its role in the problem. An exception is the use of *x* for mass fraction composition.
3. To formulate a specific material balance, it is vital to first define the system under analysis. But who defines the system? You, the person formulating it! As shown in example 3, before writing a material balance it is strictly necessary to define the system and its boundaries. Only then can we write a material balance equation and determine which flows are entering and leaving the system. For an example, see Fig. 7.13.
4. For nonreactive systems (this chapter) it is better to use mass units for all streams (Fig. 7.13). As was pointed out earlier, mass is conservative. Recall the example of ethanol and water that showed that volume was not necessarily additive. As we will see in the next chapter (material

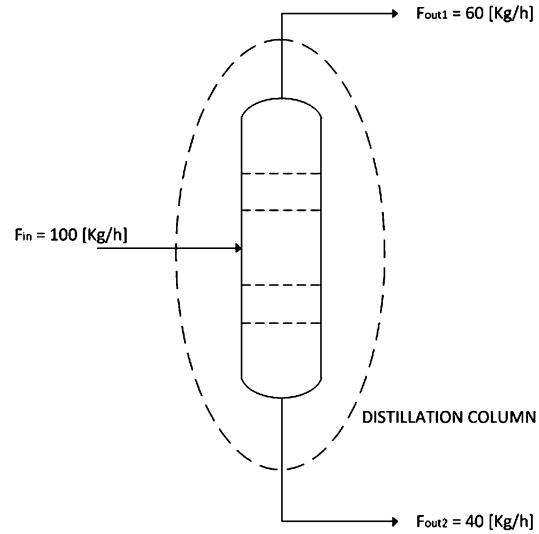

Fig. 7.13 Distillation column

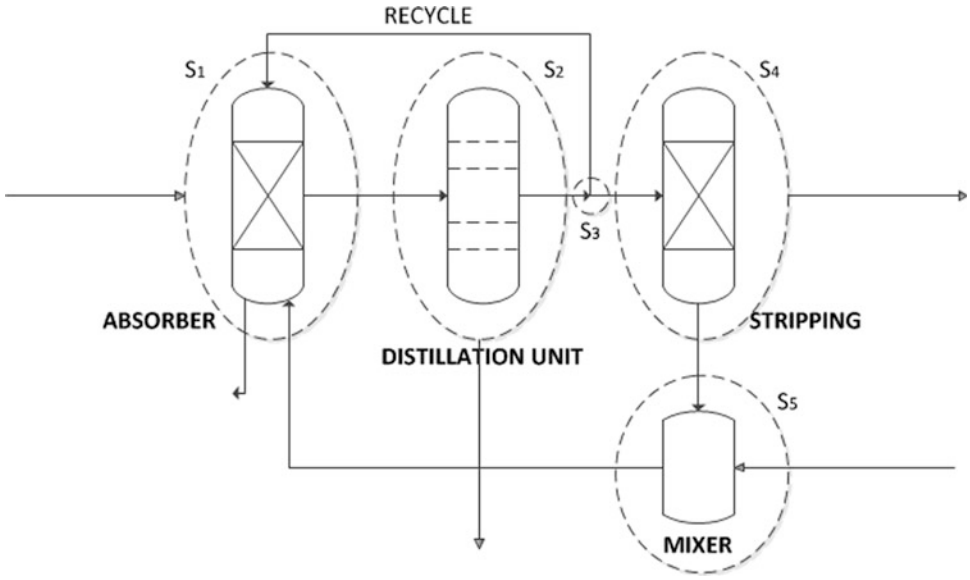

Fig. 7.14 Complex process including different units and a recycle

balance for reactive systems), moles should be used when the material balance includes reactions (due to stoichiometry).

5. Subdivide the whole process into its different units and analyze each unit step by step (Fig. 7.14). In complex processes that includes several units it is advisable to analyze one unit at a time. Normally you cannot solve the material balance problem for each unit independently, but to formulate the whole mathematical problem, it is better to go step by step (unit by unit).

As depicted in Fig. 7.14, the whole process is divided into five systems (S_1, S_2, . . ., and S_5) for further study. In addition, we can consider the total system (S_T), but of these six systems only five are independent.

6. In each unit of the processes, identify all the independent material balance equations. If you have n components, then you can write n independent material balance equations. In example 3, the feed and output streams had two components and then we were able to write two independent equations.
7. Minimize as much as possible the number of variables. For example, if you have two components in a stream, say H_2O and solids, you can use two variables, one for the mass fraction w/w of H_2O (X_{H2O}) and one for the solids (X_{SOLIDS}), and add one relationship that says $X_{H2O} + X_{SOLIDS} = 1$, or just use one variable (better), say X_{H2O}, knowing that you can calculate "mentally" X_{SOLIDS} from $X_{SOLIDS} = 1 - X_{H2O}$. As in example 3, we obtained the mass fraction of H_2O from $X_{H2O1} = 1 - X_{A1}$. An example is given in Warm-up Example 2 in Sect. 7.8.3.
8. If a number of substances is being maintained in fixed proportions in the process, it is better to consider them all together, enclosing them in one variable (see proposed problem 13 in Sect. 7.10). For example, if a food material (e.g., tomato juice) is to be concentrated by removing water, then all components (e.g., carbohydrates, proteins, fats) will remain together in the concentrated stream; then it would be most efficient to call all of them solids and use just one variable for it all.
9. A substance that enters the process and leaves without transformation is very important as a reference substance (or tie substance). Again, in example 4, we chose component A to formulate one of the material balance equations. The advantage in doing that was that component A was involved in just two streams and facilitated calculations.

First, for now, these nine points are simply a list of recommendations, but we will analyze these recommendations later when we focus on developing an integral and general procedure for approaching, formulating, and solving material balance problems. Second, we will exemplify the relevance of these tips on warm-up examples and when solving problems in Sect. 7.9 (solved exercises).

7.8 Designing and Structuring a General Procedure to Formulate and Solve Material Balance Problems

In this and the following chapter, we will dedicate all our efforts to teaching students mainly material balance under steady-state conditions in continuous mode. This option has the advantage of simplifying the mathematics involved and, in addition, at the same time does not limit us in our attempt to cover a very broad range of interesting examples in chemical and bioprocess engineering.

7.8.1 Developing a General Procedure for Material Balance Problems

As shown in example 4, following a simple and intuitive procedure the solution of a material balance problem seems straightforward. We need to acknowledge that example 4 is simple and not representative of real material balance problems, but on the other hand, it showed us that a sound fundamental strategy could help to simplify the solution. Systematically, our experience with several generations of freshmen has taught us that the following procedure (Fig. 7.15) helps students and professionals have clarity when facing flow diagrams, from the simple to the intricate.

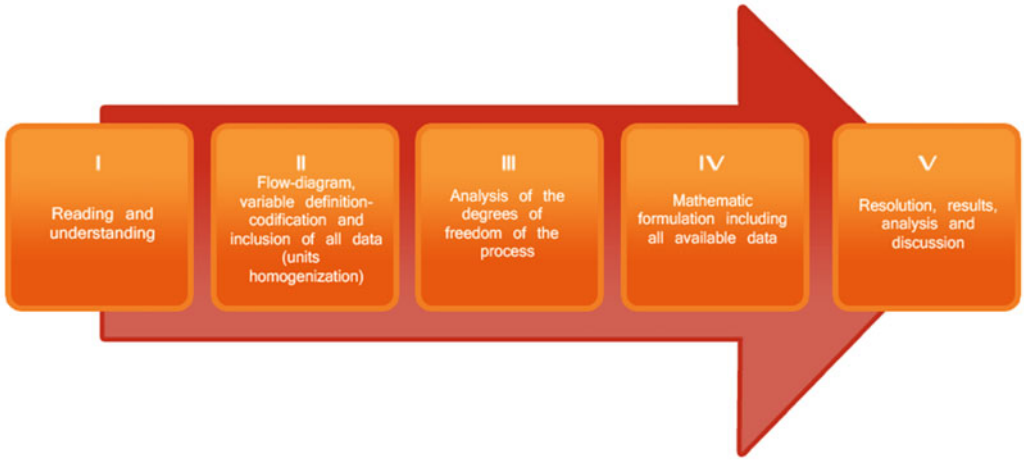

Fig. 7.15 General strategy and procedure to solve materials balance problems

This scheme may seem very detailed and often unnecessary for simple material balance problems. However, as will be seen again and again throughout the examples, this procedure is vital in solving most real plant material balance problems. In some problems, you might feel very confused and possibly a bit tormented by the intricate flow diagrams, but if you follow the proposed procedure step by step, most, if not all, problems can be correctly formulated and solved. We will be very emphatic that for you, as a freshman, but probably for most engineering students as well, all material balance problems should be tackled with a clear strategy, e.g., the strategy proposed here. This will be further shown in warm-up examples and in Sect. 7.9 (solved exercises).

7.8.2 Understanding the Procedure to Approach, Formulate, and Solve Material Balance Problems

In this section we present and describe, step by step, this schematic procedure to approach material balance problems.

The content presented below is essential and indispensable for your training as a future process engineer. A clear understanding on how to approach, formulate, and solve problems of material balance is extremely important and vital for a process engineer. If, after this chapter you develop the capacity and methodology to solve most problems of material balance, then the purpose of the chapter will have been fulfilled and you can consider yourself halfway to your goal of becoming an engineer!

As was emphasized in Chap. 5, the most important thing in solving a problem is to have a strategy and a method, i.e., a set of rules that guarantees the optimal decision and procedure. In addition, here again, at this stage, the most critical aspect is not to obtain a final solution but to be able to mathematically express and formulate the verbal statement of a material balance problem. Often we have been surprised when freshmen possessing a very basic background can approach, formulate, and solve challenging and complex material balance problems that include multiple units (e.g., Sect. 7.10, exercise 28). Our own challenge is to provide you with a method and empower you to solve real-world problems.

7.8.2.1 Procedure Description and Analysis

As depicted in Fig. 7.15 the proposed procedure consists of five steps in series that are described and analyzed below. Later, in the warm-up examples, we will apply it to problems.

STEP I

Reading and understanding

This first step is critical. Sometimes with long statements and when complex flow diagrams are included it is easy to get confused. If the problem statement includes the flow-diagram then a good strategy will be to read the statement and at the same time try to match it with the flow-diagram. Most of the time, it will be easier to follow the statement with the diagram and then you will start to understand and feel the problem. If the diagram is not provided, then you need to draw one right away: do not wait until **Step II** of this procedure. An idea is to start constructing the flow-diagram step by step reading each phrase of the problem at a time. You will have a good understanding if you match the problem statement with the flow-diagram. In addition, at this stage of your engineering career we strongly recommend you to familiarize yourself (just to gain a general understanding) with the main function of some common equipment used in process engineering. As we will see in the following examples and sections 7.9 and 7.10 some of the equipment items are: Distillation columns, Crystallizers, Evaporators, Presses, Settlers, and Humidifiers. In chapter 8 (materials balance for reactive systems) you will be exposed to reactors and fermenters.

STEP II

Flow-diagram, variable definition-codification and inclusion of all available data

Possibly most problems will include a flow-diagram but at this stage we recommend not only drawing a flow-diagram but also include all the available data and, in addition, include (or define) all the variables. All available means, all, not only the information related to mass flow rate of the streams and compositions, but also any given relationship (e.g. $F_1/F_2 = 2$, the ratio of two flow streams). Because we will be adding codified variables in the flow-diagram, then it is strictly necessary to define each variable of the problem. As mentioned the right codification is very important. Then, we will have a clear understanding of all variables involved at its role in the problem and which ones are known and unknown, in addition, all given relationships. This step will be done with a complete flow-diagram and a list with well-defined and codified variables. If the flow-diagram is not attached to the problem, then we must devote ourselves to "decipher" the problem verbal-statement and construct (draw) the flow-diagram, and if the flow-diagram is attached, in some cases we might re-draw the flow-diagram to have breadth and clarity of each of the units and processing lines. In some specifics cases, it is necessary to add “units” to the flow-diagram, for example, when a recycle stream, a by-pass or a side-stream join another stream is better to add a mixer for clarity (see examples and definitions of recycle and side-stream in Step III).

STEP III

Analysis of the degrees of freedom of the process

Firstly, what are the degrees of freedom? For example if you are solving the equation X + Y = 5, we can say that it has infinite solutions because for a specific value for X you get a specific value for Y, but for a

different value of X you get a new value for Y. This situation occurs because you have 2 variables (X and Y) and just 1 equation, then you have 1 degree of freedom, meaning that you have the freedom to fix the value of one variable and then solve the problem for the unknown value of the other variable. In this simple case we can state that the degrees of freedom (*DF*) are:

DF = Number of variables (unknowns) – Number of equations 7.16

Where the problem is set and ready to be solved when *DF* = 0 (e.g. 2 variables and 2 equations).

In materials balance problems we face a similar situation. We will be expressing our materials balance with equations. Although, in materials balance we will face a slightly different situation because equations are not given to us: we need to formulate them. In addition, some variables will be specified and also some relations will be given. Although conceptually, it is the same the situation, the definition of *DF* in materials balance problems is slightly different and will include additional terms.

Why is the analysis of degrees of freedom important? To mathematically solve the materials balance problem we need to solve a set of equations and as mentioned to set the problem we need to reach the state of *DF* = 0. For example if the degrees of freedom are 2, then, we have the freedom to fix two variables in the process. Even though, when *DF* > 0 you have the freedom to fix some variables (e.g. if *DF* = 4 you can fix four variables) your freedom is limited. Yes, you will be able to fix four variables but with limitations. Why? For example all mass flow rates should be positive numbers, all mass fractions should be positive, the sum of all mass fractions in a stream should be 1, etc. We will further analyze these limitations.

Definition of degrees of freedom in a materials balance problem

As we have seen through examples 1, 2, and 3 in materials balance problems we have variables and equations but also we could have variables that are previously specified and some relationships could be given. Then to analyze the degrees of freedom in materials balance problems we will define:

DF: Degrees of freedom of the process.

NV: Total number of variables in the process (mass flow rates and mass fractions).

NMB: Total number of independent materials balance equations in the whole process.

NSV: Total number of independent specified variables (e.g. Feed flow = 100 kg/h)

NR: Total number of given relationships among the variables (e.g. $F_1 + F_2 = 150$ kg/h)

Therefore the degrees of freedom in a materials balance problem can be generalized by the following equation:

$$DF = NV - (NMB + NSV + NR) = (NV - NSV) - (NMB + NR) \quad 7.17$$

The right hand side of equation 7.17 could be interpreted as the number of unknowns (*NV – NSV*) less number of independent equations (*NMB + NR*).

Examining equation 7.17 we can identify three different cases as follows:

a) *DF* > 0, the problem is under-specified. To solve it you need to provide some extra information (e.g. if *DF* = 1, then you have the freedom to fix one of the process variables, for example if the feed flow rate is unknown you can assign a value, say 100 kg/h and then *DF* will be 0). When you assign a value to a variable, means that you are adding an equation e.g. $F_1 = 100$.

b) *DF* = 0, the problem is correctly specified, and you can proceed to solve it.

c) *DF* < 0, the problem is over-specified, and then some of the information is redundant and must be discarded. In addition, it is necessary to check if the redundant information is consistent and check which information will be discarded.

It is not necessarily trivial or straightforward to determine the respective values for *NV*, *NMB*, *NSV* and *NR*, thus we will devote the next section to explain how to determine each value in a given process.

a) Total number of variables (*NV*), Total number of specified variables (*NSV*) and Total number of relationships *(NR)*. As expressed in the previous step (step II) all variables (known or unknown) will be included in the flow-diagram. Then from the complete flow-diagram developed in step II we can get *NV*, *NSV* and *NR*.

b) Total number of independent materials balance equations (*NMB*). It is necessary to determine which are all the equations (without writing them) that can be formulated. Frequently a process is composed of multiple units, and sometimes including by-passes, side-streams and recycles. Inferring from section 7.6 if a process-unit has a total of n_i components entering and going out of the unit then the number of independent materials balance on the unit is n_i. Strictly, you can formulate $n_i + 1$ equations, one per each component and, in addition, a total mass balance, but only n_i of them are truly independent. It is important is to recall that you have the freedom to select which n_i equations you will be using from the $n_i + 1$ total equations. For example you can include or not within the n_i equations the total mass balance.

In the same manner, if the complete process has *P* units and each unit has n_i components (where *i* represents the number of the unit, from 1 to *P*), then the total number of independent materials balance equations is*:*

$$NMB = \sum_{i=1}^{i=P} n_i = n_1 + n_2 + \ldots + n_P \qquad 7.18$$

Where:

NMB: Number of independent materials balance equations.

P: Number of process units.

n_i: Number of components in unit *i* (i = 1, 2, 3, ..., *P*)

As will be shown in warm-up example 2 if the process includes by-passes, side-streams, and/or recycle streams (see Figure 7.16) it is advisable to include an additional process-unit (mixer) in your flow-diagram showing the mixture of one stream and, for example, a recycle stream. This advice is critical because you need to consider this "additional unit" when determining the number of independent materials balance equations (*NMB*). In addition, if a stream is divided (e.g. side-stream), all new streams will have the same composition but you need to add one total material balance equation. When the side-stream joins the process again, it is necessary to add a new process-unit (mixer).

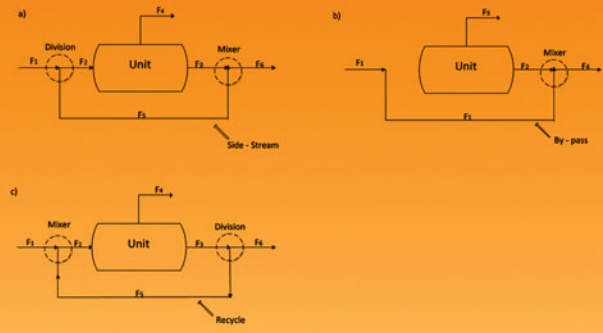

Fig. 7.16 A process depicting side-stream (**a**), by-pass (**b**) and recycle (**c**)

As depicted in Figures 7.16a, b and c we have added dotted line where two streams are mixed. When writing the materials balance equations it is necessary to consider this situation as a new unit: mixer. As an example Figure 7.16a is better drawn as shown in Figure 7.17.

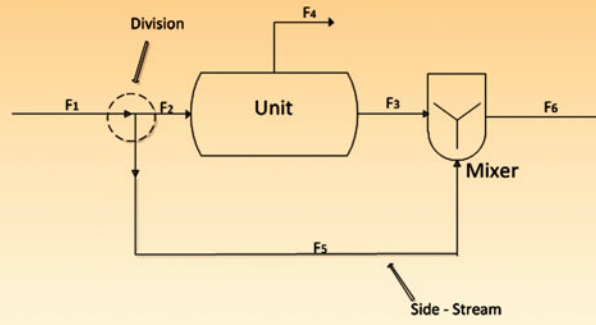

Fig. 7.17 Process with a side-stream

As you can comprehend from Figures 7.16 and Figure 7.17 we can state that:

Each side-stream and a recycle stream in the process will add a division (one total mass balance) and a mixer. The mixer will add a number of material balance equations according to the total number of components involved in it.

Each by-pass will add a mixer (as shown in Figure 7.17) where the additional number materials balance equations will be directly related with the total number of components involved in it.

Additional considerations on the degrees of freedom. We already have all the tools to determine the values for *NV*, *NMB*, *NSV* and *NR,* then we can determine *DF*. Now we need to understand that its use has some limitations. For example if we have some degrees on freedom in your process, then, certainly you will be able to fix some unknown variables in your process but with some limitations. For example if a process-unit has three degrees of freedom and with one stream at the input and two streams at the output you cannot fix all three streams arbitrarily, there is a mass balance equation that relates these three streams, then, in this case, you can fix two of these three streams. We are not implying that you cannot use all the *DF*, just to state that some constraints limit your freedom in the choices of values and variables used as *DF*. This concept will be further analyzed and exemplified in the warm-up examples and reinforced in section 7.9. For example in warm-up example 2 we will show how to analyze the degrees of freedom in a multiple unit process including a side-stream.

As a teaching and learning-process strategy, in this chapter, most of the problems will be correctly specified or under specification (i.e. $DF \geq 0$). As mentioned before if $DF < 0$, not only is it required to discard the redundant information but also is necessary to check if the redundant information is consistent within the problem.

STEP IV

Mathematical formulation including all available data

Firstly, we already have a complete flow-diagram (step II) with all variables and available information. Secondly, with the analysis of degrees of freedom (step III) we have determined how many independent materials balance equations (*NMB*) there are in the process. Thirdly, given that we already know what is the number of independent materials balance in the process, then we need to determine which of the equations will be used. For example in each unit we can choose whether or not to use the total mass balance. In addition, we need to decide whether or not we will use the global mass balance. Given that normally we have information on the input and output streams and its composition appears reasonable, first, to explore the possibility to use the global mass balance. Therefore we need to write all the selected equations and if $DF > 0$ then specify some variables to get $DF = 0$ to set and solve the problem.

STEP V

Solution, results, analysis and discussion

Normally, in mathematical terms the problem to be solved will be a system of equations. Although sometimes, it might be hard to solve, our main goal is to empower you until the point that you can correctly formulate the whole materials balance problem. In addition, if you are able to solve it, great! You will eventually learn the procedures and tools for solution of large systems of equations.

Once you have the results is necessary to check if they are according to your expectations and more than that, hopefully test if they are correct. In warm-up examples 1 and 2 we will show you some cases were results are properly tested. In addition, this concept will be further analyzed in section 7.9 (solved problems).

Example 5. Analyzing the degrees of freedom. A separator is fed with a stream with solids and H_2O (Fig. 7.18), and at the output there are two streams, both containing solids and H_2O.

One of your best friends and also your classmate in Introduction to Chemical and Bioprocess Engineering states that in the system (separator) you can identify four degrees of freedom. He argues that there are six variables (three streams and, e.g., three solid mass fractions). Thus, NV = 6, and because the streams contain two components, he argues that you can formulate two independent material balances (NMB = 2). In addition, there are no specified variables and no relationships are given. Therefore, NSV = 0 and NR = 0. At this point, thanks to your knowledge of material balance, you fully agree with your classmate because applying (7.17) you get

$$\mathrm{DF} = \mathrm{NV} - \mathrm{NMB} - \mathrm{NSV} - \mathrm{NR} = 6 - 2 - 0 - 0 = 4.$$

Then your friend says that he will specify four variables (NSV = 4) to solve the problem. He chooses the solid mass fraction at the feed stream ($x_{S1} = 0.1$) and the flow rate of the three streams ($F_1 = 100$ kg/h, $F_2 = 80$ kg/h, and $F_3 = 20$ kg/g) and tells you to quantitatively solve the problem. Then, first, you dutifully follow the procedure presented in this book, reading and rereading the problem and drawing a flow diagram (steps I–III) including all data, defining all variables (Fig. 7.19), and determining the degrees of freedom.

Step I–III

F_1: Mass flow rate of feed stream (kg/h)
x_{S1}: Solid mass fraction composition in feed stream
F_2: Mass flow rate of first output stream (kg/h)
x_{S2}: Solid mass fraction composition in first output stream
F_3: Mass flow rate of second output stream (kg/h)
x_{S3}: Solid mass fraction composition in second output stream

According to your friend you have $\mathrm{DF} = \mathrm{NV} - \mathrm{NMB} - \mathrm{NSV} - \mathrm{NR} = 6 - 2 - 4 - 0 = 0$. Therefore, the problem is set and ready to be solved.

Step IV

Mathematical formulation including all available data

First, we will write the material balance in generic terms without any data as follows:

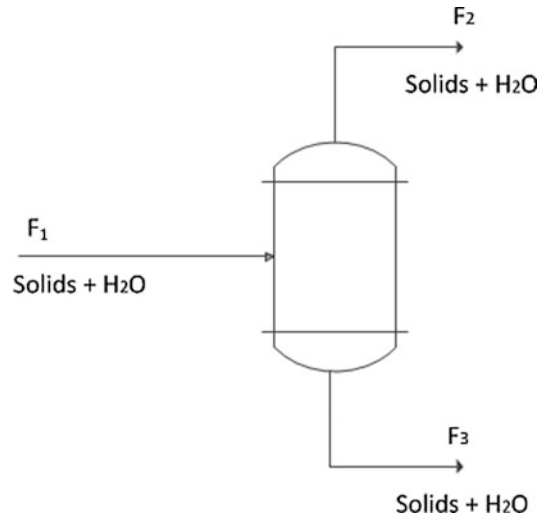

Fig. 7.18 Analysis of the degrees of freedom in a process

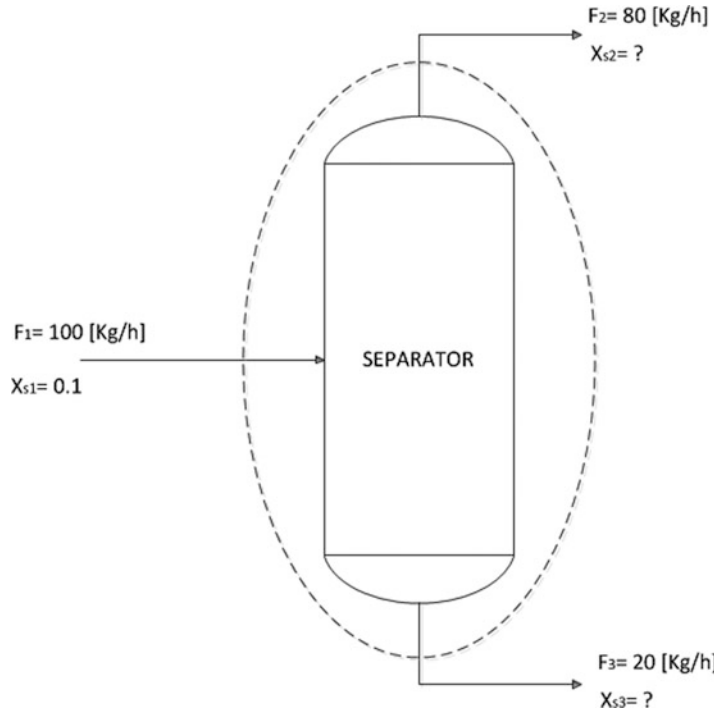

Fig. 7.19 Flow diagram including all variables and available data

Mass balance for solids

$$F_1 x_{S1} - F_2 x_{S2} - F_3 x_{S3} = 0. \tag{7.19}$$

Total mass balance

$$F_1 - F_2 - F_3 = 0. \tag{7.20}$$

Therefore, replacing the specified variables given by your classmate in (7.19) and (7.20) we get

$$100 \times 0.1 - 80 \times x_{S2} - 20 \times x_{S3} = 0, \tag{7.21}$$

$$100 - 80 - 20 = 0. \tag{7.22}$$

We did follow the advice given by our friend, and it seemed correct, but we have one equation (7.21) and two variables (x_{S2} and x_{S3}). Clearly 7.22 is not an equation. What went wrong? Strictly speaking, we have one degree of freedom, and we cannot solve the problem!

Our friend told you that he will be specifying four variables, and we believed him. The question is whether or not he really specified four variables. The definitive answer is no. Why not? He specified x_{S1}, F_1, F_2, and F_3. It seems that he is specifying four variables but really he is only specifying three. If he specifies F_1 and F_2, he does not have the freedom to fix F_3 because F_3 is already fixed by the total mass balance ($F_1 - F_2 - F_3 = 0$). As was mentioned when we were analyzing step III of the proposed procedure, there are some limitations in assigning the degrees of freedom. Yes, if we have four degrees of freedom, then we will be able to specify four variables, but with limitations, as shown in this example. The lesson here is to be cautious and select the variables to be specified in such a way that they are independent of each other. In this example F_1, F_2, and F_3 are not all independent because there is a relationship among them (total mass balance). We will return to this important aspect in Sect. 7.9.

7.8.3 Solving Material Balance Problems with the Proposed Procedure

As Einstein said, "*Example is not another way to teach, it is the only way to teach.*" Because we agree with this statement, we are fully committed to giving you a large number of examples, either solved or to be solved. In this section we will provide some warm-up examples to start practicing the proposed procedure.

It is definitely advisable to follow these two warm-up examples step by step. They are simple but will guide you in the first steps of the proposed procedure.

Warm-Up Example 1

Dryer [3]. 100 kg/h of a food are fed to a continuous dryer (H_2O removal) operated under steady-state conditions to reduce its moisture content from 88 to 12 %. Assume that for the purpose of the problem the food has two components: solids and H_2O (Fig. 7.20). (a) What is the amount of dried food obtained (kg/h)? (b) How much H_2O (kg/h) is removed in the continuous dryer?

Solution

Step I

Reading and Understanding

It might appear to be an exaggeration to follow the proposed procedure (Fig. 7.15) for such a simple problem that an advanced student could probably solve without any strategy. Experience has repeatedly shown us that, although it is simple, it is better and advisable to always follow the proposed procedure. It is a good idea to remember our experience with the shoe problem in Chap. 5. Maybe you solved it mentally and got the wrong answer. Our purpose here is to familiarize you with the proposed procedure and demonstrate its use.

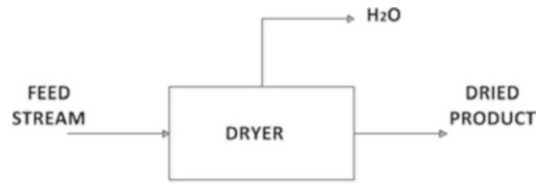

Fig. 7.20 Drying process

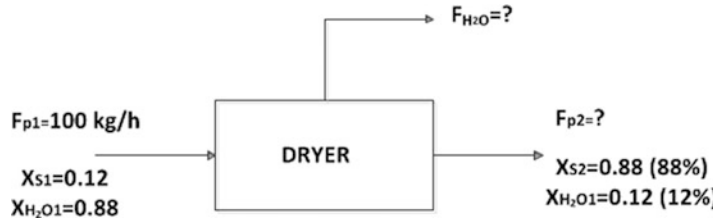

Fig. 7.21 Drying process including all variables and available data

In this case we have a simple unit operation for water removal to significantly reduce the initial product moisture content. A dried food product is much more stable than a high moisture product and has an extended shelf life.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

Variable codification is very important and crucial for your own clarity when formulating and solving problems, so we propose the following:

F_{P1}: Mass flow rate of feed stream (kg/h)
x_{S1}: Solid mass fraction composition in feed stream
x_{H2O1}: H_2O mass fraction composition in feed stream
F_{P2}: Mass flow rate of first output stream (kg/h)
x_{S2}: Solid mass fraction composition in first output stream
x_{H2O2}: H_2O mass fraction composition in first output stream
F_{H2O}: Mass flow rate of H_2O the second output stream (kg/h)

Then the total number of variables is seven (NV = 7).

NOTE: As we will see in warm-up example 2, it is not necessary here to use two variables for mass fraction composition in each stream. For example, we know that $x_{H2O1} = 1 - x_{S1}$, so we can reduce the total number of variables used here to just one mass fraction as a variable. In this simple example, it is not really critical, but in general we will try to minimize the number of variables!

Step III

Analysis of degrees of freedom in process

In this case, we have two components, solids and H_2O, and one process unit (dryer). Therefore, we can formulate two independent material balances (later we will decide which ones and then write its equations).

Thus, NMB = 2.

Observing Fig. 7.21 we notice that we have five specified variables and no relationships are given. Thus, NSV = 5 and NR = 0.

Finally, the number of degrees of freedom (DF) in this example is

$$\mathrm{DF} = \mathrm{NV} - \mathrm{NMB} - \mathrm{NSV} - \mathrm{NR} = 7 - 2 - 5 - 0 = 0; \quad \mathrm{DF} = 0. \tag{7.23}$$

Thus, the problem is set and ready to be quantitatively formulated and solved.

Step IV

Mathematical formulation including all available data

As mentioned, we can write two independent material balance equations. Given that solids are just in two of the three streams, it is a good option to write a mass balance for solids. The other good option in this case is the total mass balance. Thus:

Mass balance for solids

$$x_{S1}F_{P1} - x_{S2}F_{P2} = 0. \tag{7.24}$$

Total mass balance

$$F_{P1} - F_{P2} - F_{H2O} = 0. \tag{7.25}$$

Therefore, the complete mathematical formulation for the whole process including all available data will be.

Specified variables

$x_{S1} = 0.12$; $x_{H2O1} = 0.88$; $x_{S2} = 0.88$; $x_{H2O2} = 0.12$ and $F_{P1} = 100$ kg/h.

Therefore, substituting into (7.24) and (7.25) we get

Mass balance for solids

$$0.12 \times 100 - 0.88 \times F_{P2} = 0. \tag{7.26}$$

Total mass balance

$$100 - F_{P2} - F_{H2O} = 0. \tag{7.27}$$

Step V

Solution, results, analysis, and discussion

The solution to these equations is straightforward. From (7.26) we get $F_{P2} = 13.6$ kg/h, and then, substituting F_{P2} in (7.27), we get $F_{H2O} = 86.4$ kg/h.

Clearly to reduce the humidity from 88 to 12 %, a large amount of H_2O was removed (86.4 of 100 kg/h in the feed stream). Although the results look reasonable, it is much better to try to test whether the results are correct. For example, the feed stream (100 kg/h) contained 12 % solids. Thus, we have 12 kg/h of solids. In the output stream ($F_{P2} = 13.6$ kg/h) we have the same amount of solids (12 kg/h), and so the mass fraction of solids at the output should be 12/13.6, which is 0.88, or 88 %, solids!

Warm-up Example 2

Evaporation unit and a dryer [5]. An unusual and extremely delicate pharmaceutical product should be dried to reach a very low moisture content (3 % w/w). The process (Fig. 7.22) has two

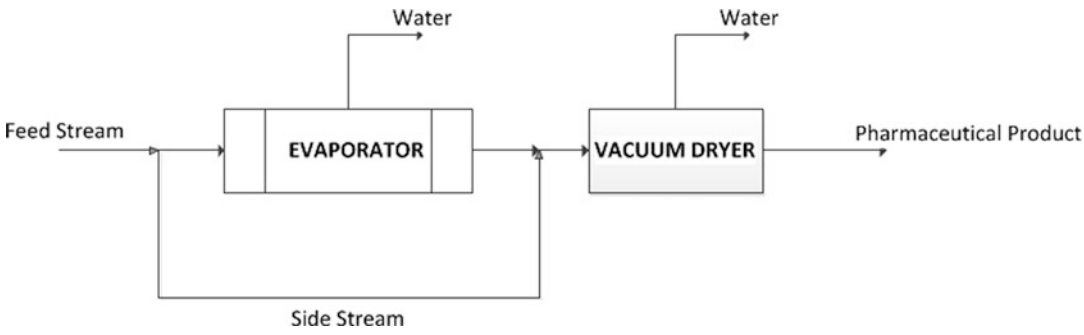

Fig. 7.22 Three step process: Evaporation, mixing and drying

units, first a single-effect evaporator and then a vacuum dryer. The feed stream has a mass flow rate of 1,000 kg/h with 12 % w/w solids (assume that the feed stream has two components: solids and H_2O). The feed stream is divided into two streams, one going directly to the single-effect evaporator and the other (side stream) joining the output stream of the evaporator that contains the solids. The output of the single-effect evaporator has two streams, one stream with just H_2O and a second stream with solids and H_2O (the one that receives and is mixed with is the side stream). This stream is fed to the vacuum dryer to remove more H_2O and reach the desired final 3 % w/w moisture content. If the side stream has a mass flow rate of 200 kg/h and the single-effect evaporator removes H_2O at a rate of 500 kg/h, then: (a) What is the mass flow rate of the pharmaceutical product? (b) What is the mass fraction of solids at the output stream of the single-effect evaporator?

Step I

Reading and understanding

This problem includes two units (single-effect evaporator and a vacuum dryer) and, in addition, a side stream. Considering that our first object in the material balance is to minimize the number of variables, we will consider one unknown mass fraction for each stream (because, as mentioned, the sum of the two mass fractions is 1). In addition, the side stream adds a division (one material balance) and a mixer (two material balances).

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

As depicted in the flow diagram (Fig. 7.23), we have included a mixer and all data. Thus, we can identify 14 variables. In addition, as shown in the flow diagram, the variables are defined as follows:

F_1: Mass flow rate of feed stream (kg/h)
x_1: Solid mass fraction composition in feed stream
F_2: Mass flow rate of side stream (kg/h)
x_2: Solid mass fraction composition in side stream ($x_1 = x_2 = x_3$)
F_3: Mass flow rate entering single-effect evaporator (kg/h)
x_3: Solid mass fraction composition in stream entering single-effect evaporator
F_4: Mass flow rate of H_2O out of single-effect evaporator (kg/h)
F_5: Mass flow rate out of single-effect evaporator (kg/h)
x_5: Solid mass fraction composition in stream out of single-effect evaporator
F_6: Mass flow rate entering the vacuum dryer (kg/h)
x_6: Solid mass fraction composition in stream entering the vacuum dryer

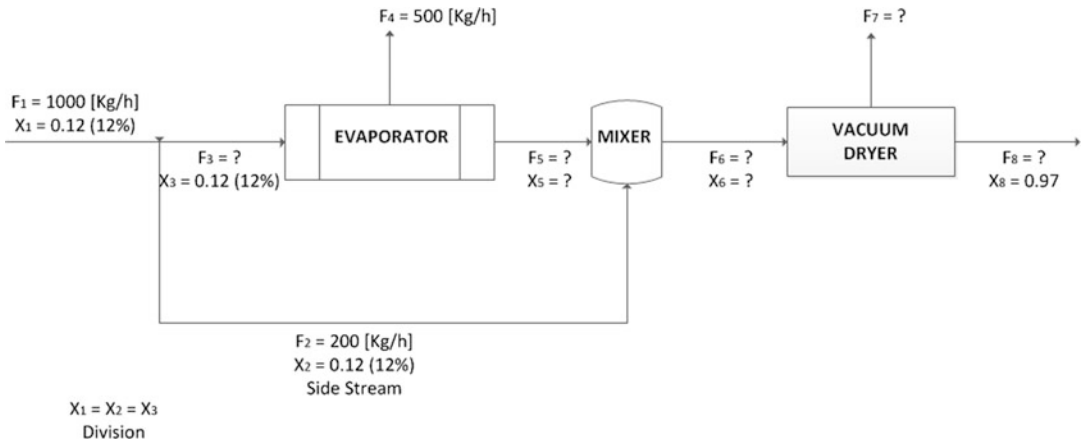

Fig. 7.23 Three step process including all variables

F_7: Mass flow rate of H_2O out of vacuum dryer (kg/h)
F_8: Mass flow rate of delicate product (3 % humidity) (kg/h)
x_8: Solid mass fraction composition at stream with delicate product

Thus, the total number of variables is 14 (NV = 14).

Step III

Analysis of degrees of freedom in process

In this process we have two units, but because the side stream is mixed with the output stream of the single-effect evaporator, we added a third unit—a mixer (Fig. 7.23). Therefore, given that we have two components in each unit, the total independent material balance is 6 ($n_1 + n_2 + n_3 = 2 + 2 + 2 = 6$). In addition, the feed stream is divided in two, and here we can add one more independent material balance equation, so NMB = 7.

As depicted in Fig. 7.23, the number of specified variables is 7 (NSV = 7). Given that there is no relationship information, we have

$$\mathrm{DF} = \mathrm{NV} - \mathrm{NMB} - \mathrm{NSV} - \mathrm{NR} = 14 - 7 - 7 - 0 = 0; \quad \mathrm{DF} = 0. \tag{7.28}$$

Thus, the problem is set and we can proceed to formulate the equations and solve the problem.

Step IV

Mathematical formulation including all available data (Fig. 7.23)

System S_1

Total mass balance at division (side stream):

$$1,000 - 200 - F_3 = 0. \tag{7.29}$$

System S_2

Total mass balance at evaporator:

$$F_3 - 500 - F_5 = 0. \tag{7.30}$$

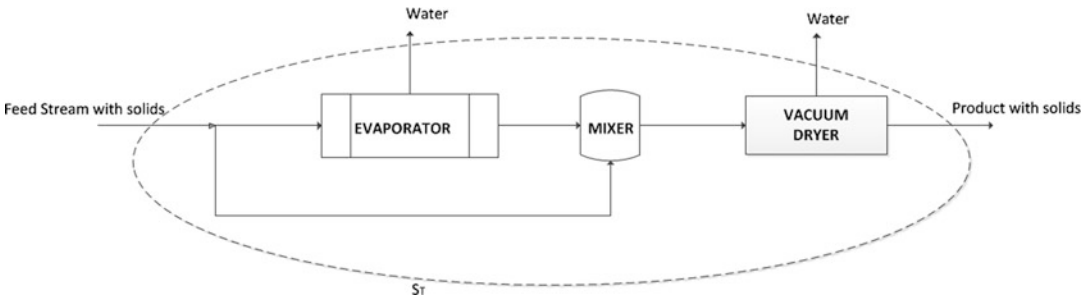

Fig. 7.24 Global material balance in the three step process

Solid mass balance at evaporator:

$$0.12 \times F_3 - x_5 \times F_5 = 0. \tag{7.31}$$

System S_3
Solid mass balance at mixer (side stream):

$$0.12 \times 200 + x_5 \times F_5 - x_6 \times F_6 = 0. \tag{7.32}$$

Total mass balance at mixer (side stream):

$$200 + F_5 - F_6 = 0. \tag{7.33}$$

System S_4
Total mass balance at vacuum dryer:

$$F_6 - F_7 - F_8 = 0. \tag{7.34}$$

Solid mass balance at vacuum dryer:

$$x_6 \times F_6 - 0.97 \times F_8 = 0. \tag{7.35}$$

Step V

Solution, results, analysis, and discussion

As shown in (7.29)–(7.35), we have formulated seven equations and we have seven unknowns. As is normal, in this material balance problem (steady state and continuous operation) the equations are simple and not necessarily hard to solve. In this case, it is relatively straightforward, starting with (7.29), and then we finally get

$F_3 = 800$ kg/h; $F_5 = 300$ kg/h; $x_5 = 0.32$ kg solids/kg solution; $F_6 = 500$ kg/h; $x_6 = 0.24$ kg solids/kg solution; $F_7 = 3.76 \times 10^2$ kg/h; $F_8 = 1.23 \times 10^2$ kg/h.

Answering the questions of the problem, (a) 1.23×10^2 kg/h of the delicate pharmaceutical product (F_8), and (b) 0.32 kg solids/kg solution (x_5).

Testing the results. If you formulate a total mass balance for solids in the whole process, then you can write (Fig. 7.24)

$0.12 \times 1{,}000 - 0.97 \times F_8 = 0$ (S_T in Fig. 7.24), and thus $F_8 = 1.23 \times 10^2$ kg/h.

Recall that it is not always simple to test the results, but at the very least you need to analyze the results and try to figure out if they make sense in your problem.

Although, strictly speaking, this is not a very difficult problem, it is a good example of how easy it is to manage a problem with several units and large numbers of variables and equations.

Please stop and take your time to quietly read the following message.

ATTENTION

As was the similar message in Chap.5, here again, we consider it crucial for you to read the statement of each solved problem and then try to solve it without looking at the proposed solution. On the first few problems, it might be a little boring to follow the proposed procedure, but we are convinced that by the end you will feel that it was worth it. Follow the proposed procedure step by step, and for all solved-problems presented here, do not skip any problems. Only once you have done this, having resolved all issues, should you proceed, with firm resolve, to all the proposed problems. And never forget the methodology!

"*I hear, and I forget. I see, and I remember. I do, and I understand*"
CHINESE PROVERB

7.9 Solved Problems

The aim of this section is to familiarize you with the proposed procedure and to reveal that most real-world problems will be hard to solve without some sort of procedure. Initially, with simple problems (simple units), some steps will be developed together, but as problems become more complex, it will be necessary to follow rigorously the proposed strategy step by step.

As we tell our students, just dutifully follow our recommendations, and, as we have stated from the beginning, the key word is PROCEDURE. A good example is problem 15. It involves 31 variables and several "hidden" relationships, and we categorized its difficulty as 10^{+}. But not to worry! If you start from problem one and advance step by step, you will be prepared for problem 15.

As stated in Sect. 7.1, one of our goals is to familiarize students with the broad application of chemical and bioprocess engineering. Although the main goal is to enable you to solve material balance problems, in addition, you will discover, through the examples, why some processes include recycle streams (e.g., problems 10 and 11) and purges (e.g., problem 15).

SIMPLE UNITS

1. **Separation process [3].** A stream with three components, A, B, and C, is fed to a process unit. At the outlet of the equipment are two streams, each containing components A, B, and C. (a) Draw a schematic representation of this process. (b) How many variables are in this process? (c) Determine the degrees of freedom of the process.

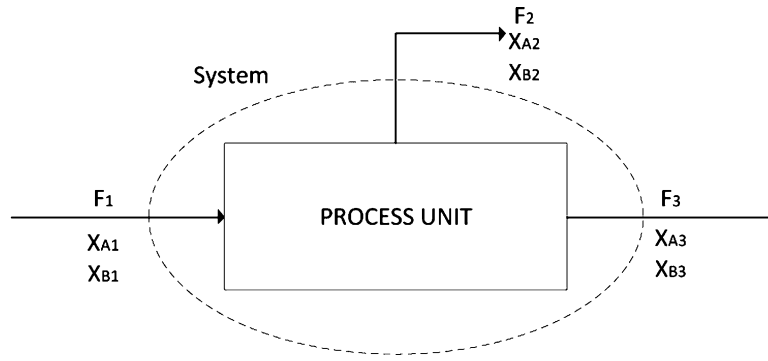

Fig. 7.25 Separation process

Step I and II

Reading and understanding a flow diagram, variable definition and codification, and inclusion of all available data

(a) Draw a schematic representation of this process (Fig. 7.25)

F_1: Mass flow rate of feed stream (kg/h)
x_{A1}: Mass fraction of component A in feed stream
x_{B1}: Mass fraction of component B in feed stream
F_2: Mass flow rate of second stream (kg/h)
x_{A2}: Mass fraction of component A in second stream
x_{B2}: Mass fraction of component B in second stream
F_3: Mass flow rate of third stream (kg/h)
x_{A3}: Mass fraction of component A in third stream
x_{B3}: Mass fraction of component B in third stream

(b) How many variables are in this process? NV = 9, and, given that there are no specified variables or any given relationship, NSV = 0 and NR = 0.

Step III

Analysis of degrees of freedom in process

Because the streams have three components, we can formulate three independent material balances. Recalling (7.13) we obtain:

(c) Determine the degrees of freedom of the process:

$$\mathrm{DF} = \mathrm{NV} - \mathrm{NMB} - \mathrm{NSV} - \mathrm{NR} = 9 - 3 - 0 - 0 = 6;\ \mathrm{DF} = 6.$$

We have already answered all the questions and will stop here. In addition, we have no data to do any further calculations. But remember, you can specify six variables, but they must be independent of each other. We learned from example 5 that you cannot arbitrarily fix the three flow streams.

2. **Separation process with a side stream [4]**. Although it normally causes confusion, the concepts of side stream and bypass are clearly different. A bypass stream does not pass through a process, i.e., the stream “dodges" the process and is passed by the following process. A side stream, on the other hand, is divided into two streams, one part going through the process and the other being “bypassed” and then joining with the stream that was processed. A generic example of a side stream is presented in this exercise.

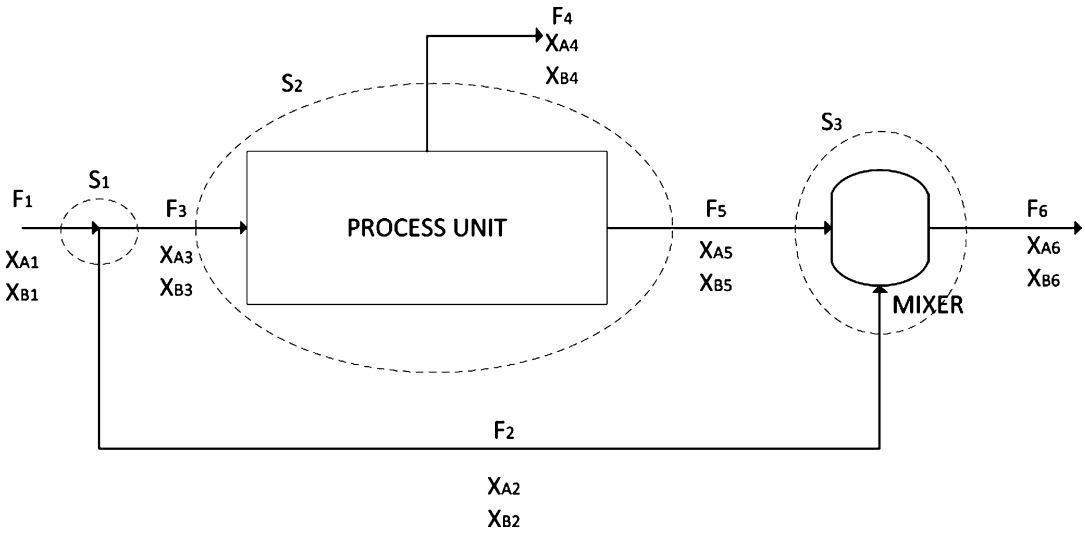

Fig. 7.26 Separation process with a side stream

Like the previous exercise (three streams and three components per stream), this one includes a side stream, where part of the feed stream is "bypassed" to join one of the streams as it leaves the equipment. (a) Draw a schematic representation of this process. (b) How many variables are in this process? (c) Determine the degrees of freedom in this process.

Step I and II

Reading and understanding a flow diagram, variable definition and codification, and inclusion of all available data

(a) Draw a schematic representation of this process.

As depicted in Fig. 7.26, we have included a mixer, as recommended in Sect. 7.8.2.1. Another interesting point for discussion is the relevance of including or not x_{A2}, x_{B2}, x_{A3}, and x_{B3} as variables. When the feed stream is separated, the flow rate of each new stream is unknown, but their concentrations are exactly the same as that of the original stream; it is simply a division. Therefore, $x_{A1} = x_{A2}$, $x_{A1} = x_{A3}$, $x_{B1} = x_{B2}$, and $x_{B1} = x_{B3}$. If we consider them variables, then we need to include these four equations. Our advice is to not consider them variables and assume that they are already considered in x_{A1} and x_{B1}.

F_1: Mass flow rate of feed stream (kg/h)
x_{A1}: Mass fraction of component A in feed stream
x_{B1}: Mass fraction of component B in feed stream
F_2: Mass flow rate of second stream (kg/h)
~~x_{A2}: Mass fraction of component A in the second stream.~~
~~x_{B2}: Mass fraction of component B in the second stream.~~
F_3: Mass flow rate of third stream (kg/h)
~~x_{A3}: Mass fraction of component A in the third stream.~~
~~x_{B3}: Mass fraction of component B in the third stream.~~
F_4: Mass flow rate of third stream (kg/h)
x_{A4}: Mass fraction of component A in fourth stream
x_{B4}: Mass fraction of component B in fourth stream

F_5: Mass flow rate of third stream (kg/h)
x_{A5}: Mass fraction of component A in fifth stream
x_{B5}: Mass fraction of component B in fifth stream
F_6: Mass flow rate of third stream (kg/h)
x_{A6}: Mass fraction of component A in sixth stream
x_{B6}: Mass fraction of component B in sixth stream

(b) How many variables are in this process? NV = 14, and given that there are neither specified variables nor any relationship given, NSV = 0 and NR = 0.

Step III

Analysis of degrees of freedom in process

Because the streams have three components, we can formulate three independent material balance equations for each process unit (two process units, systems S_2 and S_3, and six equations). In addition, we have a division (system S_1), adding one more total mass balance. Therefore, NMB = 7.

(c) Determine the degrees of freedom in this process:

$$\mathrm{DF} = \mathrm{NV} - \mathrm{NMB} - \mathrm{NSV} - \mathrm{NR} = 14 - 7 - 0 - 0 = 7;\ \mathrm{DF} = 7.$$

3. **Dryer [6]**. A wet material is passed through a dryer unit to attain some specific moisture content. Hot fresh air is fed to the dyer and at the outlet, part of the humid air is recycled. (a) How many variables can you identify? (b) Determine the degrees of freedom in this process unit.

Step I

Reading and understanding

Remember, familiarize yourself with the equipment used in process and bioprocess engineering. In this specific case it is not necessary to learn and become familiar with all classes of dryers, but at least familiarize yourself with its main function. Given that the flow diagram of the process is not included, it is important to read carefully and put the statement of the problem in a clear and schematic figure. Particularly in this problem a good schematic representation of the drying process is critical. As we will see, this type of problem presents its own special features, and the way one defines systems is important. Do not be too concerned about its difficulty because we have intentionally included this example for its difficulty to characterize the process and its systems.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

F_1: Mass flow rate of wet material (kg/h)
x_{S1}: Mass fraction of solids on wet material stream
F_2: Mass flow rate of dried material stream (kg/h)
x_{S2}: Mass fraction of solids on dried material stream
F_3: Mass flow rate of hot fresh air (kg/h)
x_{H2O3}: Mass fraction of H_2O in hot fresh air
F_4: Mass flow rate of air entering dryer (kg/h)
x_{H2O4}: Mass fraction of H_2O in air entering dryer
F_5: Mass flow rate of air leaving dryer (kg/h)
x_{H2O5}: Mass fraction of H_2O in air leaving dryer
F_6: Mass flow rate out of process (kg/h)
F_7: Mass flow rate of recycle stream (kg/h)
F_8: Mass flow rate of H_2O evaporated from wet material (kg/h)

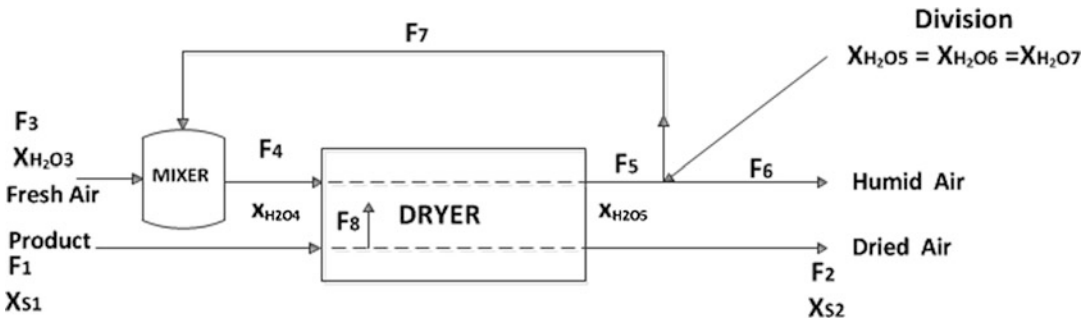

Fig. 7.27 Drying process

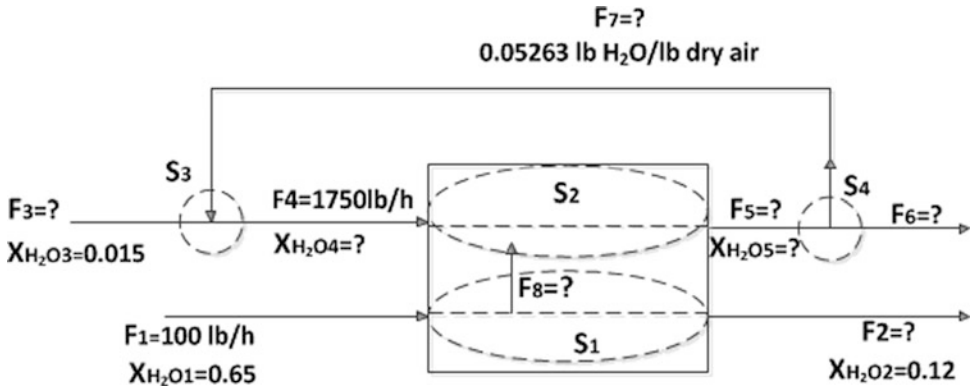

Fig. 7.28 Drying process including all variables and available data

One key feature in this example is that we have conveniently separated the drying process into two systems. Whenever you have data on the humidity of air, it is convenient to separate it into two systems, as depicted in Fig. 7.27. In addition, as explained in the previous problems, we are not considering x_{H2O6} and x_{H2O7} as variables because they are equal to x_{H2O5} (due to the division).

(a) How many variables can you identify? NV = 13, and given that there are no specified variables or any given relationship, NSV = 0 and NR = 0.

Step III

Analysis of degrees of freedom in process

As shown in the flow chart (Fig. 7.28), the whole process has been divided into four systems, where in systems S_1, S_2, and S_3 we can formulate two material balances in each one. System S_4 is a division that adds one more material balance. Therefore, NMB = 7.

(b) Determine the degrees of freedom in this process unit:

$$\mathrm{DF} = \mathrm{NV} - \mathrm{NMB} - \mathrm{NSV} - \mathrm{NR} = 13 - 7 - 0 - 0 = 6;\ \mathrm{DF} = 6.$$

4. **Dryer [5]**. 100 lb/h of a wet material with 65 % humidity w/w, and the rest can be considered a solid that is passed through a dryer where the end product reaches 12 % humidity w/w. The hot fresh air has 1.5 % humidity w/w and before entering the dryer is mixed with recycled air with 0.05263 lb

H_20/lb of dry air. If 1,750 lb/h of humid air are fed to the dryer, then: (a) What is the mass flow rate of hot fresh air? (b) What is the mass flow rate of the recycled air?

Step I

Reading and understanding

This problem is similar to the previous problem but with quantitative data. Then the number of variables should be the same (NV = 13) and the degrees of freedom will change because here we have some specified variables.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

F_1: Mass flow rate of wet material in pounds per hour (lb/h)
x_{H2O1}: Mass fraction of water in wet material stream (w/w)
F_2: Mass flow rate of dried stream (lb/h)
x_{H2O2}: Mass fraction of water in dried material stream (w/w)
F_3: Mass flow rate of hot fresh air (lb/h)
x_{H2O3}: Mass fraction of H_2O in hot fresh air (w/w)
F_4: Mass flow rate of air entering dryer (lb/h)
x_{H2O4}: Mass fraction of H_2O in air entering dryer (w/w)
F_5: Mass flow rate of air out of dryer (lb/h)
x_{H2O5}: Mass fraction of H_2O in air out of dryer (w/w)
F_6: Mass flow rate out of process (lb/h)
F_7: Mass flow rate of recycle stream (lb/h)
F_8: Mass flow rate of H_2O evaporated from wet material (lb/h)

Thus, NV = 13, and there are six specified variables and no relationships are given. Therefore, NSV = 6 and NR = 0.

Step III

Analysis of degrees of freedom in process

As shown in the flow chart (Fig. 7.28), the whole process, as in the previous problem, has been divided into four systems. In each of systems S_1, S_2, and S_3 we can formulate two material balances. System S_4 is a division that adds one more material balance. Therefore, NMB = 7. Replacing NV, NMB, NSV, and NR in (7.13), we get

$$\mathrm{DF} = \mathrm{NV} - \mathrm{NMB} - \mathrm{NSV} - \mathrm{NR} = 13 - 7 - 6 - 0 = 0;\ \mathrm{DF} = 0.$$

meaning that the problem is set and can be solved.

Step IV

Mathematical formulation including all available data

As depicted in Fig. 7.28, the process is divided into four systems (S_1, S_2, S_3, and S_4). In addition, as mentioned in systems S_1, S_2, and S_3, we can formulate two material balances in each one and write one more material balance (total mass balance) in system S_4. In addition, as shown in the flow diagram, there are seven unknowns, and we can formulate seven equations. Before formulating the equations of systems S_1, S_2, S_3, and S_4 it is necessary to have consistent units in all variables. For example, the humidity of the recycled air stream is given in pounds of water per pound of dry air (lb H_2O/lb dry air). To be consistent, it is necessary to express this as pounds of water per pound

of humid air (lb H_2O/lb humid air), where 0.05263 lb H_2O/lb dry air is equivalent to 0.05263/(1 + 0.05263) lb H_2O/lb humid air ~5.000 × 10^{-2} lb H_2O/lb humid air. Therefore, $x_{H2O7} = 5.000 \times 10^{-2} = x_{H2O5}$. The hot fresh air has 1.5 % humidity w/w. Therefore, $x_{H2O3} = 0.015$ lb H_2O/lb humid air.

System S_1
Total mass balance:

$$100 - F_2 - F_8 = 0. \tag{7.36}$$

Mass balance for H_2O:

$$0.65 \times 100 - 0.12 \times F_2 - F_8 = 0. \tag{7.37}$$

System S_2
Total mass balance:

$$1,750 + F_8 - F_5 = 0. \tag{7.38}$$

Mass balance for H_2O:

$$X_{H2O4} \times 1,750 + F_8 - 5.000 \times 10^{-2} \times F_5 = 0. \tag{7.39}$$

System S_3
Total mass balance:

$$F_3 + F_7 - 1,750 = 0. \tag{7.40}$$

Mass balance for H_2O:

$$0.015 \times F_3 + 5.000 \times 10^{-2} \times F_7 - 1,750 \times X_{H2O4} = 0. \tag{7.41}$$

System S_4
Total mass balance:

$$F_5 - F_6 - F_7 = 0. \tag{7.42}$$

Step V
Solution, results, analysis, and discussion
First, we have seven equations and seven unknowns. Sometimes, this might be a bit difficult, but it is worth trying. Solving (7.36) and (7.37) we obtain $F_2 \sim 39.773$ lb/h and $F_8 \sim 6.2 \times 10$ lb/h. Then from (7.38) and (7.39) we get $F_5 \sim 1.81 \times 10^3$ and $X_{H2O4} \sim 0.0173$. Finally, from (7.40)–(7.42) we obtain $F_7 \sim 1.15 \times 10^2$ lb/h, $F_3 \sim 1.63 \times 10^3$ lb/h, and $F_6 \sim 1.69 \times 10^3$ lb/h.
(a) What is the mass flow rate of hot fresh air? $F_3 \sim 1.63 \times 10^3$ lb/h.
(b) What is the mass flow rate of recycled air? $F_7 \sim 1.15 \times 10^2$ lb/h.

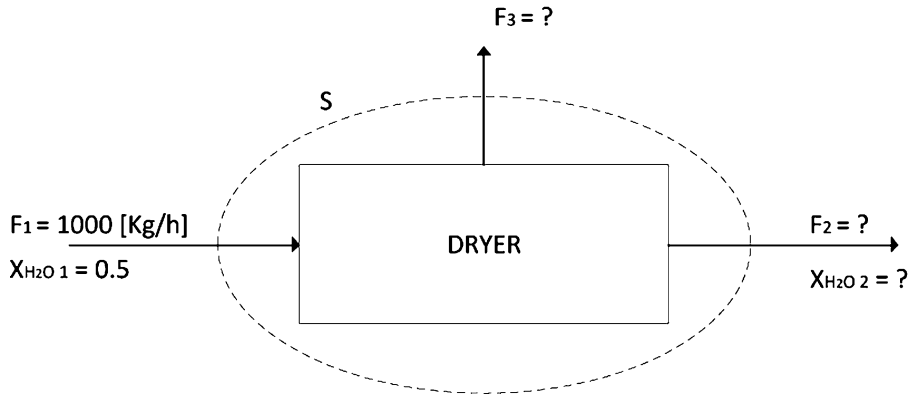

Fig. 7.29 Drying process including all variables and available data

One way to test the results is to carry a global mass balance of the whole process (not included in (7.36)–(7.42). In Fig. 7.28 the total mass balance corresponds to $F_1 + F_3 - F_2 - F_6 = 0$. Replacing each value we confirm that this equation is correctly accomplished with the obtained values for the streams. In addition, $F_3 + F_7$ should add up to 1,750 lb/h, and it does.

There are several ways to see whether at least the results are in the expected range. For example, $x_{H2O3} < x_{H2O4} < x_{H2O7}$ (recall that $x_{H2O5} = x_{H2O7}$). As you can verify, x_{H2O4} is greater than x_{H2O3} and less than x_{H2O7}

5. **Dryer analysis [6]**. 1,000 kg/h of a wet material with 50 % moisture content will be dehydrated. The manager wants a very dry product and obtains 400 kg/h end product. You do some thought and conclude that it would be impossible to obtain less than 500 kg/h dried product (just removing H_2O). The manager protests your claim that he occupied one degree of freedom, which is exactly the degrees of freedom of the process. Is he right? Why not?

Step I

Reading and understanding

As you indicate, if the material has 50 % moisture content, then if you remove all the H_2O, your dried product will have a mass flow rate of 500 kg/h. On the other hand, the manager might be right when he says that the process has 1 degree of freedom.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data (Fig. 7.29)

F_1: Mass flow rate of wet material (kg/h)

x_{H2O1}: Mass fraction of water in wet material stream (w/w)

F_2: Mass flow rate of the dried stream (lb/h)

x_{H2O2}: Mass fraction of water in dried material stream (w/w)

F_3: Mass flow rate of H_2O (kg/h)

Therefore, NV = 5, and there are two specified variables (F_1 and x_{H2O1}), and no relationships are given. Thus, NSV = 2 and NR = 0.

Step III

Analysis of degrees of freedom in process

The wet material has two components (solids + H_2O), and the process is carried out in one process unit (dryer). Therefore, we are able to formulate two independent material balances:

$$\text{NMB} = 2.$$

Thus,

$$\text{DF} = \text{NV} - \text{NMB} - \text{NSV} - \text{NR} = 5 - 2 - 2 - 0 = 1:$$
$$\text{DF} = 1.$$

Yes, the manager is right when he says that the process has one degree of freedom. Yes, we can fix one variable, but, remember, with limitations. We have three unknowns, and we can fix one variable (DF = 1).

What are the limitations in this case?

$F_3 \leq 500$ kg/h, the mass flow rate of the wet material is 1,000 kg/h with 50 % humidity, and so the amount of H_20 in F_1 is 500 kg/h. Of course, you cannot remove more than 500 kg/h.

$F_2 \geq 500$ kg/h if the amount of H_2O removed is ≤ 500 kg/h. Thus, F_2 must be ≥ 500 kg/h. Why? The total mass balance in this process is $1{,}000 - F_2 - F_3 = 0$. Then $F_2 = 1{,}000 - F3$. If $F3 \leq 500$ kg/h, then $F_2 \geq 500$ kg/h.

Therefore, the manager was right that the degree of freedom in this process was one but wrong when he set $F_2 = 400$ kg/h. In addition, we suggest that you analyze the limitations for X_{H2O2}.

MULTIPLE UNITS

6. **H_2O removed from two processes in series including a recycle [6]**. In generic terms, a recycle stream is an outlet stream that is divided into two streams, one stream returning to the process (e.g., to the feed stream) and the other stream going on to the following stage of the process, if any.

In the following generic process there are two process units in series including a recycle stream. The feed stream contains A, B, and H_2O. In each process unit, part of the H_2O is removed. At the outlet of the second process unit, the stream is divided into two streams; one of the streams is recycled to join the feed stream. (a) Draw a schematic representation of the complete process. (b) How many variables are in this process (NV)? (c) Determine the degrees of freedom in this process (DF).

Step I

Reading and understanding

This process has two units and a recycle stream. As was stated earlier, the addition of a recycle stream means that we need to consider a division (add one total mass balance) and add a mixer (in this case adding three material balances because the stream has three components).

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

(a) Draw a schematic representation of the complete process

F_1: Mass flow rate of feed stream (kg/h)

x_{A1}: Mass fraction of component A in feed stream

x_{B1}: Mass fraction of component B in feed stream

F_2: Mass flow rate of the input stream in first process unit (kg/h)

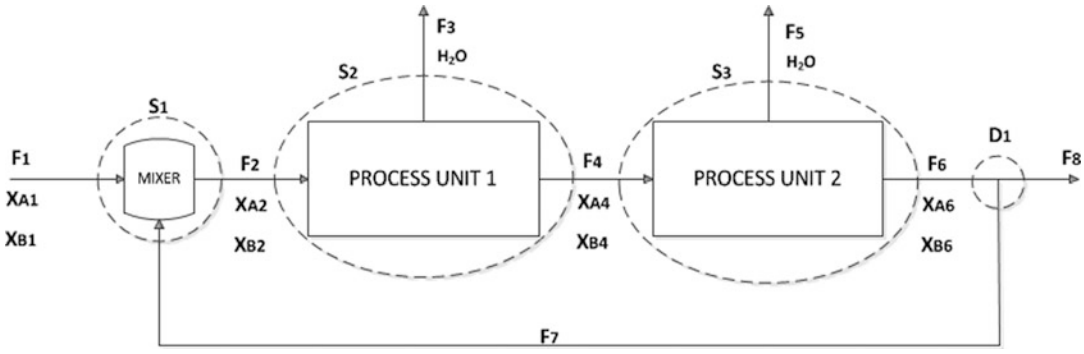

Fig. 7.30 Process in series to remove water

x_{A2}: Mass fraction of component A in input stream in first process unit
x_{B2}: Mass fraction of component B in input stream in first process unit
F_3: Mass flow rate of H_2O at output in first process unit (kg/h)
F_4: Mass flow rate at outlet of first process unit (kg/h)
x_{A4}: Mass fraction of component A at outlet of first process unit
x_{B4}: Mass fraction of component B at outlet of first process unit
F_5: Mass flow rate of H_2O at output in second process unit (kg/h)
F_6: Mass flow rate at outlet of first process unit (kg/h)
x_{A6}: Mass fraction of component A at outlet of second process unit
x_{B6}: Mass fraction of component B at outlet of second process unit
F_7: Mass flow rate of recycle stream (kg/h)
F_8: Mass flow rate at outlet of whole process (kg/h)

As shown in the division, when a stream is separated, the flow rate of each new stream is unknown, but their concentrations are exactly the same as the original stream; it is simply a division ($x_{B6} = x_{B7} = x_{B8}$). Then the outlet stream of the process and the recycle stream have already assigned variables for their concentrations of A and B (x_{A6} and x_{B6}). In addition, the water removed in each stage is pure (100 % H_2O).

(b) How many variables are in this process?

As depicted in Fig. 7.30, we have 16 variables (NV = 16), and no specified variable or relationships are given. Thus, NSV = 0 and NR = 0.

Step III

Analysis of degrees of freedom in process

With the inclusion of the mixer we now have three process units and one division. Therefore, we can formulate ten independent material balances (three per unit and one at the division), so NMB = 10.

(c) Determine the degrees of freedom in this process (DF).

$$\mathrm{DF} = \mathrm{NV} - \mathrm{NMB} - \mathrm{NSV} - \mathrm{NR} = 16 - 10 - 0 - 0 = 6;\ \mathrm{DF} = 6.$$

7. **Multieffect evaporator [8]**. Multieffect evaporators are simple evaporators connected in series in order to reduce the consumption of energy (steam) per kilogram of water evaporated from the fluid under concentration. This is achieved by reusing the vapors coming out of each effect in the following or preceding effect (depending on whether it is a cocurrent or countercurrent). Figure 7.31 is a generic schematic representation of a multieffect evaporator operated under a countercurrent, i.e., the diluted

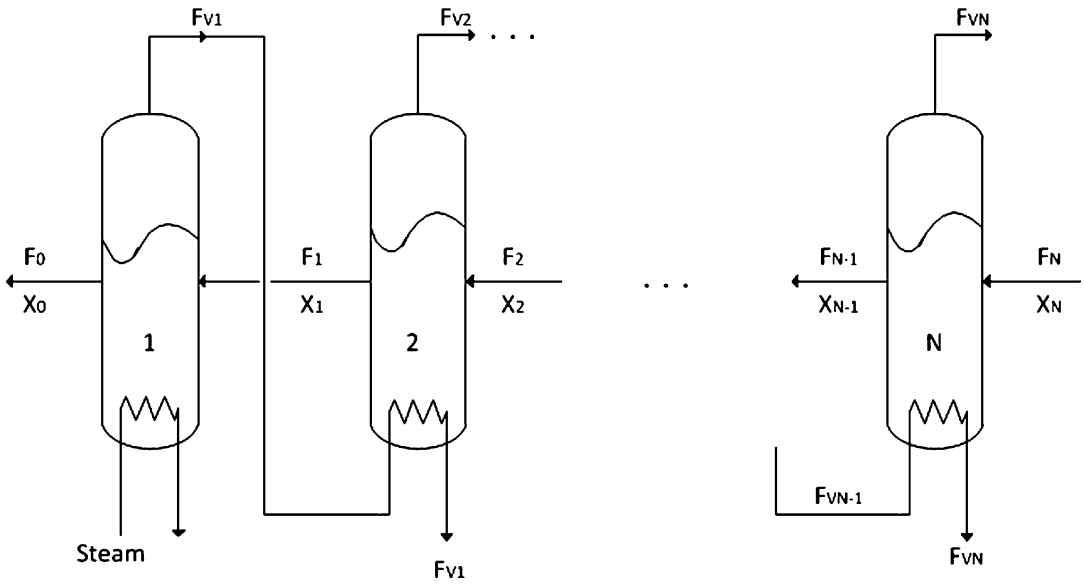

Fig. 7.31 Multi-effect evaporator

fluid is fed to the last effect (the lowest temperature). Assuming that the feed fluid is composed of solids and water: (a) How many variables are in this process? (b) How many independent material balances can be formulated? (c) Determine the degrees of freedom of the multieffect evaporator process (DF).

Step I

Reading and understanding

According to the problem statement and as shown in Fig. 7.31, the vapor generated in the following effect is used in the preceding effect as energy. In this arrangement, just the first effect uses live steam from the boiler. Then in each effect we have three streams, two for the product and the third of vapor. The stream shown below in each effect is used just as a medium to supply energy. Strictly speaking, it will not be part of the system in terms of mass balance.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

As depicted in Fig. 7.32, $S_1, S_2, \ldots, S_N$ are the systems chosen for material balance analysis (similar to the one shown in Fig. 7.33).

According to the flow diagram provided in the problem and with our additions (systems) we have

$F_0, F_1, F2, \ldots, F_N$: Mass flow rate of each stream of product (kg/h);

$F_{V1}, F_{V2}, \ldots, F_{VN}$: Mass flow rate of vapor per each effect;

$x_0, x_1, x_2, \ldots, x_N$: Mass fraction of solids in each stream of product.

Then we have $N + 1$ streams of product, N streams of vapor, and $N + 1$ concentrations, in total $3N + 2$.

(a) How many variables are in this process?

NV $= 3N + 2$. In addition, NSV $= 0$ and NR $= 0$.

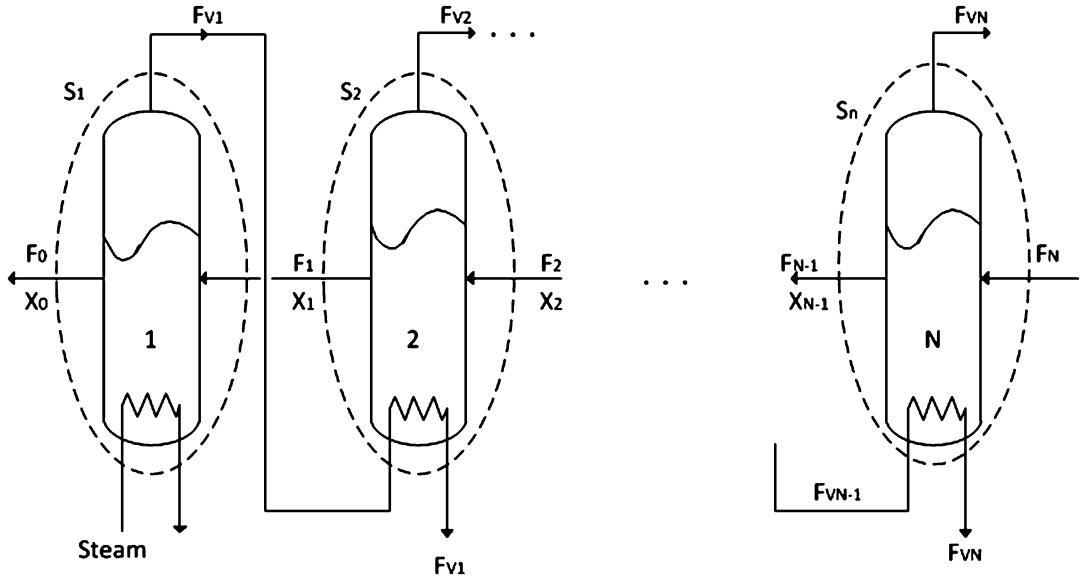

Fig. 7.32 Multi-effect evaporator showing systems $S_1, S_2, \ldots, S_N$

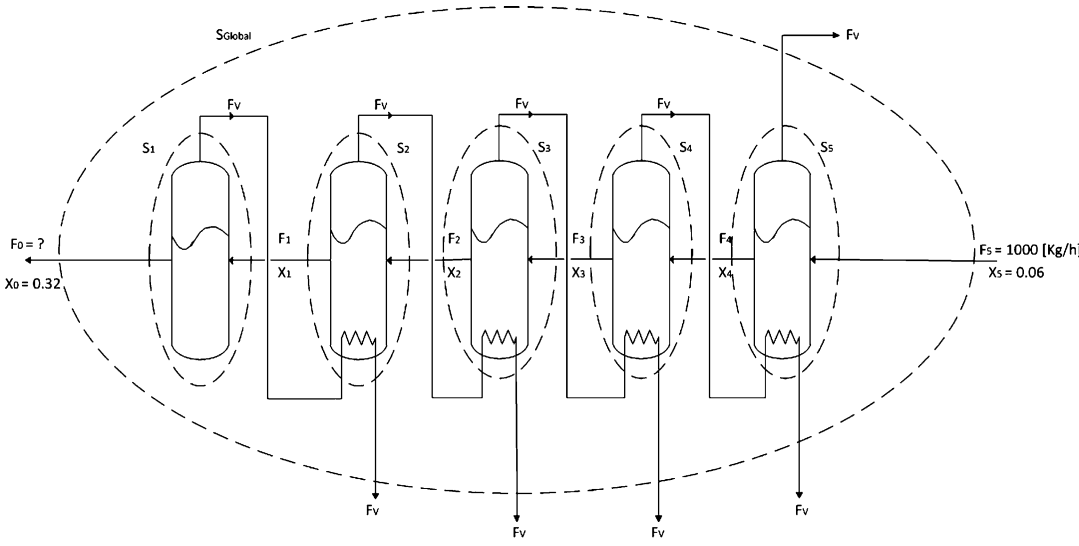

Fig. 7.33 Multi-effect evaporator for tomato paste

Step III

Analysis of degrees of freedom in process

(b) How many independent material balances can be formulated?

In each effect we can formulate two independent material balances (two components). Thus, in total we can formulate $2N$ equations. Therefore,

$$\text{NMB} = 2N.$$

(c) Determine the degrees of freedom of the multieffect evaporator process (DF).

$$\text{DF} = \text{NV} - \text{NMB} - \text{NSV} - \text{NR} = (3N + 2) - 2N - 0 - 0 = N + 2;\ \text{DF} = N + 2.$$

8. **Tomato concentrates [6]**. 1,000 kg/h tomato juice (6 % solids and H_2O) are fed to a five-effect evaporator operated under a countercurrent to reach 32 % solids at the outlet. The amount of H_2O removed in each effect is the same. (a) Determine the degrees of freedom of the process. (b) Determine the solid concentration at the outlet of each of the following effects (2, 3, 4, and 5). (c) What is the flow rate at the outlet of the first effect?

Step I

Reading and understanding

This problem is a practical application of the preceding one. Therefore, we will use a similar nomenclature to define all the variables but assume that the amount of H_2O evaporated, in each effect, is the same. Thus,

$$F_{V1} = F_{V2} = F_{V3} = F_{V4} = F_{V5} = F_V.$$

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

F_0, F_1, F_2, F_3, F_4, F_5: Mass flow rate of each stream of product (kg/h)

F_V: Mass flow rate of vapor per each effect (kg/h)

x_0, x_1, x_2, x_3, x_4, x_5: Mass fraction of solids in each stream of product

Therefore, NV = 13, NSV = 3 (F_5, x_5, and x_0), and NR = 0.

Step III

Analysis of degrees of freedom in process

In each effect we can formulate two independent material balances. Thus, NMB = 10.

(a) Determine the degrees of freedom of the process (DF).

DF = NV − NMB − NSV − NR = 13 − 10 − 3 − 0 = 0; DF = 0. Therefore, the problem is ready to be solved.

Step IV

Mathematical formulation including all available data

As we have learned from previous problems, a good choice for material balance in this case is the solid and total mass balance for each effect. In addition, we can formulate ten independent material balances. We can do that by formulating two equations per system (S_1, S_2, S_3, S_4, and S_5) and not using the global mass balance (S_T). In this situation, given that we have information at the input and output of the whole process, it appears convenient to use the global mass balance (total and solids) as two of the ten equations.

System S_T (global)

Total mass balance:

$$1,000 - F_0 - 5 \times F_V = 0. \tag{7.43}$$

Solid mass balance:

$$0.06 \times 1,000 - 0.32 \times F_0 = 0. \tag{7.44}$$

System S_5
Total mass balance:

$$1,000 - F_4 - F_V = 0. \tag{7.45}$$

Solid mass balance:

$$0.06 \times 1,000 - x_4 \times F_4 = 0. \tag{7.46}$$

System S_4
Total mass balance:

$$F_4 - F_3 - F_V = 0. \tag{7.47}$$

Solid mass balance:

$$x_4 \times F_4 - x_3 \times F_3 = 0. \tag{7.48}$$

System S_3
Total mass balance:

$$F_3 - F_2 - F_V = 0. \tag{7.49}$$

Solid mass balance:

$$x_3 \times F_3 - x_2 \times F_2 = 0. \tag{7.50}$$

System S_2
Total mass balance:

$$F_2 - F_1 - F_V = 0. \tag{7.51}$$

Solid mass balance:

$$x_2 \times F_2 - x_1 \times F_1 = 0. \tag{7.52}$$

Step V

Solution, results, analysis, and discussion

Then, from step IV, we have ten equations and ten variables. Solving first (7.44) we get $F_0 = 187.5$ kg/h, then from (7.43) we obtain $F_V = 162.5$ kg/h. After these calculations the rest is straightforward; just continue with (7.45) and so on.

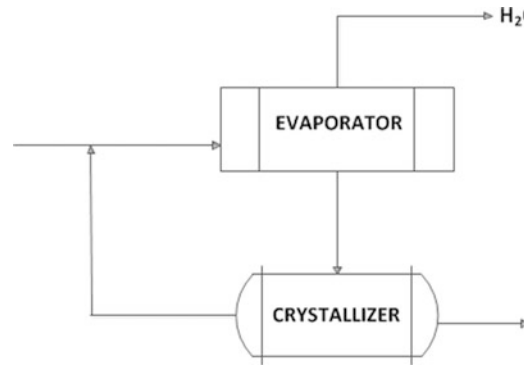

Fig. 7.34 Evaporation-Crystallization system including recycle

(b) Determine the solid concentration at the outlet of each of the following effects (2, 3, 4, and 5).

$$x_1 = 0.1714,\ x_2 = 0.1171,\ x_3 = 0.0889,\ x_4 = 0.0716$$

(c) What is the flow rate at the outlet of the first effect?

$$F_0 = 187.5\ \text{kg/h}$$

One important lesson is that, sometimes, it is not clear what the right decision is as to which equations to formulate to facilitate the mathematical solution. Several times, as in this example, the total and solid mass balance for the whole process (global) could be the trigger for a quick solution. There is no general rule on how to select the right equations in such a way as to minimize your potential struggle with the mathematical solution. On the other hand, today you have many tools, and so the mathematical solution will not necessarily pose a problem. In addition, as has been expressed in different parts of the text, the most important thing at this stage is to learn material balance.

9. **Evaporation-crystallization including recycle [8]**. As depicted in Fig. 7.34, the process is composed of an evaporation unit and a crystallizer.

The process is operated in a continuous mode under steady state. A solution with component A and H_2O will pass first to an evaporator unit to remove some H_2O and then through a crystallizer. The mother liquor (water and component A) will be recycled; and the crystals leaving the process will carry, on their surfaces, a small amount of the mother liquor. (a) How many variables you can identify in this process? (b) Determine the degrees of freedom of the process.

Step I

Reading and understanding

In this case, as discussed earlier, it is better to improve or redo the schematic diagram of the process provided in this problem. For example, given that the process includes a recycle stream, it is advisable, for clarity, to include a mixer in the flow diagram. In addition, remember to familiarize yourself with the unit operations covered in this process, evaporation and crystallization. One strategy that we suggest in this problem is to treat the stream with crystal and mother liquor as two different streams.

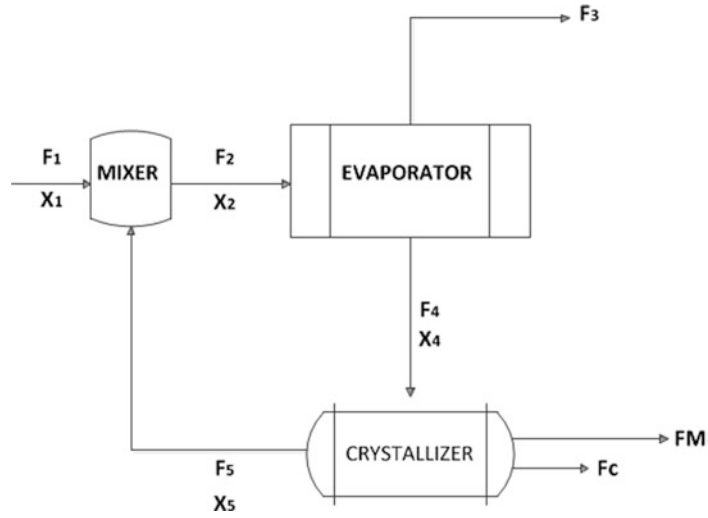

Fig. 7.35 Evaporation-Crystallization system including all variables

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

F_1, F_2, F_3, F_4, F_5 *(recycled mother liquor)*, F_C, F_{ML} *(mother liquor (F_{ML})* and crystals (F_C)): mass flow rate of streams kg/h

x_1, x_2, x_4, x_5: Concentrations w/w (where $x_5 = x_{ML}$).

We have decided to separate the output mass flow rate of the crystallizer into two streams for clarity. In the next solved exercise we will show why it is better to separate this output flow into two streams.

(a) How many variables can you identify in this process?

From Fig. 7.35, NV = 11, NSV = 0, and NR = 0.

Step III

Analysis of degrees of freedom in process

As depicted in Fig. 7.35, now the process includes three process units: a mixer, evaporator, and crystallizer. The fluid has two components, so we can formulate two equations per unit. Therefore,

$$\text{NMB} = 6.$$

(b) Determine the degrees of freedom in the process.

$$\text{DF} = \text{NV} - \text{NMB} - \text{NSV} - \text{NR} = 11 - 6 - 0 - 0 = 5;\ \text{DF} = 5.$$

10. **KNO_3 crystallization with recycle [8]**. 100 kg/h of a solution with 25 % KNO_3 will be crystallized in an evaporation-crystallization unit, as shown in the previous exercise. Upon leaving the evaporation unit, the solution has a concentration of 55 % KNO_3. At the crystallizer the temperature is dropped and the mother liquor has 38 kg of KNO_3/100 kg of H_2O. The mother liquor is recycled and the crystals are removed, carrying on the surface a small amount of mother liquor (mass of mother liquor/mass crystals = 0.03 w/w). (a) How much H_2O is evaporated? (b) What is the flow rate (kg/h) of the recycle stream? (c) What is the flow rate of pure KNO_3 crystals? (d) What is the efficiency of the process (mass of KNO_3 crystals/mass of KNO_3 fed)?

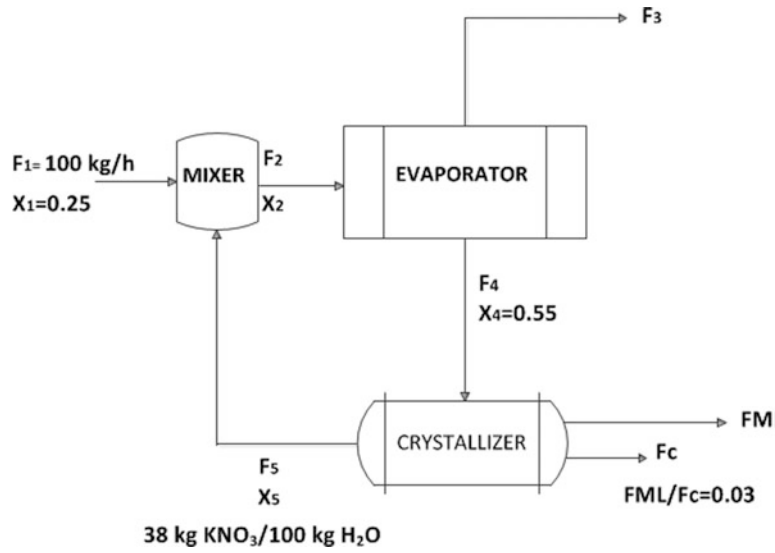

Fig. 7.36 KNO_3 crystallization with recycle including all variables and available data

Step I

Reading and understanding

First, we need to follow the advice from the previous problem and draw a good flow diagram (Fig. 7.36). In addition, consider separating the flow rate at the output of the crystallizer into two streams, crystals (F_C) and mother liquor (F_{ML}).

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

F_1, F_2, F_3, F_4, F_5, F_C, F_{ML}: Mass flow rate of streams (kg/h)

x_1, x_2, x_4, x_5: KNO_3 concentrations w/w (kg KNO_3/kg solution)

(a) How many variables can you identify in this process?

From Fig. 7.36, NV = 11, NSV = 4 (F_1, x_1, x_4, and x_5), and NR = 1 ($F_{ML}/F_C = 0.03$). We need to consider that x_5 has been given, but indirectly.

Step III

Analysis of degrees of freedom in process

Now (Fig. 7.36) the process consists of three process units, and the fluid in each has two components. Therefore, NMB = 6, and thus DF = 11 − 6 − 4 − 1 = 0. The problem can now be solved.

Step IV and V

Mathematical formulation including all available data, solution, results, analysis, and discussion

First, as mentioned, X_5 has been given, but only indirectly. We need to express the concentration as kg KNO_3/kg solution. The concentration of the mother liquor is given as 38 kg KNO_3/100 kg of H_2O and, expressed as kg KNO_3/kg, the solution will be

$$x_5 = 38\ \text{KNO}_3/(38\ \text{kg KNO}_3 + 100\text{kg H}_2\text{O}) \sim 0.28\ \text{kg KNO}_3/\text{kg solution}.$$

From Fig. 7.36 we know that we have seven unknowns, and given that NMB = 6 and NR = 1, we can formulate seven independent equations.

As in the previous problem, we need to analyze and try to discover the best equations to use. Because we have complete information on the feed stream and relations at the output of the crystallizer we will use the global mass balance (S_T). Normally, it is a good idea to explore the global mass balance because usually we have information on the feed stream and of the end product.

System S_T (global)
Total mass balance:

$$100 - F_3 - F_C - F_{ML} = 0. \tag{7.53}$$

Mass balance for KNO_3:

$$0.25 \times 100 - F_C - 0.28 \times F_{ML} = 0 \tag{7.54}$$

and

$$F_{ML}/F_C = 0.03. \tag{7.55}$$

Before continuing, here we have three equations (7.53)–(7.55) and three unknowns (F_3, F_C, and F_{ML}). Therefore,

$$F_C = 24.8 \text{ kg/h}, \; F_{ML} = 0.744 \text{ kg/h}, \; F_3 = 74.5 \text{ kg/h}.$$

At this point we can answer the following questions: (a) How much H_2O is evaporated? $F_3 = 74.5$ kg/h. (c) What is the flow rate of pure KNO_3 crystals? $F_C = 24.8$ kg/h. (d) What was the efficiency of the process (% mass of KNO_3 crystals/mass of KNO_3 fed)? We can answer this question as follows:
η: Efficiency (% mass of KNO_3 crystals/mass of KNO_3 fed). Therefore,

$$\eta = 100 \times F_C/(0.25 \times 100) = 100 \times 24.8/25 = 99.2 \; \%.$$

To answer question (b) regarding the flow rate (kg/h) of the recycle stream, we will formulate all mass balances for systems S_1, S_2, and S_3; then, using the values obtained for F_C, F_{ML}, and F_3, and selecting some specific equations, we will calculate F_5.

System S_1 (Mixer)
Mass balance for KNO_3:

$$0.25 \times 100 + 0.28 \times F_5 - x_2 \times F_2 = 0. \tag{7.56}$$

Total mass balance:

$$100 + F_5 - F_2 = 0. \tag{7.57}$$

System S_2 (Evaporator)
Mass balance for KNO_3:

$$x_2 \times F_2 - 0.55 \times F_4 = 0. \tag{7.58}$$

Total mass balance:

$$F_2 - F_4 - F_3 = 0. \tag{7.59}$$

System S_2 (Crystallizer)
Mass balance for KNO_3:

$$0.55 \times F_4 - 0.28 \times F_5 - F_C - 0.28 \times F_{ML} = 0. \tag{7.60}$$

Total mass balance:

$$F_4 - F_5 - F_C - F_{ML} = 0. \tag{7.61}$$

Notice that in total we have written nine equations (7.53)–(7.61); clearly they are not all independent. As stated, we have seven independent equations. Observing (7.56)–(7.61) we will select (7.56), (7.58), and (7.61) to calculate F_5. Inserting values and working with (7.56) and (7.58) we get

$$x_2 \times F_2 = 0.25 \times 100 + 0.28 \times F_5, \text{ and } x_2 \times F_2 = 0.55 \times F_4.$$

Therefore,

$$0.25 \times 100 + 0.28 \times F_5 = 0.55 \times F_4. \tag{7.62}$$

And (7.61) states

$$F_4 - F_5 - 24.8 - 0.744 = 0. \tag{7.63}$$

Thus, $F_5 = 39.9$ kg/h and $F_4 = 65.4$ kg/h.
(b) What is the flow rate (kg/h) of the recycle stream? $F_5 = 39.9$ kg/h.
The lesson here is that from step III we already know that we can solve the problem, but we are dealing with seven equations and seven unknowns. As shown, analyzing system S_T first was a good choice. In addition, we would like to emphasize that it would be very good if you could solve the problem, but remember that formulating a problem well and understanding the problem comprise 99 % of our goal.
Now we invite you to test the results. For example, given that the composition of KNO_3 in stream 2 comes from the mixture of the feed stream and the recycle stream, we must expect that

$$0.25 \leq x_2 \leq 0.28.$$

First we will obtain F_2 from (7.57) and then we will calculate x_2 from (7.56):

$$100 + F_5 - F_2 = 0;\ F_2 = 100 + F_5 = 1.40 \times 10^2 \text{ kg/h}.$$

$0.25 \times 100 + 0.28 \times F_5 - x_2 \times F_2 = 0$; replacing and rearranging, we get
$x_2 = 0.257$ kg KNO_3/kg solution, which, as expected, is in the right range.

11. **KNO_3 crystallization without recycle [6].** 100 kg/h of a solution with 25 % KNO_3 will be crystallized in an evaporation–crystallization unit. After the evaporation unit the solution has a

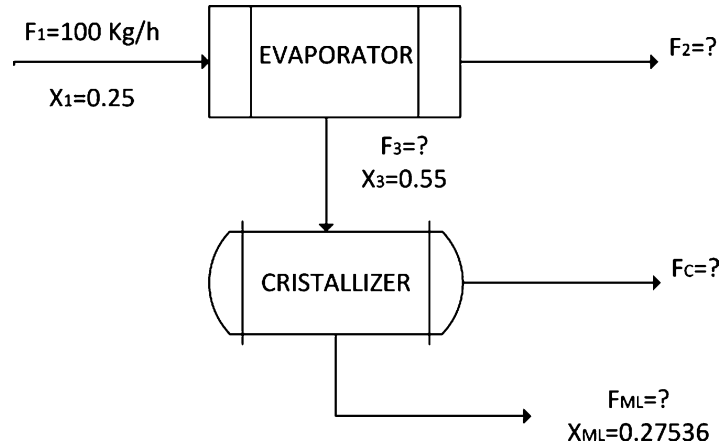

Fig. 7.37 KNO_3 crystallization without recycle including all variables and available data

concentration of 55 % KNO_3. At the crystallizer the temperature is dropped and the mother liquor has 38 kg of KNO_3/100 kg of H_2O. (a) How much H_2O is evaporated? (b) What is the flow rate of pure KNO_3 crystals? (c) What was the efficiency of the process (% mass of KNO_3 crystals/mass of KNO_3 fed)? (d) Compare and discuss these results with those of the previous example.

Step I

Reading and understanding

First, this problem is similar to the previous one but without recycle. Again we will consider separating the flow rate at the output of the crystallizer into two streams, crystals and mother liquor.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

F_1, F_2, F_3, F_C, F_{ML}: mass flow rate of streams kg/h

x_1, x_3, x_{ML}: KNO_3 concentrations w/w (kg KNO_3/kg solution)

(a) How many variables you can identify in this process?

From Fig. 7.37, NV = 8, NSV = 4 (F_1, x_1, x_3, and x_{ML}), and NR = 0. Assume x_{ML} is given indirectly (see previous problem with variable x_5).

Step III

Analysis of degrees of freedom in process

As shown in Fig. 7.37, the process consists of two process units in each of which the fluid has two components. Thus, NMB = 4, and then DF = 8 − 4 − 4 − 0 = 0. The problem can now be solved.

Step IV and V

Mathematical formulation including all available data, resolution, results, analysis, and discussion

As mentioned, x_{ML} is given indirectly and, similarly to the previous problem, we get

$$x_{ML} = 38\ \text{KNO}_3/(38\ \text{kg KNO}_3 + 100\ \text{kg H}_2\text{O}) \sim 0.28\ \text{kg KNO}_3/\text{kg solution}.$$

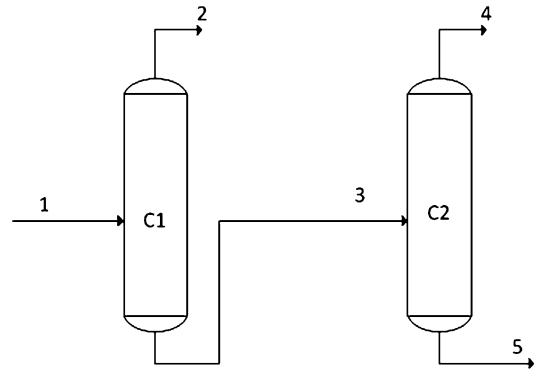

Fig. 7.38 Two distillation columns in series

System S_1 (evaporator)
Total mass balance:

$$100 - F_2 - F_3 = 0. \tag{7.64}$$

Mass balance for KNO_3:

$$0.25 \times 100 - 0.55 \times F_3 = 0. \tag{7.65}$$

From (7.65) we get $F_3 = 45.5$ kg/h, and thus, replacing F_3 in (7.64), we obtain $F_2 = 54.5$ kg/h.

System S_2 (crystallizer)
Total mass balance:

$$45.5 - F_C - F_{ML} = 0. \tag{7.66}$$

Mass balance for KNO_3:

$$0.55 \times 45.5 - F_C - 0.28 \times F_{ML} = 0. \tag{7.67}$$

Then from (7.66) and (7.67) we obtain $F_C = 17.2$ kg/h and $F_{ML} = 28.2$ kg/h.
(a) How much H_2O is evaporated? $F_2 = 54.5$ kg/h.
(b) What is the flow rate of pure KNO_3 crystals? $F_C = 17.2$ kg/h.
(c) What was the efficiency of the process (% mass of KNO_3 crystals/mass of KNO_3 fed)?

$$\eta = 100 \times F_C/(0.25 \times 100) = 100 \times 17.2/25 = 68.9\ \%.$$

(d) Compare and discuss these results with those of the previous example. First, the efficiency of the process fell from a high of 99.2 % (with recycle) to a low of 68.9 % (without recycle). Second, the decision to use or not recycle should be analyzed in economic terms because the inclusion of recycle will imply some additional equipment, like, for example, a mixer, pumps, or fittings. However, the increase in efficiency is so high that it would most likely be worth including recycle stream.

12. **Distillation [6]**. A distillation process composed of two units operating in series under steady-state conditions is fed by a solution containing components A, B, and C (in all streams) (Fig. 7.38).

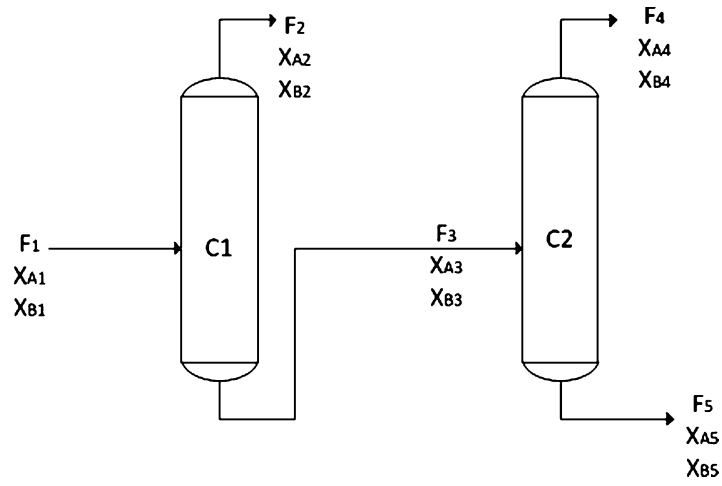

Fig. 7.39 Two distillation columns in series including all variables

(a) How many variables are in this process? (b) Determine the degrees of freedom of the process.

Step I

Reading and understanding

In simple terms, distillation is a classical unit operation in chemical and bioprocessing companies with the aim of separating mixtures based on the difference volatilities of the components. There are several applications of distillation, for example, in crude oil, water (to remove impurities), and fermented solutions.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data (Fig. 7.39)

F_1: Mass flow rate of feed stream (kg/h)
x_{A1}: Mass fraction of component A at feed stream
x_{B1}: Mass fraction of component B at feed stream
F_2: Mass flow rate of top stream of first column (kg/h)
x_{A2}: Mass fraction of component A at top of first column
x_{B2}: Mass fraction of component B at top of first column
F_3: Mass flow rate of bottom stream of first column (kg/h)
x_{A3}: Mass fraction of component A at bottom of first column
x_{B3}: Mass fraction of component B at bottom of first column
F_4: Mass flow rate of top stream of second column (kg/h)
x_{A4}: Mass fraction of component A at top of second column
x_{B4}: Mass fraction of component B at top of second column
F_5: Mass flow rate of bottom stream of second column (kg/h)
x_{A5}: Mass fraction of component A at bottom of second column
x_{B5}: Mass fraction of component B at bottom of second column

(a) How many variables are in this process? NV = 15, NSV = 0, and NR = 0.

Step III

Analysis of degrees of freedom in process

As shown in Fig. 7.39, the process consists of two process units, and each stream has three components. Thus, NMB = 6, and then DF = 15 − 6 − 0 − 0 = 9.

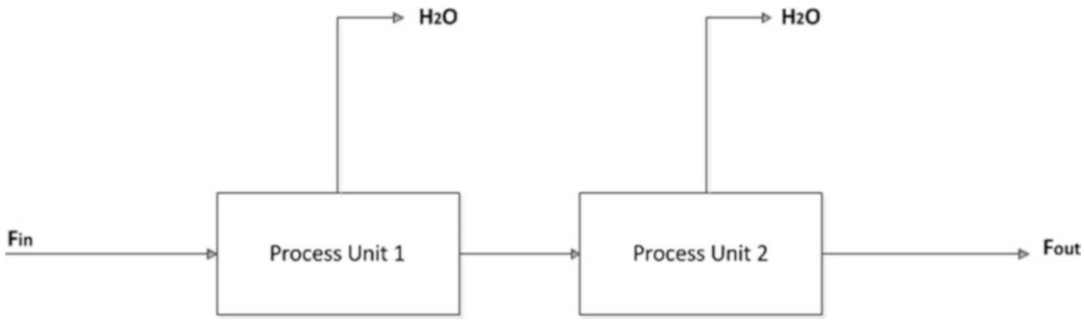

Fig. 7.40 Concentration process with two units in series

(b) Determine the degrees of freedom of the process.

$$\text{DF} = 9.$$

13. **Are there enough data? [8].** In the following concentration process, units 1 and 2 remove H_2O. At the feed the mass flow rate of the wet material is 100 kg/h with a solid concentration of 10 % w/w (the remainder is H_2O). At the output of unit 2 the solid concentration is 80 % (w/w) (Fig. 7.40). (a) How much H_2O was removed in the whole process? (b) What is the mass flow rate of the end product (kg/h)?

Step I

Reading and understanding

A simple observation suggests that there are not enough data to answer the questions. An interesting reflection here is that to completely solve a problem, you need DF = 0, but occasionally, although DF > 0, you can still answer the questions of the problem. If DF > 0, then you cannot calculate the value of all variables, but you might be able to calculate some of them, and if they are the ones requested in the problem statement, then you can solve the problem.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

Our first step will be to complete the flow diagram provided with the problem statement.

F_1: Mass flow rate of wet material (kg/h) (first unit)
x_{S1}: Mass fraction of solids on wet material
F_2: Mass flow rate of semidried material (kg/h) (first unit)
x_{S2}: Mass fraction of solids on semidried material
F_3: Mass flow rate of H_2O (kg/h) (first unit)
F_4: Mass flow rate of H_2O (kg/h) (second unit)
F_5: Mass flow rate of product (kg/h) (second unit)
x_{S5}: Mass fraction of solids on end product.

Thus, NV = 8, NSV = 3, and NR = 0.

Step III

Analysis of degrees of freedom in process

As shown in Fig. 7.41, the process consists of two process units, and each stream has two components. Thus,

$$\text{NMB} = 4.$$

and so DF = 8 − 4 − 3 − 0 = 1.

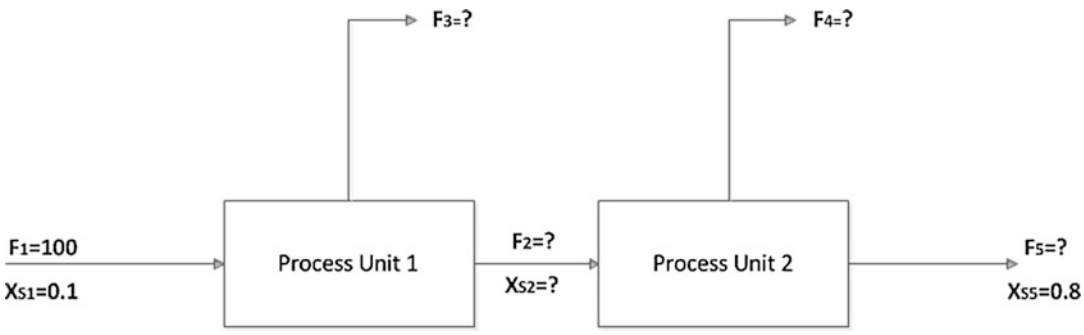

Fig. 7.41 Concentration process with two units in series including all variables and available data

As expressed in step I, we cannot calculate all the unknowns (five in total). We can formulate four independent equations, and we have five unknowns. But as stated, we might be able to calculate some of them.

Step IV
Mathematical formulation including all available data
As a first attempt, we will explore an analysis of system S_T.

System S_T
Total mass balance:

$$100 - F_3 - F_4 - F_5 = 0. \tag{7.68}$$

Solids mass balance:

$$0.1 \times 100 - 0.8 \times F_5 = 0. \tag{7.69}$$

Step V
Solution, results, analysis, and discussion
From (7.69) we get $F_5 = 12.5$ kg/h, and replacing F_5 in (7.68) we get $F_3 + F_4 = 87.5$ kg/h, where $F_3 + F_4$ represents all the H_2O removed in the process.

(a) How much H_2O was removed in the whole process?

$$F_3 + F_4 = 87.5 \text{ kg/h}.$$

(b) What is the mass flow rate of the end product in kilograms per hour?

$$F_5 = 12.5 \text{ kg/h}.$$

As expressed and discussed in step I, we cannot calculate all the unknowns but we might be able to answer the specific questions. For example, we cannot calculate F_3 and F_4 individually, but the question was related to the total amount of H_2O removed ($F_3 + F_4$), and, as shown, it was possible to calculate that.

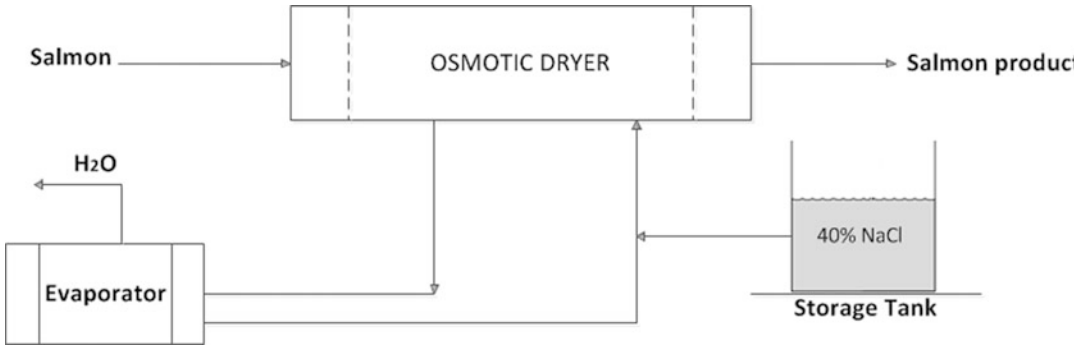

Fig. 7.42 Osmotic dehydration of salmon

14. **Osmotic dehydration of salmon [10].** The salting of salmon through a osmotic-dehydration process has been experimentally tested at the pilot plant scale. In the osmotic-dehydration unit, salmon exchange water and receive salt from the solution. The experimental process is depicted in Fig. 7.42. The object of the process is to obtain a salt (NaCl) concentration of 3 % w/w on the salmon product. 1,000 kg/h of salmon are fed to the osmotic-dehydration unit and the salmon composition is 67 % H_2O, 12 % fat, 19 % proteins, 2 % inert solids (all w/w), and no salt (0 %). Preliminary results show that H_2O in salmon product (after processing) is 65 % w/w. As shown in Fig. 7.42, to maintain NaCl concentration at 30 %, in the osmotic-dehydration unit, part of the brine is concentrated in a single-effect evaporator and then mixed with a brine solution with 40 % NaCl that comes from a storage tank. (a) What is the mass flow rate that comes from the storage tank? (b) How much H_2O is evaporated in the single-effect evaporator?

Step I

Reading and understanding

First, what is an osmotic dehydration process? In osmotic dehydration the product (in this case salmon) is in direct contact with a low-water-activity solution (e.g., concentrated salt) in which a two-way mass transfer is established: (a) water is transferred from the product to the solution and (b) in the opposite direction, solute (in this case salt) is transferred from the solution to the salmon tissue. Now it is clear why it is necessary to inject salt into the system (storage tank) because the salmon product carries 3 % salt at the end of the process.

In terms of minimizing variables, it would be advisable to put together as one variable fat, proteins, and inert material.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

F_1: Mass flow rate of feed stream (kg/h)
x_{S1}: Solids mass fraction at feed stream (fat, proteins, inert, but excluding salt) (w/w)
x_{H2O1}: H_2O mass fraction at feed stream (w/w)
x_{NaCl1}: NaCl mass fraction at feed stream (w/w)
F_2: Mass flow rate salmon product (kg/h)
x_{S2}: Solids mass fraction at outlet stream (fat, proteins, inert, but excluding salt) (w/w)
x_{H2O2}: H_2O mass fraction at outlet stream (w/w)
x_{NaCl2}: NaCl mass fraction at outlet stream (w/w)
F_3: Mass flow rate of brine solution (out of osmotic unit) (kg/h)

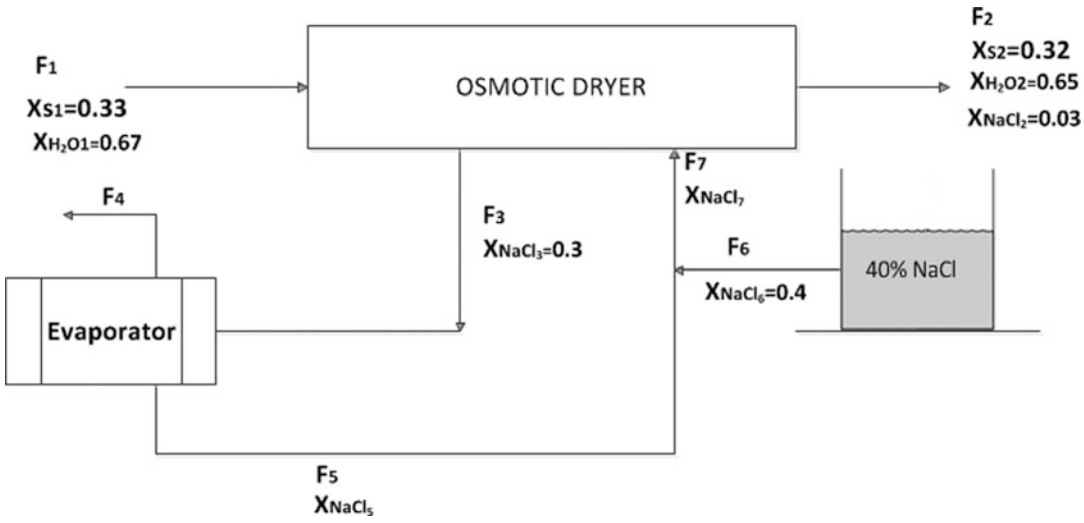

Fig. 7.43 Osmotic dehydration of salmon including all variables and available data

x_{NaCl3}: NaCl mass fraction of brine solution (out of osmotic unit) (w/w)
F_4: Mass flow rate of H2O from single effect evaporator (kg/h)
F_5: Mass flow rate of concentrated brine from evaporator (kg/h)
x_{NaCl5}: NaCl mass fraction of concentrated brine from evaporator (w/w)
F_6: Mass flow rate of brine solution from storage tank (kg/h)
x_{NaCl6}: NaCl mass fraction of brine solution from evaporator (w/w)
F_7: Mass flow rate of mixed brine solution (kg/h)
x_{NaCl7}: NaCl mass fraction of mixed brine solution (w/w)

To minimize the number of variables, we have defined solids as fat, proteins, and inert materials (all together).

Thus, NV = 17, NSV = 9 (F_1, x_{S1}, x_{H2O1}, x_{NaCl1}, x_{H2O2}, x_{S2}, x_{NaCl2}, x_{NaCl3}, and x_{NaCl6}), and NR = 0. We have included x_{S2} as a known variable at the output stream because the mass fraction of H_2O is 0.65 (65 %) and the mass fraction of NaCl is 0.03 (3 %). Thus, $x_{S2} = 1 - 0.65 - 0.03 = 0.32$.

Step III

Analysis of degrees of freedom in process

As shown in Fig. 7.43, the process consists of two process units (osmotic dehydrator and an evaporator) plus an "added" mixer. In the osmotic-dehydration unit we have three components, and so we can formulate three independent material balance equations. In the evaporator and mixer we have two components (H_2O and NaCl), so we can formulate two equations for each equipment item. Therefore, NMB = 7.

$$\mathrm{DF} = \mathrm{NV} - \mathrm{NMB} - \mathrm{NSV} - \mathrm{NR} = 17 - 7 - 9 - 0 = 1.$$

Thus, we cannot calculate all the unknowns (of which there are eight).

Step IV

Mathematical formulation including all available data

As expressed in step III, we can formulate seven independent equations (NMB = 7) and the process has eight unknowns. We cannot calculate all unknowns but, as shown in the previous problem (problem 13), we might be able to calculate the unknowns that are related to questions (a) and (b) i.e., F_4 and F_6, respectively.

First, we will write three mass balances for the system S_T because in these material balances, both F_4 and F_6 variables are involved.

System S_T

Total mass balance:

$$1,000 + F_6 - F_2 - F_4 = 0. \tag{7.70}$$

Solid mass balance:

$$0.33 \times 1,000 - 0.32 \times F_2 = 0. \tag{7.71}$$

NaCl mass balance:

$$0.4 \times F_6 - 0.03 \times F_2 = 0. \tag{7.72}$$

Step V

Solution, results, analysis, and discussion

From step IV we have three equations (7.70)–(7.72) and three unknowns (F_2, F_4, and F_6). Within these three unknowns are the required calculations of F_4 and F_6.

From (7.71) and (7.72) we can directly obtain F_2 and F_6, respectively. Therefore, $F_2 = 1.03 \times 10^3$ kg/h and $F_6 = 77.34$ kg/h. Replacing F_2 and F_6 in (7.70) we get $F_4 = 46.09$ kg/h.

This is an intricate problem. In our experience, students "suffer" a lot with this problem, but if you follow the proposed procedure, not only will it seem very simple, but indeed it is simple.

15. **Glutamic acid purification [10⁺]**. Glutamic acid ($C_5H_9NO_4$) is a nonessential amino acid (the human body is capable of producing it), but it is extensively used as a food additive due to its taste-enhancing properties and because it gives food an umami, or savory, taste. Umami is a term introduced by Kikunake Ikeda in 1908 and since 1985 has been recognized as the fifth basic taste. A processing plant for the purification of glutamic acid starts with a flow rate of a liquor that contains 1,000 lb/h of water, 5.0 lb/h of impurities, and an unknown amount of glutamic acid. A schematic representation of the process is given in Fig. 7.44.

The liquor is concentrated in the evaporator where 950 lb/h H_2O are removed. Then to favor crystallization a diluted solution of 85.14 lb/h containing 10/11 parts H_2O and 1/11 parts of the substance w/w (considered as impurity) is added to the precipitator.

In the centrifuge, crystals are separated from the residual liquor. Crystals are wet with a layer of liquor, where the liquor layer represents 11.09 % w/w of the clean crystals. Then the residual liquor is recycled and mixed with the liquor fed to the process, and part of the residual liquor purged to avoid the accumulation of impurities in the process. Finally, crystals are fed to a dryer where all the remaining water is removed. The dryer is fed with 490 lb/h dry air with a humidity of 0.001 lb H_2O/lb dry air and leaving the dryer with a humidity of 0.011 lb H_2O/lb dry air. The final product contains 1 % impurities. (a) What is the efficiency of the process, defined as follows: (kilogram per hour of glutamic acid in final product/kilogram per hour of glutamic acid fed) × 100. (b) What is the flow rate and composition of purge.

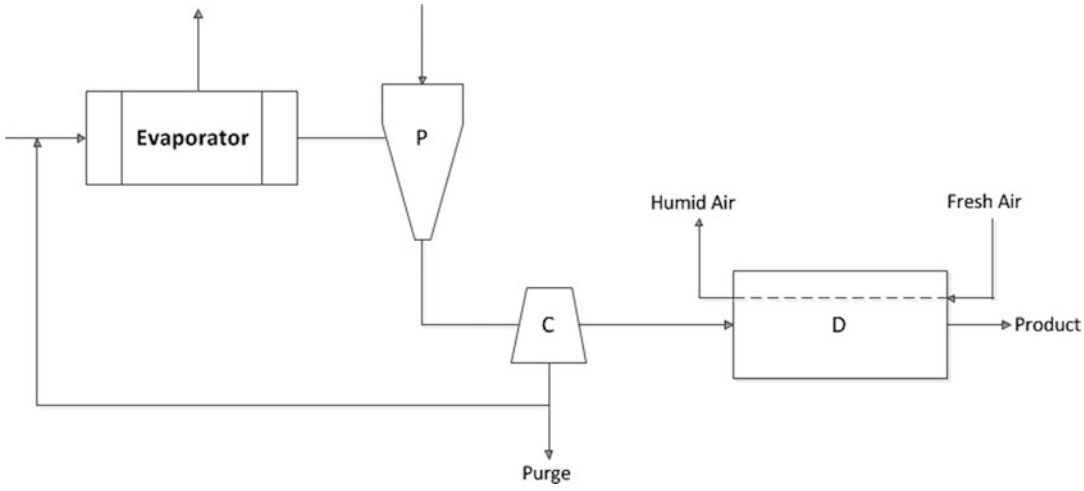

Fig. 7.44 Glutamic acid purification

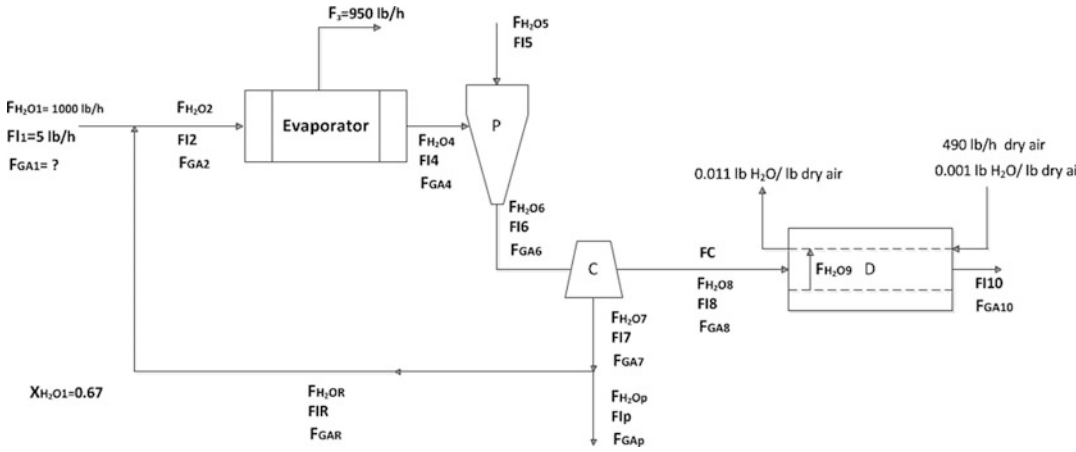

Fig. 7.45 Glutamic acid purification process including all variables and available data

Step I

Reading and understanding

First, the data are not provided in a typical way. For example, in the feed stream the information on H_2O and impurities are given as a mass flow rate. In addition, at first glance, there are several variables. As was stressed earlier, it will be critical to complete Fig. 7.44 and then carefully determine the degrees of freedom. Then we will need to look at the whole picture and devise a good strategy to solve the problem. We might be able to calculate all the unknowns, but remember that our main object is to answer the questions.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

As depicted in Fig. 7.45, we have decided to use the mass flow rates as variables for each component (not the composition) due to the way the data were provided in the problem statement, where $F_{(i)j}$ is

the mass flow rate of component i [water (H_2O), glutamic acid (GA), and impurities (I)] in stream j (1, 2, 3. . .10, R, P), where R stands for recycle and P for purge.

Then we can determine that NV = 31, NSV = 4, and NR = 10. It is clear from Fig. 7.45 that we have in total 31 variables and 4 of them are specified, but why NR = 10?

From the problem statement we know that the diluted solution added to the crystallizer contains 10/11 parts water and 1/11 parts substance w/w (two relationships). Also, H_2O, impurities, and glutamic acid are in the same proportion in all streams with the mother liquor (recycle, purge stream 7, and mother liquor in stream 8. From that we get six independent relationships as follows:

$$F_{IP}/F_{H2OP} = F_{IR}/F_{H2OR}, F_{IP}/F_{H2OP} = F_{I7}/F_{H2O7},\ F_{IP}/F_{H2OP} = F_{I8}/F_{H2O8},$$
$$F_{GAP}/F_{H2OP} = F_{GAR}/F_{H2OR},\ F_{GAP}/F_{H2OP} = F_{GA7}/F_{H2O7}, F_{GAP}/F_{H2OP} = F_{GA8}/F_{H2O8}.$$

In addition, the liquor layer represents 11.09 % w/w of the clean crystals (one relationship), and the final product contains 1 % impurities (one relationship). Therefore, in total we have ten relationships.

Step III

Analysis of degrees of freedom in process

Observing the flow diagram we can formulate three independent material balances in each piece of equipment (mixer, evaporator, crystallizer, and centrifuge), for a total of 12 equations. The dryer can be separated into two systems, and so we can formulate three equations for the mother liquor and crystals and one for the air. Finally, at the purge there is a division, and so we can formulate one more independent equation.

Therefore, NMB = 17, and DF will be

$$\mathrm{DF} = 31 - 17 - 4 - 10 = 0.$$

Step IV and V

Mathematical formulation including all available data, solution, results, analysis, and discussion

As was mentioned earlier, we need to focus, first, on solving the problem; it is not necessary to answer all unknowns (27 in total). Then, we will first calculate the unknowns related to the questions. What are they?

(a) What is the efficiency of the process, defined as follows: (kg/h glutamic acid in final product/kg/h glutamic acid fed) × 100. Thus, efficiency = $100 \times (F_C + F_{GA10})/F_{GA1}$.

(b) What is the flow rate and composition of the purge, thus the mass flow is ($F_{H2OP} + F_{IP} + F_{GAP}$) and the compositions are $x_{H2OP} = F_{H2OP}/(F_{H2OP} + F_{IP} + F_{GAP})$; $x_{IP} = F_{IP}/(F_{H2OP} + F_{IP} + F_{GAP})$; $x_{GAP} = F_{GAP}/(F_{H2OP} + F_{IP} + F_{GAP})$.

This indicates that our strategy should be focused on calculating F_{GA1}, F_C, F_{GA10}, F_{H2OP}, F_{IP}, F_{GAP}.

First, looking at system S_T (global) for H_2O, we have three unknowns, F_{H2O5}, F_{H2OP}, and F_{H2O9}, where we can formulate the following equations.

Global mass balance for H_2O

$$1,000 - 950 + F_{H2O5} - F_{H2O9} - F_{H2OP} = 0. \tag{7.73}$$

Relationship for F_{H2O5}:

$$F_{H2O5} = (10/11) \times 85.14; \text{therefore, } F_{H2O5} = 77.40 \text{ lb/h}.$$

Mass balance for H_2O in dryer

H_2O entering with air + H_2O from mother liquor − H_2O leaving with air = 0, and given that the mass flow rate of dry air does not change,

$$490 \times 0.001 + F_{H2O9} - 490 \times 0.011 = 0; \text{thus, } F_{H2O9} = 4.90 \text{ lb/h}.$$

Replacing F_{H2O5} and F_{H2O9} in (7.73) we get $F_{H2OP} = 1.23 \times 10^2$ lb/h.

Now our focus should be to obtain F_{GA1}, F_C, F_{GA10}, F_{IP}, F_{GAP}

Global mass balance for impurities:

5 + $F_{I5} - F_{I10} - F_{IP} = 0$, where $F_{I5} = (1/11) \times 85.14 = 7.740$ lb/h. Therefore, 5.0 + 7.740 − $F_{I10} - F_{IP} = 0$; then

$$12.7 - F_{I10} - F_{IP} = 0. \tag{7.74}$$

The final product contains 1 % impurities. Thus,

$F_{I10}/(F_C + F_{GA10}) = 0.01$. Note that $F_{GA10} = F_{GA8}$. Therefore, we can write

$$F_{I10}/(F_C + F_{GA8}) = 0.01. \tag{7.75}$$

Crystals are wet with a layer of liquor, where the liquor layer represents 11.09 % w/w of the clean crystals. Thus,

$(F_{GA8} + F_{I8} + F_{H2O8})/F_C = 0.1109$, where $F_{I8} = F_{I10}$ and $F_{H2O8} = F_{H209} = 4.9$ lb/h. Then

$$(F_{GA8} + F_{I10} + 4.9)/F_C = 0.1109. \tag{7.76}$$

Finally, from $F_{IP}/F_{H2OP} = F_{I8}/F_{H2O8}$ and $F_{GAP}/F_{H2OP} = F_{GA8}/F_{H2O8}$, and recalling that $F_{H2O8} = F_{H209} = 4.9$ lb/h, $F_{GA10} = F_{GA8}$ and $F_{H2OP} = 1.23 \times 10^2$ lb/h, we can write

$F_{IP}/122.5 = F_{I8}/4.9$, but $F_{I8} = F_{I10}$. Thus,

$$F_{IP}/122.5 = F_{I10}/4.9, \tag{7.77}$$

$$F_{GAP}/122.5 = F_{GA8}/4.9. \tag{7.78}$$

We have five equations (7.74)–(7.78) and five unknowns (F_{I10}, F_{IP}, F_C, F_{GA8}, and F_{GAP}). Therefore,

$$F_{I10} = F_{I8} = 0.47,\ F_{IP} = 12.25,\ F_C = 48.96,\ F_{GA8} = F_{GA10} = 0.0397,\ \text{and } F_{GAP} = 0.9924.$$

Now doing a global mass balance for glutamic acid we obtain

$$F_{GA1} - F_C - F_{GA10} - F_{GAP} = 0. \tag{7.79}$$

Replacing F_C, F_{GA10} and F_{GAP} we get $F_{GA1} - 4.9 \times 10 - 0.040 - 0.99 = 0$, then $F_{GA1} = 50$.

(a) Efficiency $= 100 \times (F_C + F_{GA10})/F_{GA1} = 100 \times (4.9 \times 10 + 0.040)/50 = 98\ \%$.
(b) Flow rate of purge $= (F_{H2OP} + F_{IP} + F_{GAP}) = 122.5 + 1.2 \times 10 + 0.99 = 1.36 \times 10^2$ lb/h.
(c) $X_{H2OP} = F_{H2OP}/(F_{H2OP} + F_{IP} + F_{GAP}) = 122.5/1.36 \times 10^2 = 0.902\ (90.2\ \%)$,

$$X_{IP} = F_{IP}/(F_{H2OP} + F_{IP} + F_{GAP}) = 1.2 \times 10/1.36 \times 10^2 = 0.0902\ (9.02\ \%),$$
$$X_{GAP} = F_{GAP}/(F_{H2OP} + F_{IP} + F_{GAP}) = 0.99/1.36 \times 10^2 = 0.0073\ (0.73\ \%).$$

One lesson from this example is that we need to be flexible in the way we define variables. In this case, the input data were given in a particular way and then we defined the variables accordingly. In addition, in this example and then in real-world problems, it is critical to have a good flow diagram and read and follow the problem statement together with the diagram to get a feel for the problem.

7.10 Proposed Problems

Before taking on the exercises, it is advisable that the student become familiar with the function and purpose of the equipment in each process stage related to the problem. Then, after the problem has been critically analyzed and solved, it is recommended that you search for information (e.g., on the Web) and briefly describe the equipment used in each problem and then add an example of its industrial utilization.

1. **Salt solution [3]**. A processing plant requires a stream with a flow rate of 100 kg/h with a salt concentration of 1 % (w/w). For this purpose, the plant has unlimited quantities of water and a stream with 3 % salt (w/w). In what proportion should the water and stream be mixed with 3 % salt to obtain a desired stream of 100 kg/h with 1 % salt?
   A: 2:1 (kg/h water stream /kg/h stream with 3 % salt)
2. **Apple juice [3$^+$]**. As a general rule, fresh juices have better color and flavor than processed juices. An undisputed advantage of processed juice is its extended shelf life and convenience. An important aspect to consider when processing fruit juice is trying to retain its natural properties, such as color, flavor, and aromas. 100 kg/h of an apple juice with moisture of 92 % (w/w) will be mixed with 200 kg/h of an apple juice with 94 % moisture (w/w). After being well mixed, they are passed through an evaporator to obtain a concentrated product with 50 % solids (w/w). How much water should be removed in the evaporator?
   A: 260 kg/h
3. **Mixing separation process [5]**. Two streams with three components, A, B, and C, are fed to a process unit. At the outlet of the equipment there is one stream containing components A, B, and C. In a similar way as was analyzed in problem 1 (Sect. 7.9) we can determine that this system has six degrees of freedom. (a) Can we arbitrarily fix the composition of the three streams (say, x_{A1}, x_{B1}, x_{A2}, x_{B2}, x_{A3}, and x_{B3})? (b) If your answer to (a) is no, explain.
   A: (a) No. (b) For example, if you fix x_{A1} and x_{A2}, then you can fix x_{A3}, but you are limited to a value between x_{A1} and x_{A2}
4. **Vacuum dryer [3]**. Usually, a vacuum dryer is used for materials (e.g., foods) that could get damaged if exposed to high temperatures. Furthermore, the vacuum prevents oxidation and hazardous conditions in certain materials that may be explosive in the presence of oxygen. It is also used when one must achieve very low humidity levels. In addition, dehydration time is normally lower when compared with atmospheric dryers. To further increase the solid

concentration of the previous product (use the data from exercise 2), it will be further processed in a vacuum dryer to obtain a final product with 66.67 % solids (w/w). How much water should be removed in the vacuum dryer?
A: 10 kg/h

5. **Single-effect evaporator [5]**. A stream of water and component A have a flow rate of 10 L/h, where the density of A is 1.2 kg/L and its concentration is 0.20 kg $A/L_{solution}$. The stream is passed through a single-effect evaporator where 20 % by weight of the input flow is removed (water). (a) What is the mass flow rate at the entry of the evaporator? (b) What is the density of the solution at the entry of the evaporator? (c) How many kilograms of water are evaporated per hour? (d) What is the density of the solution at the outlet of the evaporator? (e) What is the concentration of component A at the outlet w/v?
A: (a) ~10.34 kg/h, (b) ~1.034 kg/L, (c) ~2.067 kg/h, (d) ~1.042 kg/L, (e) 0.252 kg $A/L_{solution}$
6. **Humid air [2]**. Humid air contains 0.025 lb water vapor per pound of dry air. How many pounds of water vapor does humid air contain per pound of humid air?
A: ~0.024 pounds of water vapor per pound of humid air
7. **Continuous process [5]**. A continuous process is fed with 10 lb/min of humid air containing 0.02 pounds of water vapor per pound of dry air. At the outlet, the air contains 0.03 g of water vapor per gram of humid air. How much water was removed or added to the air in 1 h of operation?
A: 6.43 lb water were added to the air in 1 h.
8. **Continuous process [6].** Humidification is an operation directed at increasing the amount of vapor present in a gas stream. The vapor may be increased by passing the gas through a liquid that evaporates into the gas. The transfer process through the gas stream takes place by diffusion at the interface. A continuous process is fed with A lb/min of humid air containing **a** pounds of water vapor per pound of dry air. At the outlet, the air contains **b** grams of water vapor per gram of humid air, where **b** > **a** and both <<<< 1. (a) Is the equipment a dryer or a humidifier? (b) How much water was removed or added in 1 h of operation?
A: (a) Humidifier. (b) 60A(b(1 + a)-a)/((1 + a)(1-b)) pounds of water were added in 1 h. You can check this formula using the data from exercise 7.
9. **Air mixture [4]**. 10 lb/min of humid air containing 0.02 pounds of water vapor per pound of dry air are mixed with 10 lb/min of humid air that contains 0.03 grams of water vapor per gram of humid air. What is the humidity of the air mixture at the outlet on a dry basis?
A: ~0.0255 pounds of water vapor per pound of dry air
10. **Air mixture [5]. A** lb/min of humid air containing **a** pounds of water vapor per pound of dry air are mixed with **B** lb/min of an air that contains **b** grams of water vapor per gram of humid air. What is the humidity of the air mixture at the outlet on a dry basis?
A: (Aa + Bb(1 + a))/(A + B(1 + a − b − ab)) pounds of water vapor per pound of dry air. You can check this formula using the data from exercise 9.
11. **Humidification [4].** Humidification is an operation directed at increasing the amount of vapor present in a gas stream. The vapor may be increased by passing the gas through a liquid that evaporates into the gas. The transfer process through the gas stream takes place by diffusion at the interface. In a certain process, it is necessary to increase the humidity of a gas stream. (a) How much water is required to increase the humidity of 100 kg of a gas having an original humidity of 1 % (w/w, wet basis) until it reaches a humidity of 2 % (w/w, wet basis)? (b) What is the mass of the final gas stream with 2 % humidity after the process?
A: (a) 1.20 kg; (b) 1.01×10^2 kg
12. **Humidification [6].** In a certain process, it is necessary to increase the humidity of a gas stream. (a) How much water is required to increase the humidity of **A** kg of a gas stream having an

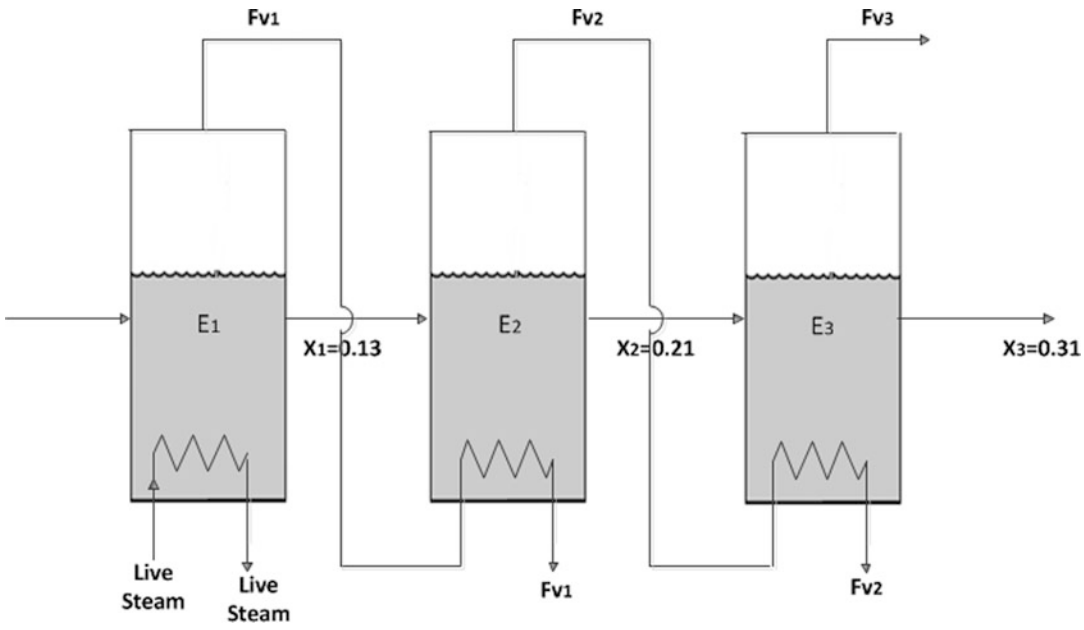

Fig. 7.46 Multi-effect evaporator

original humidity of **a** % (w/w, wet basis) until it reaches a humidity of **b** % (w/w, wet basis) where **b** > **a**? (b) What is the mass of the final gas stream with **b** % humidity after the process?
A: (a) A(b − a)/(100 − b); (b) (A(100 − a))/(100 − b). You can check this formula using the data from exercise 11.

13. **Multieffect evaporator [5]**. In an evaporation process, the fluid is heated to its saturation temperature, and then additional energy is applied to start the liquid evaporation. Evaporation occurs at constant temperature and requires a large amount of energy so that the molecules in the liquid state pass to the vapor state. Unlike dryers, what is achieved in the evaporator is concentration, where normally a diluted "juice" with 5–10 % solids is concentrated to 30–50 % solids. Indeed, evaporation is a concentration process and not a dehydration process. Tomato juice will be concentrated in a three-stage multieffect evaporator (Fig. 7.46). 1,000 kg/h of tomato juice are fed to the system with an initial concentration of 6 % solids (w/w). The object of the operation is to obtain a commercial concentrate of 31 % solids (w/w). The outlet solid concentration of stages 1–3 are 13, 21, and 31 % (w/w), respectively. (a) How much water was removed in each stage of the system? (b) What is the flow rate of the concentrate at the outlet in kg/h?
Note: It is reasonable to assume that tomato juice is composed of two phases, solids (mainly soluble carbohydrates) and water.
A: (a) ~538.5, ~175.8, and ~92.17 kg/h, respectively; (b) ~193.6 kg/h
14. **Tomato concentrates [6]**. The agro industry is a very important industrial sector worldwide, especially for countries like New Zealand and Chile. In addition, process optimization has always been a noble objective of engineers entrusted with the responsibility of developing and improving processes throughout the food industry. In trying to optimize the quality of tomato concentrate [31 % solids (w/w)], some modifications have been proposed to the operation of a three-effect evaporator (problem 13). A good and creative friend of yours is proposing to include a side stream so not all the juice passes through the three stages of the multieffect evaporator. He is

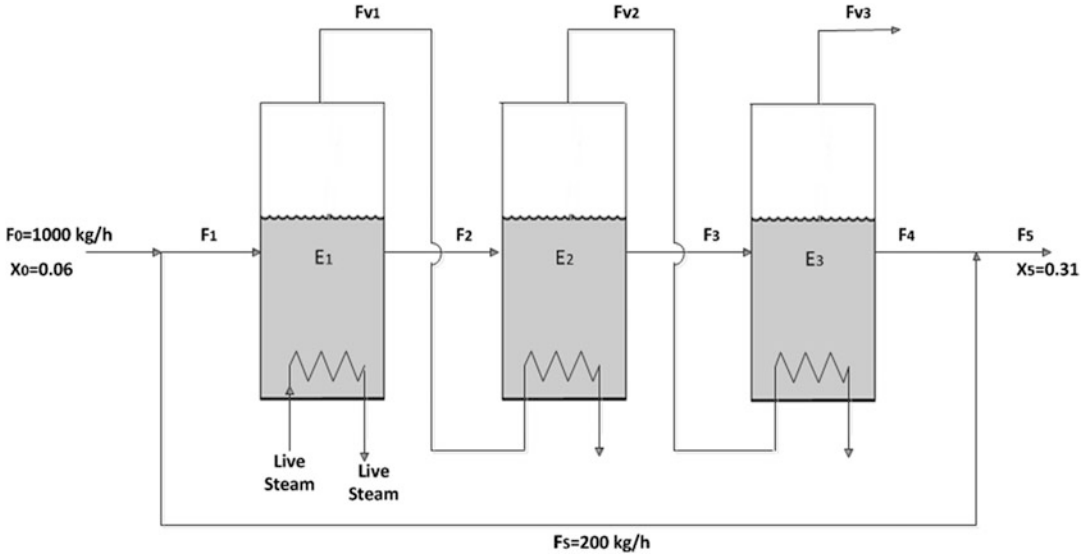

Fig. 7.47 Multi-effect evaporator for tomato concentration

proposing to have a side stream of 200 kg/h, as shown in Fig. 7.47, and is assuring you a very high quality of the tomato concentrate in terms of color and consistency. He argues that the process, including the side stream, has 15 variables (NV), 9 independent material balances (NMB), and 6 degrees of freedom. He points out that so far, he has used just three of them ($X_0 = 0.06$, $F_0 = 1{,}000$ kg/h, and $X_5 = 0.31$), so he still has three degrees of freedom. Where will he decide to have one of the degrees of freedom to be the side stream of 200 kg/h? What do you think?

A: Your friend is right in his analysis of the degrees of freedom, but, as was stated in Sect. 7.8.2.1, there are limitations on the value that can be assigned, in this case to the side stream. Yes, your friend can fix the side stream, but with limitations. As calculated in problem 13, the output flow [31 % solids (w/w)] is ~193.6 kg/h. Then your friend's proposition is impossible to implement because the side stream should be less than 193.6 kg/h. What is the maximum theoretical value of the mass flow rate of the side stream? ~142 kg/h

15. **New idea [5]**. After listening to your powerful arguments (problem 14), your creative friend acknowledges that you are right, but he comes back with a new idea. He still wants to have a side stream of 200 kg/h, but he is now aware of your technical and correct analysis. Now, for the same inputs and outputs (feed stream = 1,000 kg/h, and input and output solids of 6 and 31 %), he has designed a five-effect evaporator with a side stream of 200 kg/h, but this time the side stream will be mixed with the output of stage 3 (Fig. 7.48). In addition, the amount of removed vapor will be the same in each effect. (a) What do you think? (b) If you are now convinced of your friend's proposition, then what are the values of the solid concentration in each stage?

A: (a) We do not know whether or the new idea is a good one, but at least it is feasible. b) 7.5 %, 10.03 %, 15.15 %, 11.62 % (after mixing with side stream), and 16.9 %

16. **Strawberry jam with pectin [5]**. Pectin is a fiber that is normally found in acid fruits. One of their many uses is as a gelling agent, and it is used for the manufacture of marmalades. It is sold as a powder (white). You have been asked to perform the calculations needed to prepare 100 kg of strawberry jam (65 % (w/w) solids). Since this is the first time you will prepare a jam, you will be using a recipe you inherited from your grandmother. According to the recipe, the main ingredients

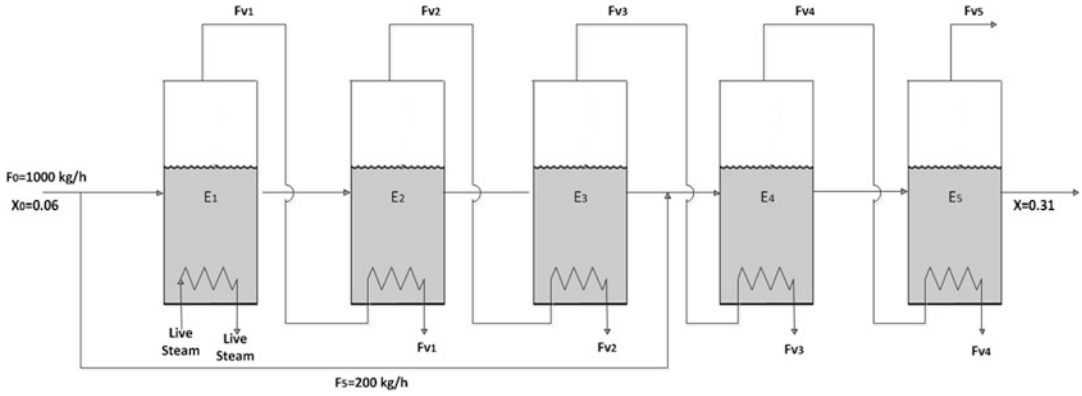

Fig. 7.48 Multi-effect evaporator for tomato concentration with a side-stream

are crushed fruit, sugar, and a gelling agent, as mentioned, usually pectin. The recipe indicates the addition of 120 g of pectin per 100 kg of jam and 55 parts sugar per 45 parts crushed fruit. The commercial sugar has 5 % w/w moisture, and the strawberry has 12 % w/w solids. If it can be assumed that the amount of water in the pectin is negligible, then: (a) How many kilograms of strawberries are needed to prepare 100 kg of jam? (b) How many kilograms of water are evaporated in the "cooking" process?

A: (a) ~50.64 kg. (b) ~12.66 kg

17. **Quince jam [5]**. Now you are asked to prepare some quince jam. Since experimentation is the mother of science, again you use the notes of your beloved grandmother. In the detailed recipe preparation steps, first you need to wash the quinces, peel and cut them into chunks, removing the heart, put them in a pot with a little water, and cook for 30 min. Then grind it all up and add the same amount of quince pulp and sugar. Finally mix well and cook for about 10 min. Unfortunately, there is no indication about the composition of the quince pulp and how much pectin is needed. But you have been learning to be practical and not complicated, so you use the same amount of pectin that was recommended in the recipe for strawberry jam –120 g of pectin per 100 kg of jam. After some research you find out that the composition of quince pulp is 92 % w/w H_20 and the rest is solids. Assume that the sugar is 5 % w/w of H_2O and that the amount of H_2O in the pectin is negligible. If the jam is 62 % w/w solids, then how many kilograms of sugar are needed to prepare 100 kg of quince jam?

A: ~60 kg

18. **Generic formula [6⁺]**. Given that most jams follow a similar recipe, you want to develop a generic formula to avoid doing these calculations each time. If the fruit pulp has **a** % w/w H_2O, then the sugar has **b** % w/w H_2O, the pectin has no H_2O, and **c** g of pectin are added per **g** kg of jam; finally, **e** parts pulp are used for **f** parts sugar. Thus: (a) How many kilograms of sugar are needed to prepare **g** kg of jam with **h** % w/w solids? (b) How many kilograms of fruit are needed to prepare **g** kg of jam with **h** % w/w solids?

A: (a) $\frac{g\left[1-\frac{100-h}{100}\right]-\frac{c}{1{,}000}}{\frac{e}{f}+1-\frac{ae}{100f}-\frac{b}{100}}$ (kg) You can check this formula using the data from exercise 17.

(b) $\frac{e}{f}\times\frac{g\left[1-\frac{100-h}{100}\right]-\frac{c}{1{,}000}}{\frac{e}{f}+1-\frac{ae}{100f}-\frac{b}{100}}$ (kg)

19. **Side stream [5]**. Tomato juice will be concentrated in a continuous one-stage evaporator. The process includes a side stream to avoid excessive browning in the final product

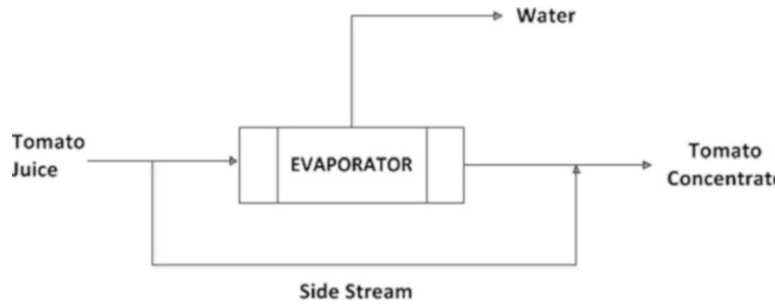

Fig. 7.49 Single-effect evaporator with a side-stream

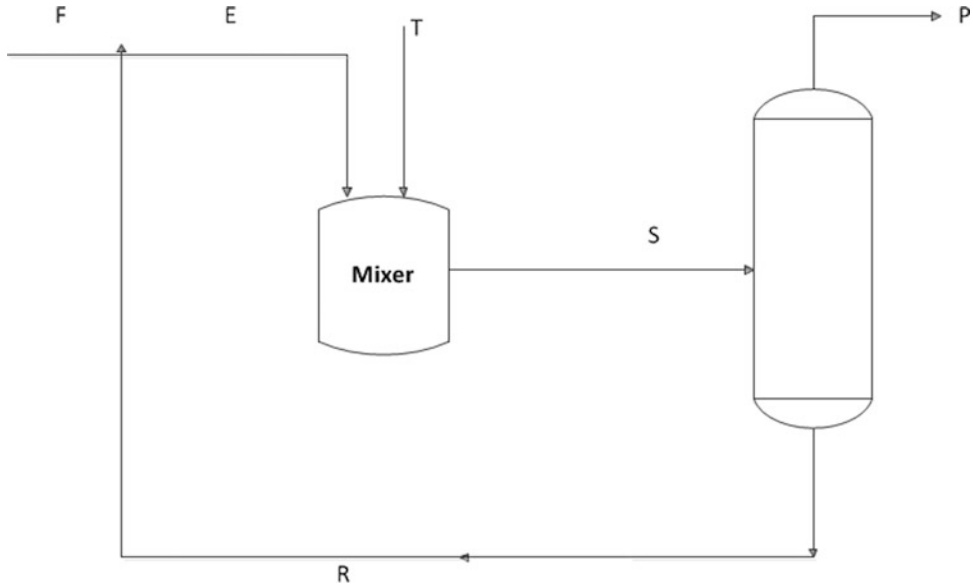

Fig. 7.50 Schematic diagram for a new process

(Fig. 7.49). The process is fed with 1,000 kg/h tomato juice (6 % w/w solids) and the side stream flow rate corresponds to 12 % of the feed flow. If you have to obtain a concentrate with 30 % w/w of solids, then: (a) How many kilograms per hour of concentrate are obtained? (b) What is the percentage w/w of solids at the evaporator outlet?

A: (a) 200 kg/h; (b) 66 % w/w

20. **Separation [4]**. A stream of 100 kg/h with 40 % w/w A, 30 % w/w B, and 30 % w/w water should be separated into two flows, one containing A and possibly water (but not B) and a second flow containing B and possibly water (but not A). In addition, the ratio of the stream containing A to the stream containing B is 2:3. (a) What is the flow rate of each stream? (b) What is the composition of each flow?

    A: (a) 60 kg/h (the one containing B) and 40 kg/h (the one containing A). (b) 50 % w/w B and 50 % water; 100 % A

21. **New process [5]**. The process shown in Fig. 7.50 is the base of the production in MBT Enterprises. The manager gives you the following information to carry out a complete analysis of the material balance of the whole process. The feed flow of the process is $F = 15$ m^3/h ($\rho = 1$ g/mL). This flow is composed of water and solids, both with a density of 1 g/mL. At the

outlet of the mixer, the flow rate is 20,000 kg/h and the added water (T) is 2,000 kg/h. The manager believes that this is enough information for you to carry out the material balance analysis. You carefully analyze the given information and then tell the manager that in fact you don't have enough information and argue that the process has more degrees of freedom than the manager thinks. Specifically, you ask for two more data points. Without objection, the manager does two additional measurements and then tells you the following information: the solid concentration on flow E is 20 % w/w and the solids concentration in flow F is 18 % w/w.
(a) How many degrees of freedom did the process have before and after obtaining the additional data? (b) Was it reasonable to ask for more information? (c) Complete all the information for each flow (including composition).
A: (a) First, two degrees of freedom, so 0. (b) Yes, you received three pieces of data (flows F, S, and T) but five were needed, (c)

Stream	Flow rate (kg/h)	% w/w of solids
F	15,000	18
E	18,000	20
T	2,000	–
S	20,000	18
P	17,000	15.9
R	3,000	30

22. **Fish meal [4]**. Ten tons of fish meal are processed in a dryer where the moisture (w/w) is decreased from 15 to 8 % (w/w). In the drying process, the hot air is at 350 °C and has a humidity of 0.012 g H_2O/g dry air. At the outlet, the air is at 80 °C with a humidity of 0.075 g H_2O/g humid air. Determine the volume of air fed.
A: ~2.0×10^7 L
23. **Evaporation-crystallization [5]**. Crystallization is the process of forming crystals from a solution. Strictly speaking, it is one of the techniques used to separate a "solid" from a solution. To perform crystallization, usually the temperature is lowered, thus decreasing the solubility of the solid in the solution. An alternative process is to remove water by evaporation. A solution contains 20.0 % w/w Na_2SO_4, 15.0 % w/w Na_2CO_3, with the rest being water. This solution is passed through an evaporator that removes 10.0 % of the original mass and then crystallizes 20.0 % Na_2SO_4 and 30.0 % Na_2CO_3. The crystals formed are Na_2CO_3 per ten molecules of H_2O and Na_2SO_4 per ten molecules of H_2O. Calculate the composition of the residual solution.
A:

Component	Composition % w/w
Na_2SO_4	23.3
Na_2CO_3	15.3
H_2O	61.5

24. **Stream mixing [8]**. Two streams of unknown flow rate are mixed to form one stream of **a** kg/h. If a soluble salt is added to the first stream, it reaches a salt concentration of 4.76 % w/w, and the salt content in the outlet mixed stream is 0.62 % w/w. What is the ratio of these two streams?
A: 7
25. **Distillation [4]**. Distillation is a technique used for separation of components in a single liquid stream. This separation is based on the different boiling points of the components. At the industrial level, one of the best known processes is the production of gasoline. Different types of distillation include, for example, simple distillation and fractional distillation. A mixture of

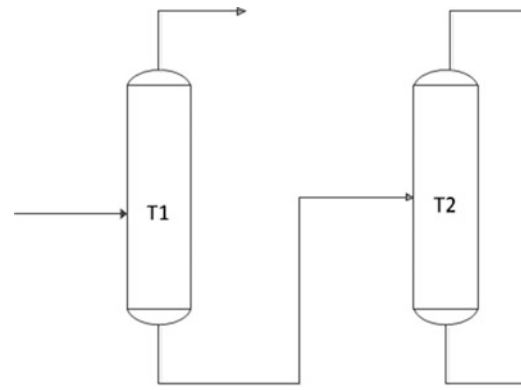

Fig. 7.51 Distillation process with two columns in series

benzene, toluene, and xylene is separated in a fractional distillation process into two columns. The first column delivers benzene as a product at the top and toluene and xylene as product at the bottom. Then the bottom product is fed to a second column. The second column delivers toluene at the top and xylene as the bottom product. A flow diagram of the process is shown in Fig. 7.51. If the feed of the first column is 25,780 lb-mol/day of a mixture whose molar composition is 37.54 % benzene, 22.34 % toluene, and 40.12 % of xylene, then what is the composition w/w at the bottom of the first column?

A: 66.13 % xylene and 33.88 % w/w toluene

26. **Distillation [5].** A process of two distillation columns arranged in series is designed to separate a mixture of three components—benzene, toluene, and xylene. The product will be three streams, each one rich in one of these chemical species. 1,275 kmol/h of a mixture of 30 mol% benzene (B), 25 % toluene (T), and the remaining xylene (X) are fed to the process. In the first column, at the bottom, the product contains 99 % xylene and no benzene. This stream recovers 98 % of the total xylene that was fed. The product at the top of the first column is fed to the second column. In the second column, the top product contains 99 % benzene and no xylene. The benzene recovered in this stream represents 96 % of the benzene fed to the column. What is the molar composition in each output stream?

A:

Column	Top % molar	Bottom % molar
1	54.1	–
	44.28	1
	1.62	**X: 99**
2	**B: 99**	4.56
	1	**T: 92.03**
	–	3.41

Benzene, **T**oluene and **X**ylene

27. **Distillation [4].** A media culture that for practical purposes can be considered as a mixture of ethanol (C_2H_5OH) and water will be distilled. The process basically consists of two columns connected in series (Fig. 7.52). The alcohol distillate (end product) obtained through the top of the second column must contain 94 % w/w ethanol. The feed flow of the first column is 100 kg/h, and the flow rate at the top is 20 kg/h. The alcohol concentration at the bottom of the first column is 2 % w/w. The flow at the bottom of the second column is 5 kg/h with 6 % w/w alcohol. (a) What is the composition of the feed flow? (b) What is the composition of the top flow of column 1? (c) How many kilograms per hour of end product are obtained?

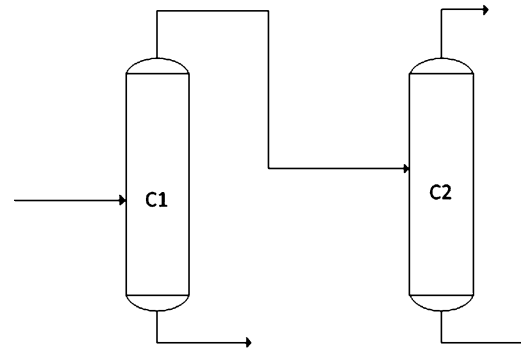

Fig. 7.52 Distillation process with two columns in series

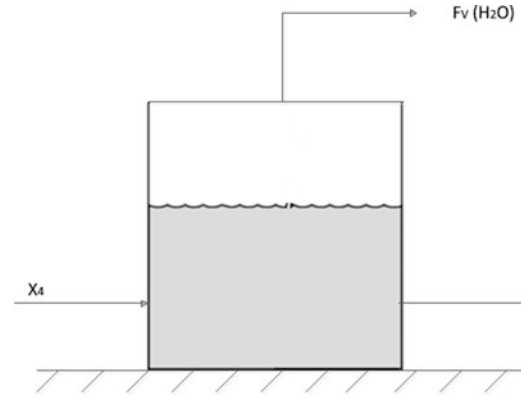

Fig. 7.53 Single-effect evaporator at laboratory scale

A: (a) 16 % w/w alcohol and 84 % w/w H_2O. (b) 72 % w/w alcohol and 28 % w/w H_2O. (c) 15 kg/h

28. **Multieffect evaporator [5]**. In order to design an industrial process to evaporate much of the water present in the product $\boldsymbol{X_4}$, an experiment was conducted in a laboratory-scale evaporator (simple) as shown in Fig. 7.53:
It was experimentally shown that this evaporator is capable of evaporating 80 % of the water initially contained in the product. The $\boldsymbol{X_4}$ product contains 80 % w/w water, and it is expected that the final product will not be more than 10 % w/w water and no less than 6 % w/w. As the water content of the product obtained in the laboratory equipment is greater than 10 %, in practice, it will be necessary to build an evaporator as depicted in Fig. 7.54:
From experience, we know that each evaporator added to the process is less efficient than the preceding one. The percentage of water that is removed at each stage is determined by the formula $X = 90 - 10i$, where X is the percentage of H_2O removed in stage i and i is the stage (1,2,3,. . ., N). (a) How many evaporators (stages) do there have to be so that the product meets the requirements established for the $\boldsymbol{X_4}$ end product? (b) What is the final concentration of H_2O in the end product?
A: (a) 3 (b) 8.76 % w/w. If you add a fourth evaporator, the product $\boldsymbol{X_4}$ will not meet the requirements. Verify this!
29. **Dryer [9]**. 10,000 lb/h of a solid material having a moisture content of 15 % w/w is dried to reach a moisture content of 2 % w/w (Fig. 7.55). The fresh air (Z) contains 0.012 lb of H_2O/lb dry air at a temperature of 70 °F. The air leaving the dryer is 100 °F with a humidity of 3 % w/w. Part of this

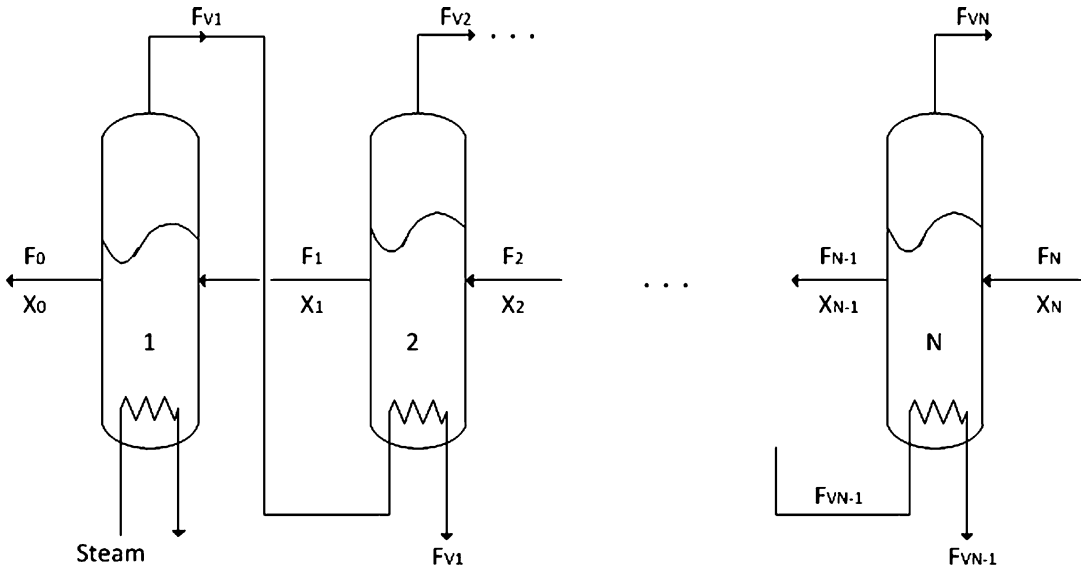

Fig. 7.54 Multi-effect evaporator

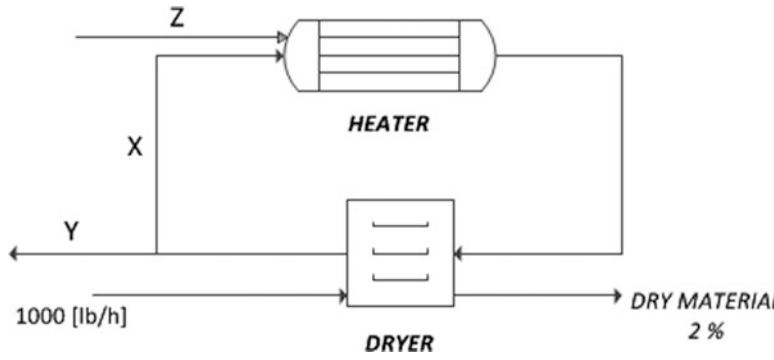

Fig. 7.55 Dryer including recycle

air is recycled (X) and mixed with fresh air (Z), then the mixture passes through a heater and leaves it at 200 °F. Then the mixed stream leaving the heater is fed to the dryer. The ratio of the recycle stream (X) to the purged stream (Y) is 2:3. The process operates at 1 atm. (a) Calculate the mass flow rate of fresh air (Z). (b) Calculate the air humidity at the dryer inlet. (c) Calculate the volume flow rate of fresh air (Z) in ft^3/h.

A: (a) ~70,925 lb/h. (b) 1.92 %. (c) 967,514.1 ft^3/h (2.74×10^7 L/h)

30. **Purification [4]**. We have designed a process for drying and purifying a particular commodity. The raw material contains A, B, and water. The final product must contain at least 98 % w/w of component A. The process consists of two dryers in series and at the last stage is a separator. The industrial process requires a production of 1,000 kg/h of component A. If the raw material contains 50 % w/w of component A and each dryer removes 70 % of the water entering the dryer, and at the last stage the separator removes all of B, then: (a) What is the composition w/w of the raw material? (b) How many kilograms of water are removed in each dryer?

A: (a)

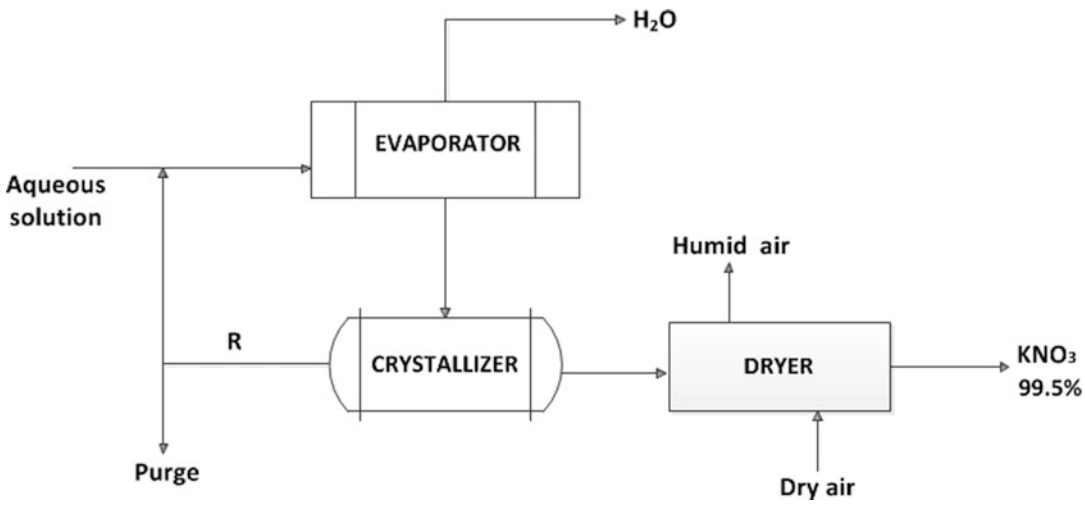

Fig. 7.56 KNO_3 purification in a system including evaporation, crystallization and dehydration

	Composition % w/w
A	50.00
B	38.66
H_2O	11.34

(b) First dryer: 158.7 kg/h, second dryer: 47.62 kg/h

31. **KNO_3 purification [9]**. KNO_3 is an odorless, white, and hygroscopic crystalline powder. It can be obtained from nature or from the neutralization of K_2CO_3 (or KOH) with aqueous HNO_3. Its most common uses are in fireworks, explosives, glass, fertilizers, food preservation, toothpaste, gunpowder, and as a diuretic. In a similar process to the glutamic acid purification (problem 15, Sect. 7.9), it is desirable to obtain KNO_3 with 99.5 % purity from 1,000 kg/h of an aqueous solution containing 15 % KNO_3 and 1.5 % insoluble impurities. The proposed process is depicted in Fig. 7.56. The fresh feed and the recycle are mixed and sent to an evaporator in which KNO_3 is concentrated to 50 % (~150 °C). This solution is cooled to 38 °C in a crystallizer yielding two separate streams: one consisting of pure crystals of KNO_3 and carrying a certain amount of liquor, the other composed of only liquor. This latter stream is in part recycled and the rest is purged to prevent the accumulation of impurities. This stream contains 0.6 kg KNO_3/kg H_2O and 12 % impurities. The wet crystals are completely dehydrated in a dryer. (a) How many degrees of freedom does this process have? (b) What is the product mass flow rate (kg/h)? (c) What is the mass flow rate of the purge? (d) What is the efficiency of the process (kg KNO_3 in the product/kg KNO_3 fed)?
A: (a) DF = 0. (b) 110.8 kg/h. (c) 120.4 kg/h. (d) 0.735 (73.5 %)
32. **Crystallization and interpolation [9]**. According to the dictionary, to interpolate is to introduce (something additional or extraneous) between other things or parts; interject; interpose; intercalate. In mathematics, of course, there are several ways or methods to interpolate. The simplest one, and in certain specific cases a good method, is linear interpolation, where you trace a straight line between two data points to estimate an unknown datum (see example below).
1,000 lb/h of a saturated $NaHCO_3$ solution at 50 °C are fed to a crystallizer. The target of the operation is to crystallize 50 lb/h of $NaHCO_3$. Determine the temperature at which the

Table 7.1 Solubility of $NaHCO_3$ in water as a function of temperature

Temperature (°C)	Solubility (g $NaHCO_3$/100 g H_2O)
60.0	16.40
50.0	14.45
40.0	12.70
30.0	11.10
20.0	9.60
10.0	8.15

crystallizer should be operated. From a manual of properties, we have the following data for the solubility of $NaHCO_3$.

A: 14 °C (obtained through a linear interpolation). (Note *linear interpolation* refers to the estimation of a value between two discrete data points. For example, if you want to estimate the solubility of $NaHCO_3$ at 14 °C, then using linear interpolation you should proceed as follows using the data reported in Table 7.1:

Temperature (°C)	Solubility (g $NaHCO_3$/100 g H_2O)
20	9.6
14	**S**
10	8.15

Graphically:

$$\frac{9.6 - S}{20 - 14} = \frac{9.6 - 8.15}{20 - 10},$$

$\boldsymbol{S} = 8.73$ [g $NaHCO_3$/100 g of H_2O].

33. **Waste treatment [7]**. A waste conditioning plant for post-coprocessing features three stages: two streams are fed to the process. The first stream (F_1) contains 80 % sawdust, 5 % heavy metals, and 15 % H_2O. The second stream (F_2) contains 72 % plastic, 8 % heavy metals, and 20 % water. Stream F_2 is three times larger than stream F_1. These two streams (which correspond to the feed) are joined by a third stream (a recirculation stream), and then the output stream is the input of a crusher. The output stream from the crusher is fed to an electromagnetic filter, which removes part of the heavy metals. The stream that leaves the electromagnetic filter is composed of 21 % water, and thus is the input of a trommel that has three output streams. The first output stream is a fine solid substitution fuel (FSSF), which is 40 (ton/net shift), takes 66.04 % of the total sawdust from the feed stream and 7 % of the total plastic of the feed stream and has a heavy metal content of 0.05 %. The second output stream is a thick solid substitution fuel (TSSF) of 140 (ton/net shift), and its content of heavy metals is 0.07 %. The third is the recycle stream, 7 (ton/h), with 0.8 % sawdust and 0.09 % heavy metals. Of the 8 h that correspond to a working day, the plant works just 5 h (the net shift is therefore 5 h) and there are three shifts per day. Answer the following questions: (a) What is the mass flow rate (ton/h) of the feed streams? (b) What is the composition of the FSSF and TSSF streams? (c) What is the amount of effective working hours per day? (d) What is the mass flow rate ton/year of the heavy metal recovery? (e) What is the percentage of heavy metals removed at the electromagnet filter? (f) If the calorific power of the TSSF is 4,100 kcal/kg, determine how much energy in kilojoules per month can be obtained. (g) If the calorific power of the FSSF is 3,800 kcal/kg, express this stream in kilowatts (kW).

A: (a) $F_1 = 9.7$ ton/h; $F_2 = 29.09$ ton/h (b)
$$F_{6,\text{FSSF}} = 8 \text{ ton/h} \quad F_{7,\text{TSSF}} = 28 \text{ ton/h}$$
$$x_{6,\text{a}} = 64.04\ \% \quad x_{7,\text{a}} = 9.41\ \%$$
$$x_{6,\text{HM}} = 0.05\ \% \quad x_{7,\text{HM}} = 0.07\ \%$$
$$x_{6,\text{p}} = 18.33\ \% \quad x_{7,\text{p}} = 69.57\ \%$$
$$x_{6,\text{w}} = 17.59\ \% \quad x_{7,\text{w}} = 20.95\ \%$$

(c) $5\left[\frac{\text{h}}{\text{shift}}\right]3\left[\frac{\text{shift}}{\text{day}}\right] = 15\left[\frac{\text{h}}{\text{day}}\right]$

(d) $F_4 = 2.79\left[\frac{\text{ton}}{\text{h}}\right]\left(\frac{15}{1}\left[\frac{\text{h}}{\text{day}}\right]\right)\left(\frac{365}{1}\left[\frac{\text{day}}{\text{year}}\right]\right) = 1.53 \times 10^4\left[\frac{\text{ton}}{\text{year}}\right]$

(e) $\eta = \frac{F_4}{F_3} \times 100 = \frac{2.79[\text{ton/h}]}{2.82[\text{ton/h}]} \times 100 = 98.9\ \%$

(f) $28\left[\frac{\text{ton}}{\text{h}}\right]\left(\frac{1{,}000}{1}\left[\frac{\text{kg}}{\text{ton}}\right]\right)\frac{4{,}100}{1}\left[\frac{\text{kcal}}{\text{kg}}\right]\left(\frac{4.184}{1}\left[\frac{\text{kJ}}{\text{kcal}}\right]\right)\left(\frac{15}{1}\left[\frac{\text{h}}{\text{day}}\right]\right)\left(\frac{30}{1}\left[\frac{\text{day}}{\text{month}}\right]\right) = 2.2 \times 10^{11}\left[\frac{\text{kJ}}{\text{month}}\right]$

(g) $8\left[\frac{\text{ton}}{\text{h}}\right]\left(\frac{1{,}000}{1}\left[\frac{\text{kg}}{\text{ton}}\right]\right)\frac{3{,}800}{1}\left[\frac{\text{kcal}}{\text{kg}}\right]\left(\frac{4.184}{1}\left[\frac{\text{kJ}}{\text{kcal}}\right]\right)\left(\frac{1}{3{,}600}\left[\frac{\text{h}}{\text{s}}\right]\right) = 3.5 \times 10^4[\text{kW}]$

34. **Banana milk []**. To prepare a delicious banana milk on an industrial scale, there is an experimental process that consists of five unit operations. A stream of banana (whose components are fruit, peel, and moisture) is fed to a peeler, where 99.9 % of the peel is removed. This stream should become pulp, so it is sent to an extruder press that mills the banana. Meanwhile, a stream of the pure additive R is added, which serves to avoid pulp oxidation. The resulting flow of 30 kg/h has a composition of 2 % w/w of additive R and is then fed to an agitated mixer. To the same mixer is fed a dairy stream of 200 kg/h that contains 70 % milk (the rest is water) and a sweet stream that contains 30 % w/w sugar (the rest is water). For each 10 kg/h of dairy flow, 1 kg/h of the sweet flow is fed. The stream that leaves the agitated mixer has 6 % fruit and 0.005 % peel and enters a thermal process in which 5 % of the water and 3 % of the milk evaporate. The resulting flow of the heat treatment (hot banana milk) finally joins the mixer, where the flavoring S is added. The final banana milk contains 30 % water. Calculate: (a) the composition and flow of the initial stream of bananas, (b) the annual flow of peels that are separated in the process (ton/year), (c) the mass lost during the thermal process (kg/h), (d) the composition of the final product stream and its flow (kg/h), and, (e) in the case where the density of the final product is 1.3 (g/mL), calculate how many 0.2 L packages of banana milk are produced per month.

(a)
$$F_2 = 41.886 \text{ kg/h}$$
$$x_{2,\text{f}} = 35.81\ \%$$
$$x_{2,\text{c}} = 29.84\ \%$$
$$x_{2,\text{w}} = 34.35\ \%$$

(b) $F_3 = 12.4875\left[\frac{\text{kg}}{\text{h}}\right]\left(\frac{24}{1}\left[\frac{\text{h}}{\text{day}}\right]\right)\left(\frac{365}{1}\left[\frac{\text{day}}{\text{year}}\right]\right)\left(\frac{1}{1{,}000}\left[\frac{\text{ton}}{\text{kg}}\right]\right) = 109.39\left[\frac{\text{ton}}{\text{year}}\right]$

(c) $F_9 = 8.61$kg/h (d)
$$F_{12} = 279.9\text{kg/h}$$
$$x_{12,\text{f}} = 5.36\ \%$$
$$x_{12,\text{c}} = 0.006\ \%$$
$$x_{12,\text{w}} = 30\ \%$$
$$x_{12,\text{R}} = 0.214\ \%$$
$$x_{12,\text{a}} = 2.14\ \%$$
$$x_{12,\text{l}} = 48.52\ \%$$
$$x_{12,\text{S}} = 13.76\ \%$$

(e) $279.9\left[\frac{\text{kg}}{\text{h}}\right]\frac{1}{1.3}\left[\frac{\text{L}}{\text{kg}}\right]\left(\frac{24}{1}\left[\frac{\text{h}}{\text{day}}\right]\right)\left(\frac{30}{1}\left[\frac{\text{day}}{\text{month}}\right]\right)\frac{1}{200}\left[\frac{\text{box}}{\text{mL}}\right]\left(\frac{1{,}000}{1}\left[\frac{\text{mL}}{\text{L}}\right]\right) = 775{,}091\left[\frac{\text{box}}{\text{month}}\right]$

35. **Liquid industrial waste [10+]**. One of the three streams that enter a mixer has a composition of 5 % w/w solids, 10 % w/w bacteria, and 7 % w/w heavy metals. The resulting stream from the mixer has a mass flow rate of 1,000 kg/h and a composition of 9.6 % w/w bacteria. Then this stream is fed to a settler, whose dense stream is 20 % of the input flow. The clarified stream is fed to a sorter, which generates a recycle of 10 %, which is the second stream that enters the mixer. The flow of the clarified stream that leaves the plant (leaves the sorter) has a composition of 1 % w/w solids. The dense flow has a composition of 20 % w/w solids and is then fed to a centrifuge that removes 30 % w/w moisture at a mass flow rate of 50 kg/h, which is the third stream that enters the mixer. The output stream of the centrifuge has 7 % of bacteria and is fed to an evaporator, which eliminates 60 % w/w humidity, generating a final stream with 40 % w/w solids. Calculate: (a) the composition and mass flow rate of the clarified stream that leaves the plant (leaves the sorter), (b) the composition and mass flow rate of the stream that leaves the evaporator, (c) the composition and mass flow rate of the input and output streams of the settler, and (d) the Composition of the third stream that enters the mixer.

(a)
$$\begin{aligned} F_6 &= 720\frac{\text{kg}}{\text{h}} \\ x_{6,\text{s}} &= 1\ \% \\ x_{6,\text{b}} &= 10.63\ \% \\ x_{6,\text{HM}} &= 7.8\ \% \\ x_{6,\text{w}} &= 80.57\ \% \end{aligned}$$

(b)
$$\begin{aligned} F_9 &= 90.75\frac{\text{kg}}{\text{h}} \\ x_{9,\text{s}} &= 40\ \% \\ x_{9,\text{b}} &= 11.57\ \% \\ x_{9,\text{HM}} &= 4.96\ \% \\ x_{9,\text{w}} &= 43.47\ \% \end{aligned}$$

(c)
$$\begin{aligned} F_2 &= 1{,}000\frac{\text{kg}}{\text{h}} \quad F_3 = 800\frac{\text{kg}}{\text{h}} \quad F_4 = 200\frac{\text{kg}}{\text{h}} \\ x_{2,\text{s}} &= 4.8\ \% \quad x_{3,\text{s}} = 1\ \% \quad x_{4,\text{s}} = 20\ \% \\ x_{2,\text{b}} &= 9.6\ \% \quad x_{3,\text{b}} = 10.63\ \% \quad x_{4,\text{b}} = 5.48\ \% \\ x_{2,\text{HM}} &= 7.04\ \% \quad x_{3,\text{HM}} = 7.8\ \% \quad x_{4,\text{HM}} = 4.02\ \% \\ x_{2,\text{w}} &= 78.56\ \% \quad x_{3,\text{w}} = 80.57\ \% \quad x_{4,\text{w}} = 70.5\ \% \end{aligned}$$

(d)
$$\begin{aligned} F_5 &= 50\frac{\text{kg}}{\text{h}} \\ x_{5,\text{s}} &= 7.4\ \% \\ x_{5,\text{b}} &= 1.0\ \% \\ x_{5,\text{HM}} &= 7.0\ \% \\ x_{5,\text{w}} &= 84.6\ \% \end{aligned}$$

36. **Chemical process [].** You are a process engineer at a factory that produces chlorine soda. On the process line, a mixer is fed with 450 kg/h of a stream that contains a solution of 5 % w/w NaOH and 10 % w/w NaCl, and another stream of 550 kg/h that contains a solution of 7 % w/w NaOH and 8 % w/w NaCl. The resulting stream is fed to an evaporator in which the output stream is 20 % w/w NaOH. This stream is fed to a crystallizer/separator that produces two streams to eliminate the excess of NaCl solution. The scrap stream removes 70 % of the NaCl and contains no NaOH. The stream of product leaving the crystallizer/separator is composed of 30 % NaOH. Calculate: (a) What is the performance of the process, expressed in kilograms of solution of product/kilograms of solution fed]? (b) What is the composition and mass flow rate of the product stream? (c) How much water evaporated in the process? (d) What is the waste stream composition?

(a) $\eta = \frac{F_7}{F_3} = \frac{203.3\frac{\text{kg}_{\text{PRODUCT}}}{\text{h}}}{1{,}000\frac{\text{kg}_{\text{FED}}}{\text{h}}} = 0.2033 \rightarrow 20.33\ \%$

(b)
$$\begin{aligned} F_7 &= 203.3\frac{\text{kg}}{\text{h}} \\ x_{7,\text{NaOH}} &= 30\ \% \\ x_{7,\text{NaCl}} &= 13\ \% \\ x_{7,\text{w}} &= 57\ \% \end{aligned}$$

(c) $F_5 = 695\frac{\text{kg}}{\text{h}}$

(d)
$$\begin{aligned} F_6 &= 101.7\frac{\text{kg}}{\text{h}} \\ x_{7,\text{NaCl}} &= 61.3\ \% \\ x_{7,\text{w}} &= 38.7\ \% \end{aligned}$$

37. **Drying apples [4].** A dryer is fed with 45 kg/h of apples with a moisture content of 85 % w/w; at the output the moisture content of the apples is 17.65 % w/w on a dry basis. The fresh air at the input has a moisture content of 0.03 kg H_2O/kg dry air, and at the outlet it has moisture of 5.21 % w/w on a wet basis. Determine the input mass flow of air (wet) to be used for this operation.
A: 1,529.1 kg humid air/h (1,484.56 kg dry air/h)
38. **Inoculum [5]**. A batch fermentation uses as inoculum a portion of the previous batch, equivalent to one-tenth the total volume. The process proceeds as follows: once a batch is done, a valve at the bottom of the fermenter is opened until the inoculum needed for the next batch is left. At that time, the valve is closed, and then the other valve is opened to feed fresh substrate at a concentration of 40 g/L to complete the volume of the batch. Fermentation stops when reaching a biomass and substrate concentration of 10 g/L. (a) Calculate how many grams of substrate are consumed per gram of biomass generated. (b) How many grams of substrate are consumed per gram of biomass generated if at the end of the fermentation the substrate concentration is 0?
A: (a) 3 g substrate/g biomass. (b) 4 g substrate/g biomass
39. **New juice [6]**. A small jam factory is developing a new product, a fruit juice. The product will be prepared with a mixture of fruit pulp to make the juice. Pulps of pineapple and strawberry will be mixed, five parts of strawberry for three parts of pineapple. To the mixture will be added sugar that make up 10 % of the total weight of the pulp (without sugar). For the opening of a restaurant, the jam factory will sell 150 jars of juice (1 L each). To produce 1 L of juice, 200 g pulp mix (with sugar) are needed. (a) How many kilograms of raw materials should the factory buy? (b) If the factory wants to prepare a second batch and the stock of raw materials is 32 kg strawberry pulp, 26 kg pineapple pulp, and 9 kg sugar, what is the maximum amount (liters) of juice the factory can produce?
A: (a) Sugar 2.72 kg, strawberry 17.05 kg, and pineapple 10.23 kg. (b) 281.6 L
40. **Filter aid [9]**. To separate cells from a culture, a stream containing a concentration of 25 g cells/L is passed through a filter. To facilitate the operation, a filter aid is used. The filter aid is fed in a proportion of 0.08 kg filter aid/kg cell. It is known that the solid cake has 40 % moisture and the entire filter aid remains in the solid cake. Furthermore, the filter aid corresponds to a 15th part by weight of the solid cake. Determine the concentration of cells in the permeate stream. Assume that the density of the culture, solid cake, and permeate is equal to that of water.
A: 0.0092 g/L
41. **Dairy company [8]**. A dairy company receives 85,000 L of milk per day, which is divided into three streams to produce whole, low-fat, and skim milk. The low-fat milk and skim milk are obtained by removing part of the fat. The removed fat will be used in another process. The fat stream is equivalent to one-tenth the weight of the milk was used to produce the skim milk. It is further known that the ratio of fed milk to total milk obtained is 1.02 w/w. (a) How many kilograms of milk goes to each line? (b) How much fat is available per day for further processing? Assume that the milk fed to the process has a density of 1.034 g/cm^3 and that its fat content is 4 % w/w. In addition, the low-fat milk has a 2 % w/w fat and skim milk contains no fat.
A: (a) 20,680 kg/day (22.7 %) of whole milk, 48,253 kg/day (57.6 %) low fat milk, and 17,233 kg/day (19.7 %) skim milk. (b) 1,724 kg/day
42. **Bioethanol production [9]**. In the first phase of a production process of bioethanol (second generation) (Wiche, 2010) 31,800 kg/h of wood (50 % moisture) are fed to a mill. In the mill, wood chips are obtained that are then fed to a tank to be impregnated with sulfuric acid. In this tank, chips are mixed with sulfuric acid solution (0.25 % w/w) at a ratio of one part chips per two parts acid solution. It can be assumed that during the process, the moisture of the chips does not change. Then the mixture undergoes a separation process in which two streams are

obtained, one of impregnated wood 0.87 H_2SO_4 mg/g of moisture wood and the other a solution of H_2SO_4. Seventy-five percent of the acid solution is recycled and the rest is purged. The recycle stream is mixed with H_2SO_4 at 95 % w/w and fresh water. This latter stream is fed to a pond, where it is mixed with the chips for impregnation. Moreover, the impregnated stream from the separator passes through a steam explosion process, which is fed in a ratio of 0.7 kg steam/kg dry wood. Then, in a cyclone, the exploded wood is separated from all the steam that was fed. The steam carries 2.7 % of the dry mass of wood that was fed to the system. Determine (a) the flow of acid and fresh water required per hour, (b) the flow of steam for the explosion process, and (c) the flow of wood and its moisture after the explosion process.

A: (a) 52.7 kg H_2SO_4/h and 15,857.6 kg H_2O/h. (b) 11,130 kg steam/h. (c) 31,370.7 kg wood/h, 50.68 % moisture

43. **Biogas [5]**. To use a biogas stream (68 % mol methane and 32 % mol carbon dioxide), the stream is passed through a separation system. First, the biogas begins to accumulate in a gas cylinder (pressure of 1.02 atm) until it reaches a height of 5 m. Then a valve is opened, allowing the passage of biogas into a system where it is separated into two streams, one rich in methane (96 % molar) and stored in a rigid tank 8 m^3 at 20 °C, and another that contains only carbon dioxide and is fed to a rigid tank 240 m^3 at 30 °C, where it is mixed with air to a pressure of 1.5 atm. This mixture of air and carbon dioxide is then used to feed a greenhouse. Determine the molar flow and composition of each stream. Assume that the gas cylinder has a radius of 4.5 m and the gas temperature is 20 °C.

    A: From the gas cylinder: 13,504 mol/batch (68 % CH_4 and 32 % CO_2); methane-rich stream: 9,565.35 mol/batch (96 % CH_4 and 4 % CO_2); from separator to mixer with air: 3,938.69 mol/bath (100 % CO_2); stream to greenhouse: 14,489.25 mol/batch (27 % CO_2, 57 % N_2, and 16 % O_2).

44. **Lignin recovery [8]**. To take advantage of some of the components obtained in the production of bioethanol, Wiche and collaborators proposed the recovery of lignin, its subsequent use as fuel, and furthermore allowing the other stream to be fermented. For this, two washing steps, in series, are carried out with alkali. In each washing process, two streams are obtained, one rich in lignin and another that is continuous in the main processing. The amount of NaOH fed for the first wash is equal to 25 % of the weight of lignin fed to the process. In addition, water is added until the concentration of solids is 5 % w/v. The amount of NaOH fed in the second wash is equivalent to 42 % by weight of the lignin remaining in the process stream. In both washing steps, the amount of lignin removed is the same, in the first washing the amount of lignin is 6.4 % of the weight of the stream, and in the second washing the amount of lignin is 1.2 % of the weight of the stream. Also, lignin liquor contains 95 % of NaOH fed at each washing step, and other components in each stream are distributed as fractions of the streams. All the cellulose remains in the main processing line. Both washing steps remove a total of 90 % of the lignin fed to the process.

    ADDITIONAL DATA

Fed stream	
Flow rate	33,300 kg/h
Density	996.4 kg/m^3
Moisture	66.6 %
Lignin	10.2 %
Cellulose	20.0 %
Other components	3.2 %

(a) Determine the amount of NaOH and water to be fed for the first and second washings. (b) Characterize (the flow and composition of) both (together) lignin liquor streams. (c) If the second washing is also carried out with 5 % solids w/v, will it be necessary to add or remove water? If so, how much?

A: (a) 849.15 kg/h for the first and 784.6 kg/h for the second. (b) 151,254.8 kg/h (2.02 % lignin, 1.05 % NaOH, 0.44 % other components, and 96.48 % H_2O). (c) 1,200 kg/h of H_2O should be retired before the second washing.

45. **Double-drum dryer and spray dryer [5]**. A factory that produces dried tomatoes receives fresh tomatoes as raw material. Fresh tomatoes have about 1 % proteins and 6 % carbohydrates (the rest is H_2O). 3,000 kg/h of tomatoes are fed into a press, generating two streams, one of pulp and the other of juice. The juice stream is 40 % in weight of feed stream. The juice has 0.3 % proteins and 0.7 % carbohydrates. Then the juice is processed in a spray dryer that removes 99 % of the water, transforming it into powder. Furthermore, the pulp obtained from the press is fed to a double-drum dryer, which delivers an output stream with 7 % moisture. Then the dried product from both dryers is fed to a mixer. Determine (a) the process yield, expressed as kilograms of dry product/kilogram of tomatoes fed; (b) the composition and flow rate of the output current; and (c) the amount of water that evaporated during the process in kilograms per hour.

A: (a) 7.9 %. (b) 76.02 % carbohydrates, 12.67 % proteins, and 11.31 % H_2O, flow rate = 236.78 kg/h. (c) 2,763.22 kg/h.

References

Wiche, P., 2010. Análisis de Ciclo de Vida de la Producción de Bioetanol de Segunda Generación. Memoria de Título para el grado de Ingeniero Civil Bioquímico. Pontificia Universidad Católica de Valparaíso, Chile.

Additional Web References

Mass Balance Tutorial https://www.youtube.com/watch?v=HOvOdAVIjW0
Open Systems - Mass and Energy Balance - Steady Nozzle https://www.youtube.com/watch?v=t6FFAC4DVA4
Energy Balance Examples for Open Systems http://www.et.byu.edu/~rowley/ChEn273/Topics/Energy_Balances/Energy_Balance_Open_Systems/Open_Sys_Examples.htm
Material Balance Problem Approach https://www.youtube.com/watch?v=BJ4Tzhi48h0
Introduction to Degrees of Freedom https://www.youtube.com/watch?v=Wr8HyPgmUF0
Degree of Freedom Analysis on a Single Unit https://www.youtube.com/watch?v=6Rx2ry1P6ME
Multiple Unit System: Degree of Freedom Analysis https://www.youtube.com/watch?v=VqvjssZku5Y
Elementary Mass Balances in Chemical Engineering https://www.youtube.com/watch?v=Wpj0XJzqPcQ

Fundamentals of Material Balance (Reactive Systems)

8[4]

"It was obvious—to me at any rate—that the answer was to why an enzyme is able to speed up a chemical reaction by as much as 10 million times. It had to do this by lowering the energy of activation—the energy of forming the activated complex. It could do this by forming strong bonds with the activated complex, but only weak bonds with the reactants or products."

Linus Pauling

"Every sentence I utter must be understood not as an affirmation but as a question."

Niels Bohr

Humanity stands . . . before a great problem of finding new raw materials and new sources of energy that shall never become exhausted. In the meantime we must not waste what we have, but must leave as much as possible for coming generations.

Svante Arrhenius

8.1 Chapter Purpose and Strategy

In both chemical and bioprocess engineering, the material balance of reactive systems is very important. According to the American Institute of Chemical Engineers (AIChE), two of the main achievements of chemical engineering are the synthesis of ammonia and the production of antibiotics on an industrial scale. As was shown and discussed in Chap. 4, in 1908, Professor Fritz Haber filed for a patent on the "synthesis of ammonia from its elements," and as we will see and discuss in Chap. 10, U.S. companies with strong governmental support began producing penicillin in 1943 and on a larger scale in 1945. These two great achievements involve reactive systems.

In our view, having completed Chap. 7, you have already learned the main concepts in material balance, and we purposely left reactive systems for this chapter. As you will see, using the same approach as that proposed, analyzed, and discussed in Chap. 7 (Fig. 7.15), you will be ready to take on and solve most, if not all, material balances for reactive systems. So why did we leave reactive systems for this chapter? As stated previously, our aim in writing this book is to teach you some concepts and various subjects step by step. We would like to reinforce the concept of teaching as discovery, i.e., furnishing you with the tools and then, through examples, facilitating your process of discovery. Perhaps you've already noticed that at the beginning of Chap. 7, we mentioned and explained the concepts of side stream and recycle, but we intentionally did not explain why they are used in processing. Then, for example, the recycle concept was presented in problem 10 (Sect. 7.9), and

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then a similar problem 11 (Sect. 7.9) without recycle was analyzed, and finally in its solution (step V) we presented a discussion on the advantages and disadvantages of recycling in a process stream. But at this point, independently of our discussion, maybe you have already "discovered" at least some of the advantages and disadvantages of a recycle stream. (Did you?)

But the main reason to leave reactive systems for this chapter is because it includes some additional characteristics. Everything that we have learned about approaching and solving material balance problems is useful and necessary. But for reactive systems we need to discuss the issue of material balance in a reactor. An important feature is the advantage of the use of mole instead of mass. Why? As you recall from chemistry, the relative quantities of reagents and products in a chemical reaction are given by the stoichiometry. For example:

$$CH_4 + 2O_2 \rightarrow CO_2 + 2\ H_2O$$

The first thing to notice is that this a balanced equation. In addition, the stoichiometry indicates that one molecule (or 1 [mol]) of CH_4 (methane) reacts with two molecules (or 2 [mol]) of O_2 to form one molecule (or 1 [mol]) of CO_2 and two molecules (or 2 [mol]) of H_2O.

Later, we will revisit the concept of a balanced equation and its implications.

8.2 Why Reactive Systems Are Relevant to Material Balance

8.2.1 Chemical Process

Chemical reactors are the heart of processing plants. Unlike all the other unit operations, it is in the reactor where the molecules that make up the raw materials undergo profound changes, transforming into valuable commercial products. Chemical changes occur only in the reactor. Other operations involve the preparation of raw materials to feed the reactor or to separate and purify the products.

But this process of chemical transformation is often very complex. In addition, it creates valuable commercial products and generates other products that may be of lesser value, undesirable, or even in some cases harmful to humans and the environment. In this situation, the engineer must be able to evaluate the performance of the reactor and answer questions such as how efficient the transformation of raw materials into valuable products is. Also, what type of products are being generated? What fraction of the raw materials degrades into noncommercial products? Which will be treated by expensive processes of separation? And to what extent are contaminants occurring that cannot be disposed of in the environment?

The most important available tool of an engineer to properly design and evaluate the operation of reactive processes is material balance. This is an accounting process; unlike nonreactive processes, it involves generating terms (positive or negative). Some molecules disappear and new molecules emerge from those consumed. This accounting process is often complicated because there are many possible reactions. For example, the combination of carbon monoxide (CO) with hydrogen (H_2) can yield methanol, methane, or higher alcohols, even hydrocarbons (Fischer–Tropsch reaction). It all depends on the type of catalyst used.

The global chemical industry generates approximately 50,000 different products, on every scale imaginable, ranging from highly specific products measurable in grams to commodities or basic products for heavy industry (petrochemical, mining), measured in thousands of tons annually. Whatever the size or complexity of the process, the material balance provides the framework to guide engineers in their work.

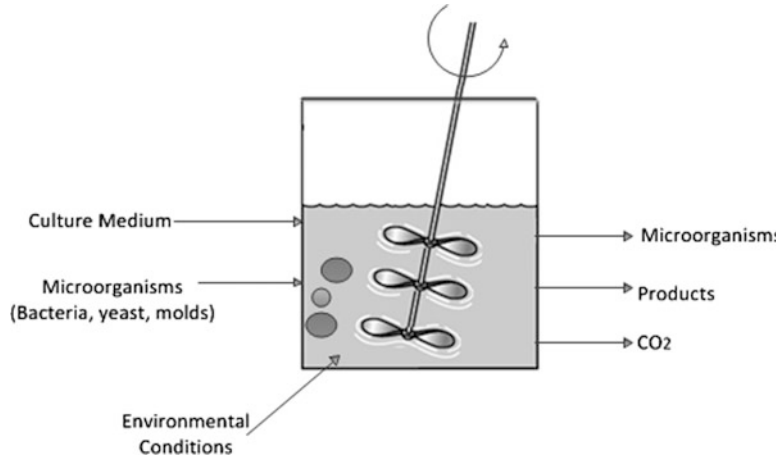


Fig. 8.1 Schematic representation of a fermentation process

8.2.2 Bioprocess

A bioprocess refers to any operation involving the transformation of a given substrate into products by the action of microorganisms or their derivatives, such as enzymes. Bioprocesses are generally conducted in reactors (e.g., bioreactors, fermenters) charged with a substrate or raw material and microorganisms to generate products. Figure 8.1 shows a schematic representation of a fermentation bioprocess.

For the bioprocess to be successful, microorganisms must be given optimal conditions for their development and growth. These conditions include the culture medium with the necessary nutrients and optimal environmental conditions (e.g., temperature, pH, agitation, oxygen). The objective of the bioprocess will depend on the desired product, which can be an organic compound or biomass (microorganisms). In most cases, the focus will be on the generation of a desired compound, e.g., ethanol, organic acids, and antibiotics. Some examples of industrial bioprocesses include the production of yogurt, beer, wine, antibiotics, bread, biological processes of wastewater treatment, and composting.

In this context, the operation of a bioprocess, like a chemical reaction, is related to heat transfer (heating/cooling), mass transfer (e.g., diffusion), kinetics, and energy balances. Considering the reactor (bioreactor) as the system under study it will be necessary to analyze changes in the different properties over time and space. It is indisputable that viewing these changes and expressing them in mathematical terms is vital for formulating material balance equations with respect to time for what enters (substrate material) and "reacts" inside the reactor (microorganisms/biomass), and what leaves the reactor. Therefore, a schematic representation is as follows (Fig. 8.2).

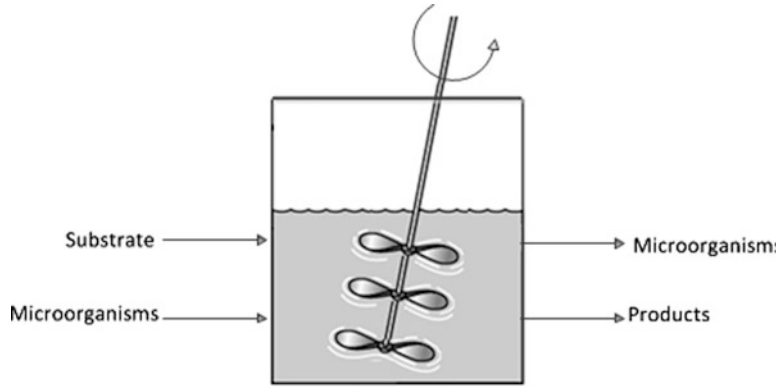
Translating the preceding scheme into a mass balance equation for microorganisms or a substrate, we obtain

$$\begin{aligned}&\text{Input} - \text{Output} \pm \text{Transformation}(\text{generation}/\text{consumption})\\ &\quad = \text{Accumulation}(\text{Deaccumulation})\end{aligned} \tag{8.1}$$

As was discussed in Chap. 7, we will face problems under steady-state conditions, but now we will be adding a term to take into account the transformation occurring inside the reactor or bioreactor:

$$\text{Input} - \text{Output} \pm \text{Transformation}(\text{generation}/\text{consumption}) = 0 \tag{8.2}$$

Fig. 8.2 Bioreactor



But do not get confused; the total mass balance remains the same as in (7.5):

$$\text{Total input mass} - \text{Total output mass} = 0. \tag{8.3}$$

8.3 Particularities of Material Balance on Reactive Systems

8.3.1 Material Balance Equations for Reactive Systems (Steady-State and Continuous Operation)

As mentioned previously, matter is neither created nor destroyed, but it is conserved. The law of conservation of matter postulates that the amount of material before and after a process is strictly the same. What the law of conservation of matter implies is that, beyond transformations, matter is always conserved. In other words, atoms react with each other as substances, but atoms are neither created nor destroyed.

Formulation of a general mass balance for a reactive and open system under steady-state conditions (process unit):

Total mass balance

Referring to Fig. 8.3, the total mass balance under steady state can be written as follows:

$$\text{Total input mass} - \text{Total output mass} = 0. \tag{8.4}$$

Thus,

$$\left(\sum_{i=1}^{i=n} \dot{m}_i\right)_{\text{in}} - \left(\sum_{j=1}^{j=k} \dot{m}_j\right)_{\text{out}} = 0, \tag{8.5}$$

where $\left(\dot{m}_i\right)_{\text{in}}$ is the mass flow rate of stream i entering the system (mass/time), and $\left(\dot{m}_j\right)_{\text{out}}$ is the mass flow rate of stream j leaving the system (mass/time).

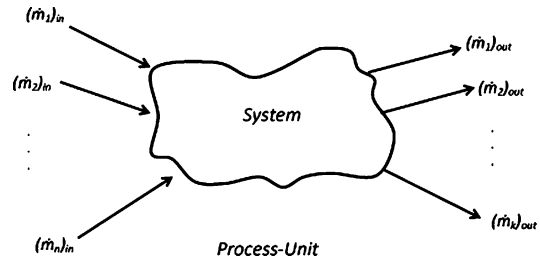


Fig. 8.3 General material balance for a reactive and open system under steady-state conditions

Mass balance for each component

It is important to note that in a reactive system, the mass for each component is not necessarily conserved (although the total mass is conserved!). As for nonreactive systems, we can write one mass balance for each component. If we have p components, then we can formulate p equations. Writing the mass balance for some specific component r, we obtain

$$\left(\sum_{i=1}^{n} x_{ri}\,\dot{m}_i\right)_{\text{in}} - \left(\sum_{j=1}^{m} x_{rj}\,\dot{m}_j\right)_{\text{out}} + (m_r)_{\text{reaction}} = 0 \tag{8.6}$$

where x_{ri} is the mass fraction of component r in stream i; x_{rj} is the mass fraction of component r in stream j; and $(m_r)_{\text{reaction}}$ is the mass of component r generated or consumed due to the reaction (this term could be positive, negative, or 0 depending on whether component r is a reagent, a product, or not part of the reaction).

The first term in (8.6) represents the addition of the masses of component r in all input streams. In the same way, the second term represents all the masses of component r leaving the system. Finally, the third term represents the amount of component r that was generated or consumed due to the reaction. If component r is not part of the reaction, then the third term in (8.6) is 0.

As with nonreactive systems, we can write one total mass balance and, in addition, one mass balance for each component (1, 2, …, p). Since we have p components, we are able to write $p + 1$ equations in total, but **ONLY *p* of them are independent!** Why? For example, if we sum up all the material balance equations formulated for each component, we will discover that the result is equal to the total mass balance.

Remember, the number of independent equations is equal to the number of components in the process unit (system).

As mentioned earlier, we have an extra term due to the reaction among compounds. Before addressing problems of material balance in reactive systems, it is necessary to briefly analyze chemical reactions and understand why the use of molar units is much more pertinent and convenient than the use of mass units for material balances in a reactor.

8.3.2 Stoichiometry

As mentioned earlier, in a balanced equation, the relative quantities of reagents and products are given in molecular or molar bases:

$$aA + bB \rightarrow cC. \tag{8.7}$$

As depicted in (8.7), a moles of compound A react with b moles of compound B to generate c moles of compound C. If we feed the reactor with a moles of A and b moles B, then we can state that the feed is a stoichiometric mixture. Remember that the equation should be balanced, but what is a balanced equation? For example, is the following equation for formaldehyde (CH_2O) production balanced?

$$CH_3OH + O_2 \rightarrow CH_2O + H_2O. \tag{8.8}$$

No, because the number of atoms per element must be the same on both sides of the equation. In (8.8), we have one carbon atom on each side and four atoms of hydrogen on each side but three atoms of atomic oxygen on the left-hand side and just two on the right-hand side. Thus, the equation is not balanced. Balancing (8.8), we obtain

$$CH_3OH + \frac{1}{2}O_2 \rightarrow CH_2O + H_2O.$$

Or, amplifying both sides of the equation, we get

$$2CH_3OH + O_2 \rightarrow 2CH_2O + 2H_2O. \tag{8.9}$$

Warm-Up Example 1. Acetylene is a gas that can be obtained by treating calcium carbide with H_2O according to the following reaction:

$$CaC_2 + H_2O \rightarrow C_2H_2 + Ca(OH)_2. \tag{8.10}$$

(a) Is the equation balanced? Why or why not? (b) How many moles of acetylene can be produced with 1 [kg] of calcium carbide?

First, to answer question (a), we need to verify whether the number atoms per element is the same on each side of (8.10).

Calcium atoms, one per side
Carbon atoms, two per side
Atoms of atomic hydrogen, two on the left and four on the right
Atoms of atomic oxygen, one on the left and two on the right

Thus the equation is not balanced.

So a balanced equation for the production of acetylene is

$$CaC_2 + 2H_2O \rightarrow C_2H_2 + Ca(OH)_2, \tag{8.11}$$

where there is one calcium atom on each side, two carbon atoms one each side, four atoms of atomic hydrogen on the left and four on the right, and two atoms of atomic oxygen on the left and two on the right.

Now, with (8.11) balanced, we can answer question (b). The balanced equation indicates that 1 [mol] of calcium carbide produces 1 [mol] of acetylene.

Moles of calcium carbide = mass of calcium carbide/molecular weight of calcium carbide = 1,000 [g]/64 [g/mol] = 15.625 [mol]. Given that, according to (8.11), to produce 1 [mol] acetylene

requires 1 [mol] calcium carbide; therefore, with 15.625 [mol] calcium carbide we can produce 15.625 [mol] acetylene.

8.3.3 Limiting and Excess Reagents and Reaction Conversion

According to the stoichiometry of a reaction, you can determine the exact number of moles of reagents A and B that are necessary to produce, say, product C. If the reaction stoichiometry is

$$5\mathrm{A} + 3\mathrm{B} \rightarrow 4\mathrm{C}, \tag{8.12}$$

then in (8.12) you need 5 [mol] of A and 3 [mol] of B to produce 4 [mol] of C, or, in general, the moles of A and the moles of B must be in a proportion of 5/3 to have a stoichiometric mixture (stoichiometry ratio, SR). If all the reagents are fed in as the stoichiometry indicates, no reagent will be limiting or in excess in the reaction.

Warm-Up Example 2. What happens if you are feeding, for example, 6 [mol] of A and 3 [mol] of B in reaction (8.12)? Then reagent A is in excess (more than the required amount) and, in addition, we can say that reagent B is limiting. As seen in this example, we can deduce

$$\text{If the molar ratio } (\mathrm{A}/\mathrm{B}) \begin{cases} > 5/3, \text{then compound A is in excess} \\ = 5/3 \text{ then compounds A and B are in a stoichiometric mixture} \\ < 5/3, \text{then compound B is in excess} \end{cases} \tag{8.13}$$

Excess
In warm-up example 2, we stated that compound A was in excess, but can we quantify its excess? Yes. How? By defining the fractional excess as

$$\textbf{Fractional excess} = (\text{Moles fed} - \text{Moles needed})/\text{Moles needed}. \tag{8.14}$$

Then, in warm-up example 2, we can calculate the excess of reagent A.
Fractional excess of reagent will be A $= (6 - 5)/5 = 0.2$ (20 %).

Conversion
If we have a stoichiometric mixture and all the reagents are consumed (disappear), then we have 100 % conversion. In this case, when we have a stoichiometric mixture, the conversion could be calculated with any of the reagents. On the other hand, if the mixture it is not stoichiometric, then the conversion must be calculated with the limiting reagent.

$$\textbf{Fractional conversion} = (\text{Moles of limiting compound reacted})/(\text{Moles of limiting compound fed}) \tag{8.15}$$

Warm-Up Example 3. 6 [mol] of A and 4 [mol] of B are fed to a reactor. The stoichiometry of the reaction is

$$2A + 3B \rightarrow 4C \tag{8.16}$$

If the conversion is 60 %, how many moles of C are produced?

First, we need to determine whether A or B is the limiting reagent. According to the stoichiometry, for each mole of A we need 1.5 [mol] of B, so for 6 [mol] of A, 9 [mol] of B are needed. Therefore, the limiting reagent is B. In addition, the conversion is 60 %. Therefore, from (8.15):

$$\text{Fractional conversion} = \text{Moles of B reacted}/\text{Moles of B fed} = 0.6 = \text{Moles of B reacted}/4.$$

Thus, 2.4 [mol] of B reacted. Given that for each mole of B that reacted we obtain 4/3 [mol] of C (8.16), then for 2.4 [mol] of B, we obtain 3.2 [mol] of C.

8.4 Material Balance in Reactors and Bioreactors

As was said and proven through examples, we will use moles as units for each stream and, in addition, we have designed a strategy specially to help you write down the equations when doing a material balance on a reactor. Warm-up examples 4 and 5 are typical chemical processes and are presented to show an easy, straightforward, and graphical way to manage material balances in a chemical reactor. Then, warm-up examples 6 and 7 are classical examples of bioprocess problems. In either case, the emphasis is not in the process itself, but in reinforcing your capabilities to manage material balance problems with reactions. As was stated previously, as you are being trained to take on and solve material balance problems on reactive systems you will be, somehow, unconsciously discovering the work of chemical and bioprocess engineers. In the next section (Sect. 8.5), we will revisit the strategy and procedure on approaching and solving material balance problems. For now, the focus is on the particularities of the material balance of chemical reactors and bioreactors (fermenters).

8.4.1 Typical Material Balance Problems in Chemical Processes

As has been stressed throughout the book, and particularly in the previous chapter, a correct procedure is, again, essential to solving material balance problems for reactive systems. Warm-up examples 4 and 5 focus on managing material balances for reactors.

Warm-Up Example 4. A reactor is fed with a mixture of oxygen (O_2) and propane (C_3H_8). Oxygen is in excess and the conversion of the reaction is 70 %. At the output of the reactor, the presence of 21 [mol] of CO_2 and 40 [mol] of O_2 has been analytically determined. (a) What is the fractional excess of oxygen? (b) How many moles of propane were fed to the reactor? (c) What is the molar composition of the output stream?

First, we need to write down the balanced equation for the reaction between oxygen and propane:

$$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O. \tag{8.17}$$

Second, we will develop a table (Table 8.1) that simulates what is happening in the reactor, i.e., a column listing all compounds (reagents and products) and then columns for input moles, reaction moles, and output moles, where x is the moles of the limiting reagent (in this case propane) and y is the excess moles of oxygen.

Table 8.1 Simulation of the reaction for the mixture of O_2 and C_3H_8 (propane) where O_2 is in excess and the fractional conversion is 70%

Compound	Input moles	Reaction moles	Output moles
O_2	$5x + y$	$0.7(5x) = 3.5x$	$(5x + y) - 3.5x = 1.5x + y$
C_3H_8	x	$0.7x$	$x - 0.7x = 0.3x$
CO_2	–	–	$0.7x \times 3 = 2.1x$
H_2O	–	–	$0.7x \times 4 = 2.8x$

Table 8.2 Completed data for the reaction of O_2 and C_3H_8 (propane)

Compound	Input moles	Reaction moles	Output moles	Molar composition (%)
O_2	75	35	40	43.47
C_3H_8	10	7	3	3.26
CO_2	–	–	21	22.83
H_2O	–	–	28	30.44
			Total = 92	100 %

Understanding the content of Table 8.1

Input moles. According to the stoichiometry, for each mole of C_3H_8 we need 5 [mol] oxygen. If we assigned x for the moles of propane, then for O_2 we need $5x$ plus the y moles in excess, thus $5x + y$.

Reaction moles. Given that O_2 is in excess, C_3H_8 is the limiting reagent, and given that the conversion is 70 % (fractional conversion = 0.7), according to (8.15),

Fractional conversion = *Moles of limiting compound reacted/Moles of limiting compound fed* = 0.7; therefore the moles of limiting compound reacted = $0.7x$, and by stoichiometry $5(0.7x) = 3.5x$ moles of O_2 will react.

Output moles. To get the output column for reagents, subtract the reaction column from the input column. To calculate the moles of products, it is necessary to see how much reagent reacts and follow the relationship given by the stoichiometry. For example, according to (8.18), for each mole of C_3H_8 we get 3 [mol] CO_2, so for $0.7x$ mol C_3H_8 that reacted, we get $2.1x$ mol CO_2. In the same way, we get $2.8x$ mol H_2O at the output.

According to the problem statement, at the output we get 21 [mol] CO_2 and 40 [mol] O_2. Thus,

$2.1x = 21$; $x = 10$ [mol] and

$1.5x + y = 40$; $y = 25$ [mol].

Completing the table, we can answer question (c) (Table 8.2).

(a) What is the fractional excess of oxygen? According to (8.14):

$$\text{Fractional excess of } O_2 = (75 - 50)/50 = 0.5(50\ \%).$$

(b) How many moles of propane were fed to the reactor? $x = 10$ [mol] propane.

Warm-up example 5. A reactor is fed with 100 [kg] of A and 150 [kg] of B, and the conversion of the reaction is 75 % (fractional conversion = 0.75). (a) What is the molecular weight of C? (b) How many moles of C are obtained at the output stream? (c) What is the molar composition at the output of the reactor?

ADDITIONAL DATA: $MW_A = 20$ [g/mol]; $MW_B = 30$ [g/mol].

$$3A + 2B \rightarrow 4C \tag{8.18}$$

Table 8.3 Simulation of the reaction of compounds A and B

Compound	Input moles	Reaction moles	Output moles
A (limiting reagent)	5	$5 \times 0.75 = 3.75$	$5 - 3.75 = 1.25$
B (in excess)	5	$(10/3) \times 0.75 = 2.5$	$5 - 2.5 = 2.5$
C (product)	–	–	$3.75 \times (4/3) = 5$

Table 8.4 Completed data for the reaction of compounds A and B in mass units

Compound	Input mass [g]	Reaction mass [g]	Output mass [g]
A (limiting reagent)	$5 \times 20 = 100$	$5 \times 0.75 \times 20 = 75$	$100 - 75 = 25$
B (in excess)	$5 \times 30 = 150$	$(10/3) \times 0.75 \times 30 = 75$	$150 - 75 = 75$
C (product)	–	–	$5 \times 30 = 150$
Total	250		250

Table 8.5 Molar composition at the out of the reactor

Compound	Input moles	Reaction moles	Output moles	Molar composition at output [%]
A (limiting reagent)	5	3.75	1.25	14.3
B (in excess)	5	2.5	2.5	28.6
C	–	–	5	57.1
Total			8.75	100

Solution

(a) According to the stoichiometry given in (8.18), 3 [mol] of A (mass of A = 60 [g]) and 2 [mol] of B (mass of B = 60 [g]) gives 4 [mol] of C. Given that mass is conserved, 120 [g] of reagent should produce 120 [g] of product (100 % conversion). Thus, the molecular weight of C should be

$$MW_C \times 4 = 120 \text{ [g]}, \text{ then } MW_C = 30 \text{ [g/mol]}.$$

(b) Here, we will focus on a strategy for analyzing the material balance for the reactor.

First, we will express the mass of A and B in moles (kg mol); thus,
Moles of A = 100 [kg]/20 = 5 [kg mol] and moles of B = 150 [kg]/30 = 5 [kg mol].

Second, we need to determine whether the fed mixture is stoichiometric. As depicted in (8.18), the stoichiometric ratio is 3/2 (moles of A/moles of B). As shown in (8.13), in this case, we can conclude that the feed ratio is not stoichiometric ($5/5 \neq 3/2$) and, in addition, because $5/5 < 3/2$, B is in excess and A is limiting the reaction. Therefore, if we are feeding 5 [kg mol] of A, according to the stoichiometry, we need $(2/3) \times 5$ [kg mol] of B, then 10/3 [kg mol] of B are needed to react with 5 [kg mol] of A. As mentioned earlier, the conversion of the reaction is referred to the limiting reagent (A in this case), then in the reactor:

Material balance for reactors

As shown in warm-up example 4, a practical way to deal with material balances for a reactor is to construct a table (Table 8.3). From now on, all the material balances for reactors will be done using this procedure.

5 [kg mol] of compound C are obtained at the output stream

Checking the mass balance. Because mass is conservative, all the mass entering the reactor should be equal to all the mass leaving it. Thus (Table 8.4),

To answer question (c) we will complete the table adding the molar composition at the output (Table 8.5):

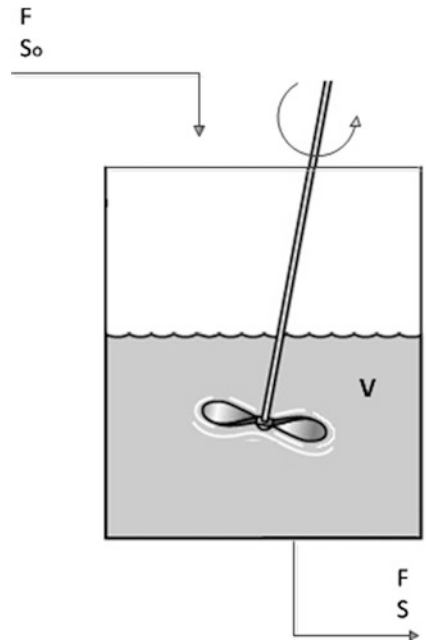


Fig. 8.4 Continuous stirred-tank reactor (CSTR)

8.4.2 Typical Material Balance Problems in Bioprocesses

Warm-up example 6. A continuous stirred-tank reactor (CSTR) of volume V is fed with a stream of F L/h and with a substrate concentration of S_0 [kg/L]. The transformation speed of the substrate within the reactor is directly proportional to its concentration ($-kS$), where k has units of 1/h. If we can assume that the concentration of the substrate at the output stream is equal to the concentration within the reactor, and the reactor is operated under steady state (Fig. 8.4), then express the substrate concentration at the output stream (S) as a function of known variables (F, V, k, and S_0).

Solution
Again, the focus is to analyze what is happening in the reactor, in this case a bioreactor. In addition, the data related to the reactions (substrate transformation) are given in mass units. Thus:

Mass balance for substrate

$$F[\mathrm{L/h}] \times S_0(\mathrm{kg/L}) - F[\mathrm{L/h}] \times S(\mathrm{kg/L}) - V(\mathrm{L}) \times k(1/\mathrm{h}) \times S(\mathrm{kg/L}) = 0. \tag{8.19}$$

Therefore, the substrate concentration at the output stream as a function of F, S_0, V, and k is

$$S = \frac{FS_0}{F + Vk}. \tag{8.20}$$

Warm-up example 7. A clever young chemical engineering student is proposing that you analyze the previous problem, but this time instead of using one reactor of volume V, he is suggesting using two reactors of volume $V/2$ each. He is clever, but not as well prepared as you, and so he asks you to investigate two alternatives, to maximize the disappearance of the substrate, as follows: (a) Use both reactors in parallel. (b) Use both reactors in series. (c) What would your recommendation be and

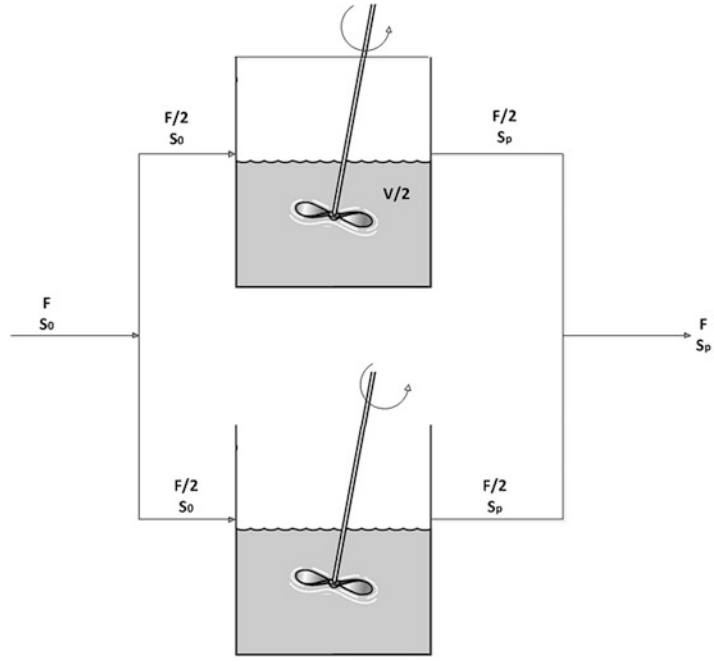


Fig. 8.5 Continuous stirred-tank reactor (CSTR)

why? (d) How would you compare the best arrangement between (a) and (b) with the previous problem (warm-up example 6)?

Solution

(a) First, we will create a schematic representation for two bioreactors ($V/2$ each) operating in parallel (Fig. 8.5):

Mass balance for substrate in each bioreactor

Bioreactor B_1

$$\frac{F}{2}S_0 - \frac{F}{2}S_P - \frac{V}{2}kS_P = 0 \tag{8.21}$$

Bioreactor B_2

$$\frac{F}{2}S_0 - \frac{F}{2}S_P - \frac{V}{2}kS_P = 0 \tag{8.22}$$

Then in both bioreactors we get the same output concentration as follows:
$S_P = \frac{FS_0}{F+Vk}$; the same answer as in the previous problem (one bioreactor of volume V).

(b) First, we will create a schematic representation for two bioreactors ($V/2$ each) operating in series (Fig. 8.6):

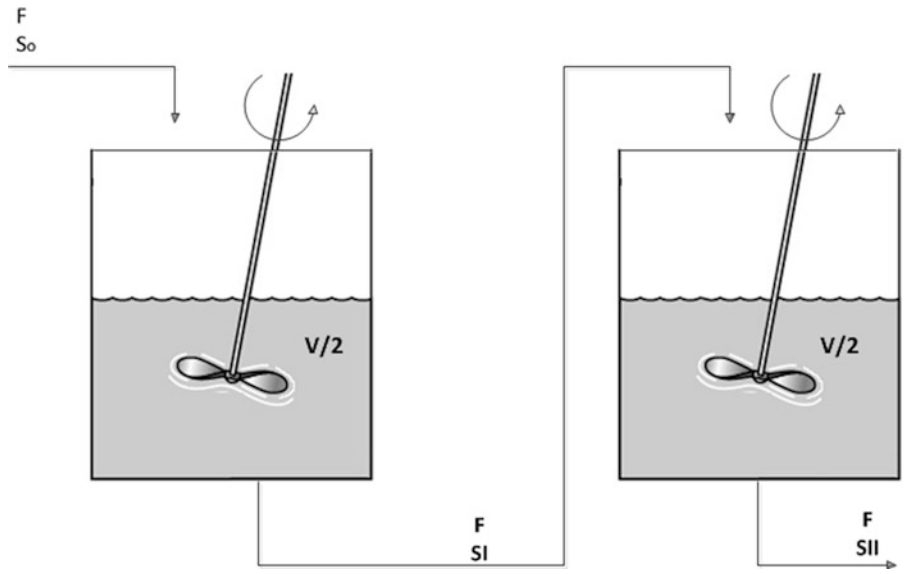


Fig. 8.6 Two continuous stirred-tank reactors operated in series

Bioreactor B_1

$$FS_0 - FS_\mathrm{I} - \frac{V}{2}kS_\mathrm{I} = 0 \tag{8.23}$$

Then

$$S_\mathrm{I} = \frac{FS_0}{F + \frac{V}{2}k}. \tag{8.24}$$

Bioreactor B_2

$$FS_I - FS_\mathrm{II} - \frac{V}{2}kS_\mathrm{II} = 0. \tag{8.25}$$

Then, substituting (8.24) in (8.25) and rearranging we get

$$S_\mathrm{II} = \frac{F^2S_0}{\left[F + \frac{V}{2}k\right]^2}. \tag{8.26}$$

(c) Given that the object is to maximize the disappearance of the substrate, we need to compare S_P with S_II and see which of these two concentrations is lower.

We rewrite S_P as follows:

$$S_\mathrm{P} = \frac{F^2S_0}{F[F + Vk]}. \tag{8.27}$$

Now, compare S_P with S_{II} to see which of the two has a larger denominator.

Denominator of S_P

$$F[F + Vk] = F^2 + FVk. \tag{8.28}$$

Denominator of S_{II}

$$\left[F + \frac{V}{2}k\right]^2 = F^2 + F\frac{V}{2}K + F\frac{V}{2}K + \left(\frac{V}{2}k\right)^2 = F^2 + FVk + \left(\frac{V}{2}k\right)^2. \tag{8.29}$$

Given that all parameters are greater than 0 and the denominator of S_{II} has an additional term, S_{II} is less than S_P.

The recommendation is to use two bioreactors in series.

(d) Two reactors of volume $V/2$ each in series is better than two reactors in parallel and better than one bioreactor of volume V.

Corollary *In warm-up examples 4 and 5 (chemical reactors), we had information about stoichiometry and conversion, and the proposed procedure was to construct a table to take into account the moles entering the reactor, moles reacting, and the moles leaving the reactors (reagents and products). This is a convenient procedure and facilitates the material balance in the reactor. On the other hand, in the bioreactor problems, we had information about the disappearance of the substrate (kinetics), and in that case it was easier just to formulate the mass balance like (8.6).*

To keep the subject as simple as possible, we have intentionally avoided kinetics. Our aim has been to teach you just a few concepts and keep challenging you; in addition, we have been using this simple, but hopefully effective, learning strategy to familiarize you with chemical and bioprocess engineering.

To further familiarize you with chemical reactors and bioreactors and to solve problems with the strategy proposed, analyzed, and revised in Chap. 7, in the next section (Sect. 8.5) we will solve three more problems of chemical processes and bioprocesses.

8.5 Formulating and Solving Material Balance Equations in Reactive Systems (Steady-State and Continuous Operation)

In Chap. 7, Sect. 7.8 ("Designing and structuring a general procedure to formulate and solve material balance problems"), we developed a detailed explanation of a general strategy for solving material balance problems for nonreactive systems. Here, we will use strictly the same strategy, although we need to take into account some particularities of the reactive systems. First, it is highly advisable to analyze the material balance with moles instead of mass. Second, we will have additional unknowns, for example, the reaction stoichiometry, reaction conversion, and molecular weights of the compounds. These new variables should be considered when doing an analysis of the degrees of freedom. Conceptually, the analysis of the degrees of freedom is the same, with the only consideration that it will be appropriate to take into account new types of variable (e.g., stoichiometry, reaction conversion).

Section 8.6 is dedicated to putting into practice the strategy developed in Chap. 7 (for details please return to Sect. 7.8.2.1 on the procedure description and analysis) but now applied to material balance for reactive systems.

8.6 Solved Problems

1. **Reactor with recycle [4]**. The system depicted in Fig. 8.7 shows a simplified scheme of a reactor-separator system that includes a recycle stream. Compounds A and B are fed to the reactor to produce compound C according to the following stoichiometric reaction:

$$A + B \rightarrow C,$$

where the molecular weights of compounds A and B are 30 and 20 [g/mol], respectively.
The system was designed to produce 1,000 [kg/h] of compound C. According to the experimental data, recorded at the laboratory, the reaction has a conversion of 60 % and the molar fraction of compound A in the feed stream of the reactor is 0.4. In addition, output stream 3 only contains compound C, and stream 4 (the recycle stream) only contains compounds A and B. (a) What is the molar composition of the fresh stream (1)? (b) What is the molar composition of the recycle stream?

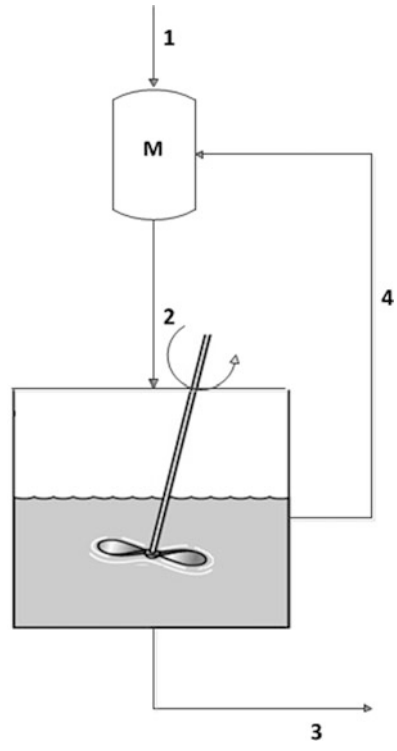


Fig. 8.7 Reactor with recycle

Solution

Step I

Reading and understanding

As was mentioned, we will follow the same strategy that was developed for material balance for nonreactive systems. Some slight differences are that here we will use moles instead of mass, and the material balance through the reactor will be done developing the table that was presented in warm-up example 4.

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

Figure 8.8 includes all the information given in the problem statement.

F_1: Molar flow rate of feed stream to system [kg mol/h]
X_{A1}: Molar fraction of compound A in feed stream
F_2: Molar flow rate of feed stream to reactor [kg mol/h]
X_{A2}: Molar fraction of compound A in feed stream to reactor
F_3: Molar flow rate of output stream of system [kg mol/h]
F_4: Molar flow rate of recycle stream [kg mol/h]
X_{A4}: Molar fraction of compound A in recycle stream
χ_R: Conversion of reaction

Thus, the total number of variables is 8 (NV = 8).

STEP III

Analysis of degrees of freedom in process

As depicted in Fig. 8.8, we can carry out a material balance for the mixture of the fresh stream and the recycle stream (two material balances) and three material balances on the reactor. Thus,

$$\text{NMB} = 5.$$

In addition, we have three specified variables, the output stream (1,000 [kg/h]), the conversion of the reaction (60 %), and the molar fraction of compound A in the feed stream to the reactor ($X_{A2} = 0.4$). Thus,

$$\text{NSV} = 3.$$

Therefore, given that NR = 0, the degrees of freedom of the system are

$$\text{DF} = \text{NV} - (\text{NMB} + \text{NSV} + \text{NR}) = 8 - (5 + 3 + 0). \text{ Thus, DF} = 0.$$

Now the problem can be quantitatively formulated and solved.

STEPS IV and V

Mathematical formulation, including all available data and solution, results, analysis, and discussion

First, to express F_3 as a molar rate, we need to calculate the molecular weight of compound C. According to the stoichiometry, 1 [mol] of A (30 [g]) plus 1 [mol] of B (20 [g]) generates 1 [mol] of C (which should be 50 [g] of C). Given that

$$\text{Moles} = \text{Mass/MW}, \ 1 \text{ [mol] of C} = 50 \text{ [g] of C/MW}_C; \ \text{MW}_C = 50 \text{ [g/mol]} = 50 \text{ [kg/kg mol]},$$
$$F_3 = 1{,}000 \text{ [kg/h]}/50 \text{ [kg/kg mol]} = 20 \text{ [kg mol/h]}.$$

Material balance for reactor (*R*)

Before creating a table, we need to determine which compound is limiting the reaction. Given that the stoichiometry establishes that for 1 [mol] of A, 1 [mol] of B is necessary, and considering that the molar fraction of A in the feeding stream of the reactor is 0.4, A is the limiting reagent.

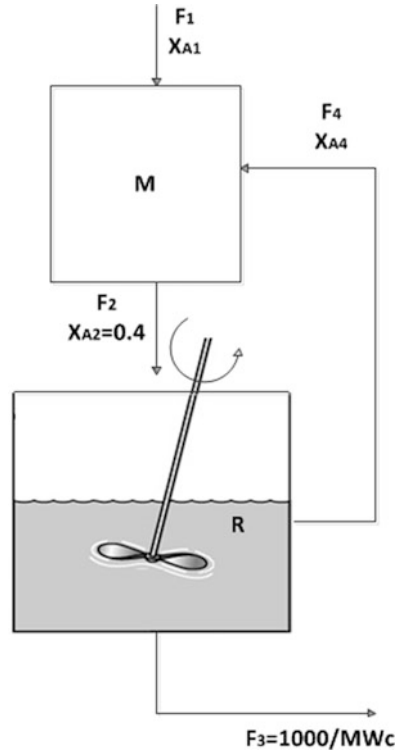


Fig. 8.8 Reactor with recycle including all variables and available data

First, remember that the conversion of the reaction is 60 % (0.6 fractional conversion). In addition, the output of C is 20 [kg mol/h]. Thus, from Table 8.6,

$$F_2 = 83 \text{ [kg mol/h]}.$$

Now completing Table 8.6, we get the following table.

From Table 8.7, we can infer that $F_4 = 43$ [kg mol/h] and

$$X_{A4} = 13.34/43.34 = 0.31.$$

Material balance for fresh stream and recycle (M)

Mole balance for compound A:

$X_{A1} \times F_1 + X_{A4} \times F_4 - X_{A2} \times F_2 = 0$, substituting the available data:

$$X_{A1} \times F_1 + 0.31 \times 43 - 0.4 \times 83 = 0 \tag{8.30}$$

Total mole balance:

$F_1 + F_4 - F_2 = 0$, substituting the available data:

$F_1 + 43 - 83 = 0$, then $F_1 = 40$ [kg mol/h], and substituting F_1 in (8.30), we get

$X_{A1} = 0.50$.

(a) $X_{A1} = 0.50$, so the molar fraction of B is also 0.50.

(b) $X_{A4} = 0.31$, so the molar fraction of B is 0.69 (1 − 0.31).

As shown in this example, the procedure developed in Chap. 7 can be fully utilized for material balances for reactive systems.

Table 8.6 Simulation of the reaction of compounds A and B

Compound	Input [kg mol/h]	Reaction [kg mol/h]	Output [kg mol/h]
A	$0.4F_2$	$0.6(0.4F_2)$	$0.16F_2$
B	$0.6F_2$	$0.6(0.4F_2)$	$0.36F_2$
C			$0.6(0.4F_2) = 20$

Table 8.7 Completed data for the reaction of compounds A and B

Compound	Input [kg mol/h]	Reaction [kg mol/h]	Output [kg mol/h]
A	33	20	13
B	50	20	30
C			20

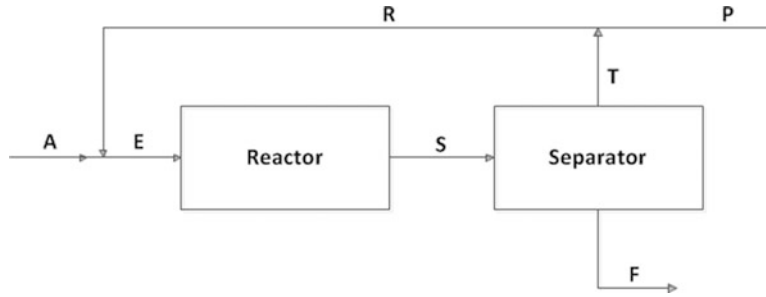


Fig. 8.9 Process for formaldehyde production

Table 8.8 The weight-by-weight composition at the output of the reactor (stream S)

Compound	% w/w
CH_3OH	25
O_2	14
CH_2O	23
H_2O	38

Table 8.9 The weight-by-weight composition in stream F

Compound	% w/w
CH_3OH	4.6
CH_2O	24.4
H_2O	71.0

2. **Formaldehyde production [8]**. Formaldehyde (CH_2O) can be obtained through the reaction of methanol (CH_3OH) and oxygen (O_2) according to

$$2CH_3OH + O_2 \rightarrow 2CH_2O + 2H_2O$$

The reaction has a conversion of 40 %, oxygen is fed in excess, and the process scheme is presented in Fig. 8.9.

The weight-by-weight composition at the output of the reactor (stream S) is as follows (Table 8.8).

In addition, the weight-by-weight composition in stream F is as follows (Table 8.9).

205 [lb/h] of O_2 are leaving the purge (P) and represent 28 % w/w of this stream. Finally, the ratio of streams R and P is 2:1. (a) What is the molar composition of the input stream to the reactor (E)? (b) What is the mass flow rate of the fresh stream A?

Solution

Step I

Reading and understanding

As usual for reactive systems, at least the material balance will be carried out on a molar basis. In addition, it is necessary to determine whether the reaction is balanced and obtain the molecular weights for methanol (CH_3OH), oxygen (O_2), formaldehyde (CH_2O), and water (H_2O):

$$2CH_3OH + O_2 \rightarrow 2CH_2O + 2H_2O.$$

The reaction is balanced because there are two carbon atoms for each side of the equation, eight atoms of hydrogen per side and four atoms of oxygen per side. In addition, the molecular weights of the compounds are as follows:

$MW_{Me} = 32$ [g/mol] or [lb/lb mol]

$MW_{O_2} = 32(g/mol)$

$MW_F = 30$ [g/mol]

$MW_W = 18$ [g/mol]

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

F_A: Mass flow rate of feed stream to system [lb/h]

$X_{A\text{-}Me}$: Mass fraction of methanol in feed stream

$X_{A\text{-}O2}$: Mass fraction of oxygen in feed stream

$X_{A\text{-}F}$: Mass fraction of formaldehyde in feed stream

F_E: Mass flow rate of E stream [lb/h]

$X_{E\text{-}Me}$: Mass fraction of methanol in stream E

$X_{E\text{-}O2}$: Mass fraction of oxygen in stream E

$X_{E\text{-}F}$: Mass fraction of formaldehyde in E stream

F_S: Mass flow rate of output stream of reactor [lb/h]

$X_{S\text{-}Me}$: Mass fraction of methanol in output stream of reactor

$X_{S\text{-}O2}$: Mass fraction of oxygen in output stream of reactor

$X_{S\text{-}F}$: Mass fraction of formaldehyde in output stream of reactor

F_F: Mass flow rate output of system [lb/h]

$X_{F\text{-}Me}$: Mass fraction of methanol in stream F

$X_{F\text{-}F}$: Mass fraction of formaldehyde in stream F

F_T: Mass flow rate output of separator [lb/h]

$X_{T\text{-}Me}$: Mass fraction of methanol in output stream of separator

$X_{T\text{-}O2}$: Mass fraction of oxygen in output stream of separator

$X_{T\text{-}F}$: Mass fraction of formaldehyde in output stream of separator

F_R: Mass flow rate of recycle stream [lb/h]

~~$X_{R\text{-}Me}$: Mass fraction of methanol in output stream of the separator~~

~~$X_{R\text{-}O2}$: Mass fraction of oxygen in output stream of the separator~~

~~$X_{R\text{-}F}$: Mass fraction of formaldehyde in output stream of the separator~~

F_P: Mass flow rate of purge stream [lb/h]

~~$X_{P\text{-}Me}$: Mass fraction of methanol in output stream of the separator~~

~~$X_{P\text{-}O2}$: Mass fraction of oxygen in output stream of the separator~~

~~$X_{P\text{-}F}$: Mass fraction of formaldehyde in output stream of the separator~~

χ_R: Fractional conversion of reaction

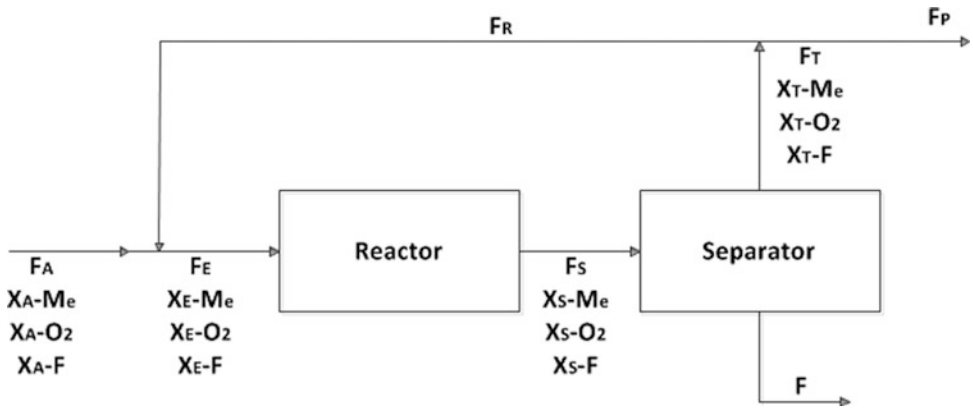


Fig. 8.10 Process for formaldehyde production including all variables

$X_{R\text{-}Me}$, $X_{P\text{-}Me}$, $X_{R\text{-}O2}$, $X_{P\text{-}O2}$, $X_{R\text{-}F}$, and $X_{P\text{-}F}$ will not be considered variables because they are equal to $X_{T\text{-}ME}$, $X_{T\text{-}O2}$, and $X_{T\text{-}F}$, respectively. Streams R and P derive from a division of stream T, so they have exactly the same composition as stream T.

Thus, the total number of variables is 22 (NV = 22).

STEP III

Analysis of degrees of freedom in process

As depicted in Fig. 8.10, we can carry out four material balances for the mixture of the fresh stream (A) and the recycle stream (S), four material balances on the reactor, four material balances on the separator, and one material balance in the division of stream T.

$$\text{NMB} = 13.$$

We have six specified variables ($X_{S\text{-}Me}$, $X_{S\text{-}O2}$, $X_{S\text{-}F}$, $X_{F\text{-}Me}$, $X_{F\text{-}F}$, and χ_R). In addition, although indirectly, we have information to calculate that F_P and $X_{T\text{-}O2}$ from *205 [lb/h] of O_2 leaving the purge (P) and represent 28 % w/w of this stream*. Therefore,

$$\text{NSV} = 8.$$

Finally, NR = 1 (F_R:F_P = 2:1), so the degrees of freedom of the system are

$$\text{DF} = \text{NV} - (\text{NMB} + \text{NSV} + \text{NR}) = 22 - (13 + 8 + 1). \textit{Thus}, \text{DF} = 0.$$

The problem can now be quantitatively formulated and solved.

STEPS IV and V

Mathematical formulation, including all available data and solution, results, analysis, and discussion

First, we calculate that F_P (purge) from *205 [lb/h] of O_2 are leaving the purge (P) and represent 28 % w/w of this stream*. Thus, we can formulate the following equations:

$$X_{P\text{-}O2} \times F_P = 205, \text{ and because } X_{P\text{-}O2} = 0.28,$$
$$F_P = 7.32 \times 10^2 \text{ [lb/h]}.$$

Global mass balance in division

$$F_T - F_P - F_R = 0 \text{ and } F_R{:}F_P = 2{:}1$$

Table 8.10 Calculation of the flow rate (lb mol/h) per each compound at the output of the reactor (stream S)

Compound	% w/w	Mass per 100 [lb]	lb mol per compound (100 [lb])	Molar fraction [%]	lb mol/h per compound at the output stream S
CH_3OH	25	25	25/32 = 0.78	19.1	34.3
O_2	14	14	14/32 = 0.44	10.7	19.2
CH_2O	23	23	23/30 = 0.77	18.7	33.6
H2O	38	38	38/18 = 2.1	51.5	92.4

Table 8.11 Simulation of the reaction of methanol (CH_3OH) and O_2

Compound	Input [lb mol/h] (stream E)	Reaction [lb mol/h]	Output [lb mol/h] (stream S)
Methanol	$2x$	$0.4(2x)$	34.3
Oxygen	$x + y$	$0.4x$	19.2
Formaldehyde	z	–	$z + 0.4(2x) = 33.6$
Water	u	–	$u + 0.4(2x) = 92.4$

Table 8.12 Completion of Table 8.11 adding the output mass per compound

Compound	Input [lb mol/h and lb/h] (stream E)	Reaction [lb mol/h]	Output [lb mol/h and lb/h] (stream S)
Methanol	57.2 (1,830.4)	22.9	34.3 (1,097.6)
Oxygen	30.64 (980.48)	11.45	19.2 (614.1)
Formaldehyde	10.72 (321.6)	–	33.62 (1,008.6)
Water	69.5 (1,251)	–	92.4 (1,663.2)
Total	(4,383.5)		(4,383.5)

Replacing F_P we obtain $F_T = 2.196 \times 10^3$ [lb/h] and $F_R = 1.464 \times 10^3$ [lb/h].

Then the pound-moles per hour of oxygen in stream T are

$0.28 \times 2.196 \times 10^3$ [lb/h]/32 [lb/lb mol] = 19.2 [lb mol/h], which is the same as the pound-moles per hour of oxygen in stream S because there is no oxygen in stream F_F.

With this information and the data given in Table 8.9, we will be able to determine the molar composition at the output of the reactor and from there to calculate the pound-moles per hour of all components because we know the pound-moles per hour of oxygen (Table 8.10).

Material balance for reactor

$$2CH_3OH + O_2 \rightarrow 2CH_2O + 2H_2O$$

The fractional conversion is $\chi_R = 0.4$ (40 %), and oxygen is in excess (methanol is the limiting reagent). Therefore, we can create the following table for the material balance on the reactor:

Thus,

$x = 28.6$ [lb mol/h]; therefore $y = 2.04$ [lb mol/h] (from $x + y - 0.4x = 19.2$);
$z = 10.72$ [lb mol/h]; and $u = 69.5$ [lb mol/h].

Finally, Table 8.11 can be completed, adding the output mass per compound (Table 8.12):

One interesting thing to notice in the table is that the total mass was conserved, as it should be.

(a) What is the molar composition of the input stream to the reactor (E)? (Table 8.13)

Table 8.13 Molar composition of the input stream to the reactor

Compound	Input [lb mol/h] (stream E)	Molar composition [%] (stream E)
Methanol	57.2	34.0
Oxygen	30.64	18.2
Formaldehyde	10.72	6.4
Water	69.5	41.4
Total	168.06	100

Table 8.14 The weight-by-weight composition of the fresh stream A

Compound	Mass [lb/h]	w/w [%]
Methanol	1,165.7	39.90
Oxygen	570.5	19.60
Formaldehyde	5.4	0.18
Water	1,177.6	40.32
Total	2,919.2	100

(b) What is the mass flow rate of the fresh stream A (F_A)?

Global mass balance in separator:

$$F_S - F_T - F_F = 0, \text{ so substituting the available data yields}$$
$$4{,}383.5 - 2{,}196.44 - F_F = 0, \text{ then } F_F = 2{,}187.06 \text{ [lb/h]}$$

Global mass balance for whole system:

$$F_A - F_P - F_F = 0$$
$$F_A - 732.143 - 2{,}187.06 = 0, \text{ so } F_A = 2{,}919.2 \text{ [lb/h]}.$$

Furthermore, if you carry additional mass balances, you can check that the w/w composition of the fresh stream A is as follows (Table 8.14):

Analysis

First, with this problem, our experience has been that the procedure developed in Chap. 7 can be directly applied to material balances for reactive systems. Second, as has been continuously stressed, it is very critical to follow the proposed procedure and work patiently and strictly follow the given order, which might get boring sometimes, but in problems with many variables it is worthwhile and rewarding to do just that. In addition, it is also important to identify, define, and correctly codify all the variables.

3. **Continuous steady-state bioreactors [6]**. In warm-up example 7, we discovered that for this specific example, it was a better option to arrange two bioreactors of volume $V/2$ in series instead of one bioreactor of volume V. A company interested in your skills wants to design a system of bioreactors with a total volume of 500 [L] and get an output substrate concentration lower than 31 [g/L].

First, you ask for technical data to do some calculations, and finally give your recommendation on how to set up the bioreactors.

Technical data

$$F = 100 \text{ [L/h]}; S_0 = 50 \text{ [g/L]}; k = 0.1 \text{ [1/h]}; V = 500 \text{ [L]}$$

To figure out what to expect, first you start calculating the output concentration of the substrate for one bioreactor with a volume of 500 [L] and obtain

$$S = \frac{FS_0}{F + Vk} = \frac{100 \times 50}{100 + 500 \times 0.1} = 33(\text{g/L}).$$

Now you think that by putting two bioreactors with a volume equal to 250 [L] in series, you will get an output concentration lower than 31 [g/L]. Unfortunately, you get

$$S_{\text{II}} = \frac{F^2 S_0}{\left[F + \frac{V}{2}k\right]^2} = \frac{100^2 \times 50}{\left(100 + \frac{500}{2} \times 0.1\right)^2} = 32(\text{g/L}), \text{ which is greater than 31 [g/L].}$$

How many bioreactors with a total volume of 500 [L] should you put in series to get the desired output concentration of 31 [g/L] or less?

Solution

Step I

Reading and understanding

From warm-up exercise 7 and the results presented in the problem 3 statement, we can infer that if we arranged more bioreactors in series (say 3, 4, …) we would lower the output concentration of the substrate, but the question is by how much? To avoid trial and error, we will try to find a general formula to relate the output concentration of the substrate (S) to the number of bioreactors arranged in series (n).

Step II

Flow diagram, variable definition and codification, and inclusion of all available data

From previous calculations we know the following:

1. Bioreactor of volume V:

$$S_{\text{I}} = \frac{FS_0}{F + Vk}.$$

2. Bioreactors of volume $V/2$ each (Fig. 8.11):

$$S_{\text{II}} = \frac{F^2 S_0}{\left[F + \frac{V}{2}k\right]^2}.$$

STEPS IV and V

Mathematical formulation, including all available data and solution, results, analysis, and discussion

Now we will deduce a formula for three bioreactors in series as follows:

First bioreactor:

$$FS_0 - FS_{\text{I}} - \frac{\text{V}}{3}kS_{\text{I}} = 0; \text{solving for} S_{\text{I}} \text{we obtain } S_{\text{I}} = \frac{FS_0}{F + \frac{V}{3}k}. \tag{8.31}$$

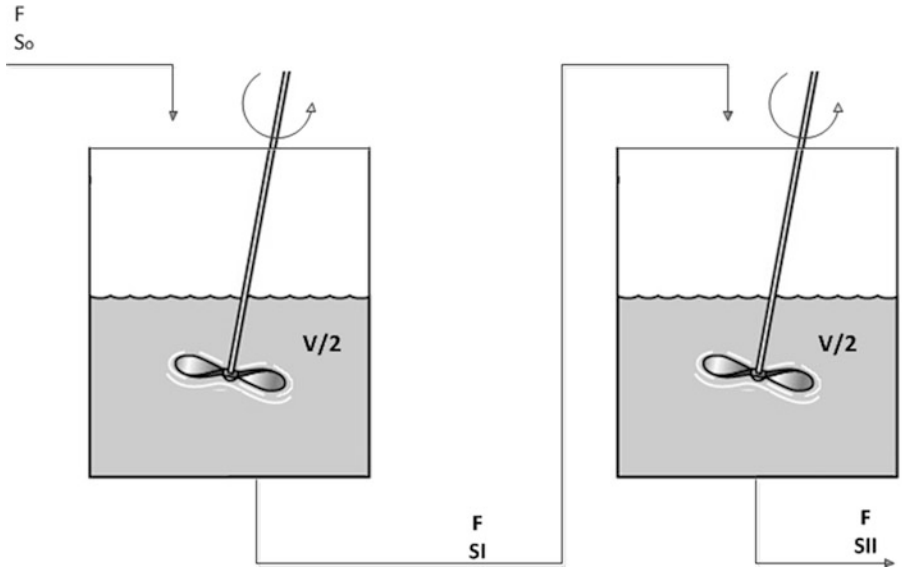


Fig. 8.11 Two bioreactors in series including all variables

Second bioreactor:

$$FS_{\mathrm{I}} - FS_{\mathrm{II}} - \frac{\mathrm{V}}{3} kS_{\mathrm{II}} = 0. \tag{8.32}$$

Substituting (8.31) in (8.32) and solving for S_{II}, we get

$$S_{\mathrm{II}} = \frac{F^2 S_0}{\left[F + \frac{V}{3}k\right]^2}. \tag{8.33}$$

Third bioreactor:

$$FS_{\mathrm{II}} - FS_{\mathrm{III}} - \frac{\mathrm{V}}{3} kS_{\mathrm{III}} = 0. \tag{8.34}$$

Substituting (8.33) in (8.34) and solving for S_{III}, we get

$$S_{\mathrm{III}} = \frac{F^3 S_0}{\left[F + \frac{V}{3}k\right]^3}. \tag{8.35}$$

Finally, by induction, we can write an equation for n bioreactors of volume V/n each in series:

$$S_n = \frac{F^n S_0}{\left[F + \frac{V}{n}k\right]^n}.$$

According to this formula we get the following output substrate concentration:

Number of bioreactors	Output substrate concentration (g/L)
1	33.3
2	32.0
3	31.5
4	31.2
5	31.04
6	**30.93**
7	30.85
8	30.78

Analysis

Then, to reach a substrate concentration lower than 31 [g/L], we need to arrange six bioreactors of volume 83.34 [L] each in series. Another alternative, possibly better, is to use just one bioreactor, but of a larger volume ($V > 500$ [L]). Please check that using one bioreactor of volume 615 [L], the output concentration of the substrate is lower than 31 [g/L].

8.7 Proposed Problems

1. **Carbon composition [2]**. Determine the carbon composition (w/w) in: (a) acetylene, (b) propane, (c) carbon dioxide, (d) carbon monoxide, and (e) ethanol.
   A: (a) 92.3 %, (b) 81.8 %, (c) 27.3 %, (d) 42.86 %, and (e) 52.1 %.
2. **Limiting reactant [3]**. A reactor is fed with a stream that contains compounds A, B, and C whose molecular weights are 30, 60, and 20, respectively. The reactor was fed with 30 [kg] of A, 40 [kg] of B, and 15 [kg] of C. The reaction is as follows:

$$A + 0.75B + 1.5C \rightarrow 3D.$$

   (a) What is the limiting reactant? (b) What is the molecular weight of D?
   A: (a) C. (b) $MW_D = 35$
3. **Stoichiometry [4]**. In a chemistry lab, the technician has the following amount of compounds A, B, and C whose molecular weights are presented in the following table:

	Amount	Molecular weight
A	1.2 [L] ($\rho = 1.4$ [kg/L])	25
B	0.8 [lb]	40
C	950 [g]	18

   The reaction is equilibrated according to the following equation:

$$aA + bB + cC \rightarrow dD,$$

   where $a + b + c = 6$, and the feed is stoichiometric.
   (a) How many moles are fed of each compound? (b) What are the numerical values of a, b, and c?
   A: (a) 67.2 [mol] of A, 9.08 [mol] of B and 52.8 [mol] of C (b) $a = 3.124$; $b = 0.422$ and $c = 2.454$

4. **Limiting reactant 2 [3]**. To obtain 100 [g] of product C ($MW_C = 28$), 300 [g] of compound A ($MW_A = 30$) and 500 [g] of compound B ($MW_B = 27$) are reacted according to the following reaction:

$$A + 2B \rightarrow 3C.$$

(a) What is the limiting reactant? (b) What is the conversion of the reaction?
A: (a) B. (b) 12.86 %.

5. **Methane [3]**. 160 [kg] of CH_4 and 180 [kg] of water are fed to a reactor. If the conversion of the reaction is 60 %, calculate how many kilograms of CO and H_2 are produced respectively?
A: CO: 168 [kg]; H_2: 36 [kg].

6. **Limestone [4]**. A limestone is composed of 85.3 % calcium carbonate ($CaCO_3$). How much limestone was used if, when reacting with hydrogen chloride (HCl) in excess, 10 [L] of carbon dioxide (CO_2) at 18 °C and 752 mmHg was obtained?
A: 48.66 [g] of limestone.

7. **Combustion [5]**. To realize a propane–oxygen combustion, we have 2 [L] of propane at 12 °C and 740 mmHg. The combustion will be carried out at 23 °C and 750 mmHg. (a) How much oxygen is needed (volume)? (b) How much air is needed if the oxygen is in 10 % excess?
A: (a) 10.25 [L] of oxygen at 23 °C and 750 mmHg, (b) 2.184 [g mol] of air.

8. **Methane and ethane [7]**. To burn a fuel that has 60 % methane (CH_4) and 40 % ethane (C_2H_6), air is fed with 50 % molar in excess. If the compounds are totally burned to form CO_2 and CO, and, in addition, 10 % of CH_4 and 15 % C_2H_6 are burned to form CO, then: (a) What is the output w/w composition of the gases? (b) How much CH_4 and C_2H_6 should be fed to produce 100 [kg/h] of CO_2? Consider that air is 20 % O_2 and 80 % N_2 (molar).
A: (a) O_2: 7.8 %; CO_2: 9.06 %; CO: 0.84 %; N_2: 82.3 %. (b) CH_4: 27.54 [kg] and C_2H_6: 18.36 [kg].

9. **Parallel reactions [8]**. 30 [cm^3] of a mixture of ethane and acetylene and 120 [cm^3] of oxygen are filled in a graduated test tube. The mixture is burned, and then the water vapor is condensed to go back to the initial conditions. The residue is formed by 81 [cm^3] of CO_2 and oxygen that was added in excess. Calculate the composition of the ethane–acetylene mixture.

$$2C_2H_6 + 7O_2 \rightarrow 4CO_2 + 6H_2O$$
$$2C_2H_2 + 5O_2 \rightarrow 4CO_2 + 2H_2O$$

A: 24 [cm^3] ethane and 6 [cm^3] of acetylene

10. **Acetylene [4]**. Acetylene is a gas that is obtained treating calcium carbide (CaC_2) with water according to the following reaction:

$$CaC_2 + 2H_2O \rightarrow C_2H_2 + Ca(OH)_2.$$

Calculate the number of hours that an acetylene lamp works that consumes 40 [L] of gas per hour at 80 °F and 740 mmHg if we have 1 [kg] of calcium carbide?
A: 9.86 h (9 h 51 min 36 s).

11. **Acetylene-2 [4]**. You have 4 [L] of water and 10 [kg] of a product that contains 80 % (w/w) of CaC_2. If the conversion of the reaction is 100 %, calculate or determine: (a) What is the limiting reactant? (b) What is the mass of the acetylene that is produced? (c) What is the residual mass of the product?
A: (a) Water. (b) 2,888.8 [g] acetylene. (c) 2,888.8 [g] (2 [kg] + 0.888 [kg] of CaC_2).

12. **Reactors in series [6]**. A stream containing 30 [kg mol] of A, 100 [kg mol] of B, and 3 [kg mol] of C is fed to the first reactor. Then the output stream is cooled down (113 [kg mol]) and fed to the second reactor, where the output of the second reactor is a total of 103 [kg mol]. In each reactor the following stoichiometric reaction takes place:

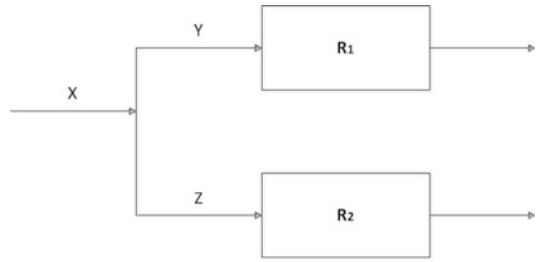


Fig. 8.12 Two reactors operated in parallel

$$A + 3B \rightarrow 2C$$

Determine: (a) the conversion of the reaction in each reactor and (b) the composition of the output flow in each reactor.
A: (a) 33.3 % and 25 %, respectively;
(b)

	First reactor		Second reactor	
	Output moles	Composition (%)	Output moles	Composition (%)
A	20	17.7	15	14.6
B	70	61.9	55	53.4
C	23	20.4	33	32
Total	113	100	103	100

13. **Haber–Bosch synthesis [4]**. The reaction known as the Haber–Bosch synthesis (Chap. 4) is the reaction of nitrogen (N_2) and hydrogen (H_2) to form ammonia (NH_3). A stream of 10 [m^3/h] H_2 at 2 [atm] and 20 °C will react with a stream of air (79 % N_2). The N_2 is 30 % in excess and the conversion of the reaction is 16 %. Determine the volumetric composition of the outlet stream.
A: N_2 26.36 %, H_2 58.26 %, NH_3 7.40 %, O_2 7.98 %.
14. **Contaminant [7]**. A reactor is charged with 1,000 [ton] of A (solid) and is continuously fed with a liquid stream of 1 [ton/h] of B with 10 % w/w of contaminant D.

$$2A + 5B \rightarrow 40C$$
$$A + D \rightarrow 24E + 2C \text{ (both reactions have 100 \% conversion)}$$

The products of the reactions are gases, and it is desirable to remove 90 % of the contaminating E from this stream, so an adsorption tower will be installed to remove contaminant E. The tower can withdraw 0.05 [kg] E/kg of adsorbent. (a) What is the molecular weight of E? (b) Calculate the load of adsorbent (kg) required if the adsorbent must be regenerated every three reactor loads.

DATA: $MW_A = 150$; $MW_B = 60$; $MW_C = 15$; $MW_D = 24$
A: (a) $MW_E = 6$, (b) 21,245.85 [ton].
15. **Reactors in parallel [6]**. Reactors R_1 and R_2 are operated in parallel, as depicted in Fig. 8.12, and the reaction is as follows:

$$2A + 3B \rightarrow C.$$

The *X* stream has a mass flow rate of 1,000 [kg/h] with 50 % molar A. If stream *X* contains A and B and the total amount of C produced is 675 [kg/h], then: (a) What is the molecular weight of C?

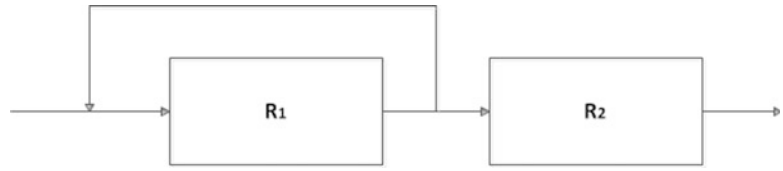


Fig. 8.13 Two reactors operated in series for a process synthesis

(b) Calculate the mass flow rate of streams *Y* and *Z*. (c) What is the molar composition at the output of each reactor? (d) What is the limiting reactant?

DATA: $MW_A = 30$; $MW_B = 40$; conversion of reaction in $R_1 = 80\ \%$; conversion of reaction in $R_2 = 100\ \%$

A: (a) $MW_C = 180$

(b) $Y = 500$ [kg/h] and $Z = 500$ [kg/h]

(c)

Compound	Reactor R_1	Reactor R_2
A	30 %	66.67 %
B	10 %	–
C	60 %	33.33 %

(d) In both reactors, the limiting reactant is *B*.

16. **Fermentation [7]**. A continuous pilot fermenter is fed with a mixture of glucose and xylose as a substrate. The microorganism *S. stipitis* consumes both sugars. Given that the pilot fermenter does not have good instruments to determine how much sugar is being consumed, the operators have been doing some simple measurements. They have determined that in 1 h 50 min, a gas tank of 20 [L] is completely filled with the CO_2 released from the reaction (at 32 °C and 1.02 [atm]). Furthermore, literature information indicates that the feed stream has a ratio xylose:glucose of 9:1. Determine how much sugar (weight) is transformed into alcohol and carbon dioxide per hour. Assume a conversion of 100 %.

$$3C_5H_{10}O_5 \rightarrow 5C_2H_6O + 5CO_2,$$
$$C_6H_{12}O_6 \rightarrow 2C_2H_6O + 2CO_2.$$

A: Xylose 35.31 [g/h], glucose 4.70 [g/h]

17. **Process synthesis [8]**. A synthesis process consists of a system with two reactors in series (Fig. 8.13). The system is fed with 150 [kg/h] (density 1.18 [kg/L]) with compounds A (2 [M]) and B (1.2 [M]). Compounds A and B react to produce compound C, according to the following reaction:

$$A + 2B \rightarrow C \text{ conversion } 35\ \%.$$

The output current of the first reactor is divided into two streams in a 2:1 ratio, where the larger stream is fed to the second reactor. The second stream is recycled to the first reactor. In the second reactor, compound A reacts with compound C, producing compounds D and E, according to the following reaction:

$$A + C \rightarrow D + 2E \text{ conversion } 70\ \%.$$

Determine the mass composition of the outlet stream of the second reactor on a dry basis.

$$MW_A = 64 \text{ [g/mol]},$$
$$MW_B = 88 \text{ [g/mol]},$$

$$MW_D = 22 \text{ [g/mol]}.$$

A: **A** 45 %, **B** 29.3 %, **C** 6.1 %, **D** 1.4 % and **E** 17.7 %.

18. **Biogas [7]**. The sulfidric acid (H_2S) content must be reduced in a stream of biogas from 4,000 ppm to 10 ppm. For this a filter filler of iron sponge is available that contains 190 [kg] of iron oxide III/m^3. The sulfidric acid from the gas reacts with iron oxide III in the sponge, producing iron sulfide III and water. If the flow of biogas to treat is 12 [m^3/h], determine the volume of the iron sponge if it is projected to be replaced once a month. Consider that the iron sponge is replaced when 85 % of the iron oxide III is consumed.
A: 0.34 [m^3].
19. **Sugars and enzymes [8]**. A student is "playing" with sugars and enzymes in the laboratory. He has a solution of 10 [g/L] sucrose, 5 [g/L] lactose, and 12 [g/L] maltose, and he is experimenting with the enzymes invertase, lactase, and maltase. After some point, he measures the concentrations, getting 0.1 [g/L] lactose, 4 [g/L] fructose, and 11 [g/L] glucose. To finish his fun evening, the student calculates the conversion of each reaction. What are his results?

A:
Invertase: 76 %
Lactase: 98 %
Maltase: 35 %

20. **Reactors in series [10]**. A system composed of two reactors in series is fed with a moles of compound A and $3a$ moles of compound B. The object of the process is to get the coveted product D.

First reactor

$$A + 2B \rightarrow C \text{ conversion} = x \text{ (decimal form, i.e. } 0 < x < 1).$$

Thus, the output stream of the first reactor is the feed stream of the second reactor.

Second reactor

$$C + B \rightarrow 3D \text{ conversion} = y \text{ (decimal form, i.e., } 0 < y < 1).$$

(a) What is the molar composition at the output of the first reactor? (b) What is the molar composition at the output of the second reactor? (c) What is the amount of D produced in grams?
The molecular weight of D is 30 and so (a) and (b) should be expressed as a function of x and y.
A: (a) and (b)

	Molar composition % output of first reactor	Molar composition % output of second reactor
A	$50(1 - x)/(2 - x)$	$100(1 - x)/(4 - 2x + xy)$
B	$50(3 - 2x)/(2 - x)$	$100(3 - 2x - xy)/(4 - 2x + xy)$
C	$50x/(2 - x)$	$100x(1 - y)/(4 - 2x + xy)$
D	–	$300xy/(4 - 2x + xy)$

(c) $90axy$ g of D.

21. **Chain reactions [8]**. The highly desirable product E is obtained through the following chain reactions:

$$A + B \rightarrow 2C \text{ conversion } 80\ \%,$$
$$C + B \rightarrow 3D \text{ conversion } 66.67\ \%,$$

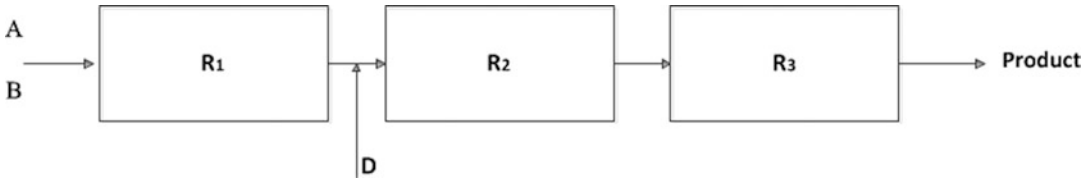


Fig. 8.14 Three reactors in series

$$D + C \rightarrow 2E \text{ conversion } 50\ \%.$$

The process is continuously operating and producing 4 [kg mol/h] of product E. If the molecular weight of compounds A, B, and E are $MW_A = 30$, $MW_B = 40$, and $MW_E = 20$, then: (a) What is the required mass of A and B to produce 100 [kg] of E? (b) How many hours will it take to produce 100 [kg] of E?

A: (a) Mass of compound A = 125 [kg] and mass of compound B = 233.2 [kg]. (b) 1 h 15 min.

22. **Mixture [5]**. 1,508 [g] of a mixture of pure calcium nitrate and barium nitrate are transformed into sulfates in the presence of sulfuric acid. Evaporating the liquid to dryness, the mixture of sulfates weighs 1,314 [g]. Calculate the composition of the mixture of nitrates.

    A: 518 [g] of $Ca(NO_3)_2$ and 990 [g] of $Ba(NO_3)_2$.

23. **Conversion [6]**. A reactor is fed with compounds A and B (stoichiometric). The reaction is as follows:

$$A + 2B \rightarrow C.$$

    If the feed stream contains 100 [mol] of A and at the output the total moles are 220, then what is the conversion of the reaction?

    A: 40 %.

24. **Three reactors in series [7]**. The system depicted in Fig. 8.14 shows three reactors in series. The system is fed with 100 [kg/h] of compound A and 180 [kg/h] of compound B. In addition, 675 [kg/h] of compound D are fed to the second reactor. The reactions areas follows:

Reactor 1:	$A + B \rightarrow 2C$	Conversion 80 %
Reactor 2:	$3D + C \rightarrow 4E$	Conversion 85 %
Reactor 3:	$E + C \rightarrow F$	

    If the system generates 27 [kg/h] of product F, then: (a) Determine the molar composition at the output of each reactor. (b) What is the limiting reactant in each reactor, where $MW_A = 20$, $MW_B = 30$, $MW_D = 25$.

    A: (**a**)

	Reactor 1	Reactor 2	Reactor 3
A	9.1	2.6	2.7
B	18.2	5.3	5.3
C	72.7	3.2	1.8
D	–	17.4	17.6
E	–	71.6	71.1
F	–	–	1.4

    (**b**) Limiting reactant for each reactor is: Reactor 1: A, Reactor 2: C, Reactor 3: C

25. **Ethylene oxide [10⁺]**. The first step in the production process of ethylene oxide (C_2H_4O) is depicted in the following flow sheet (Fig. 8.15).

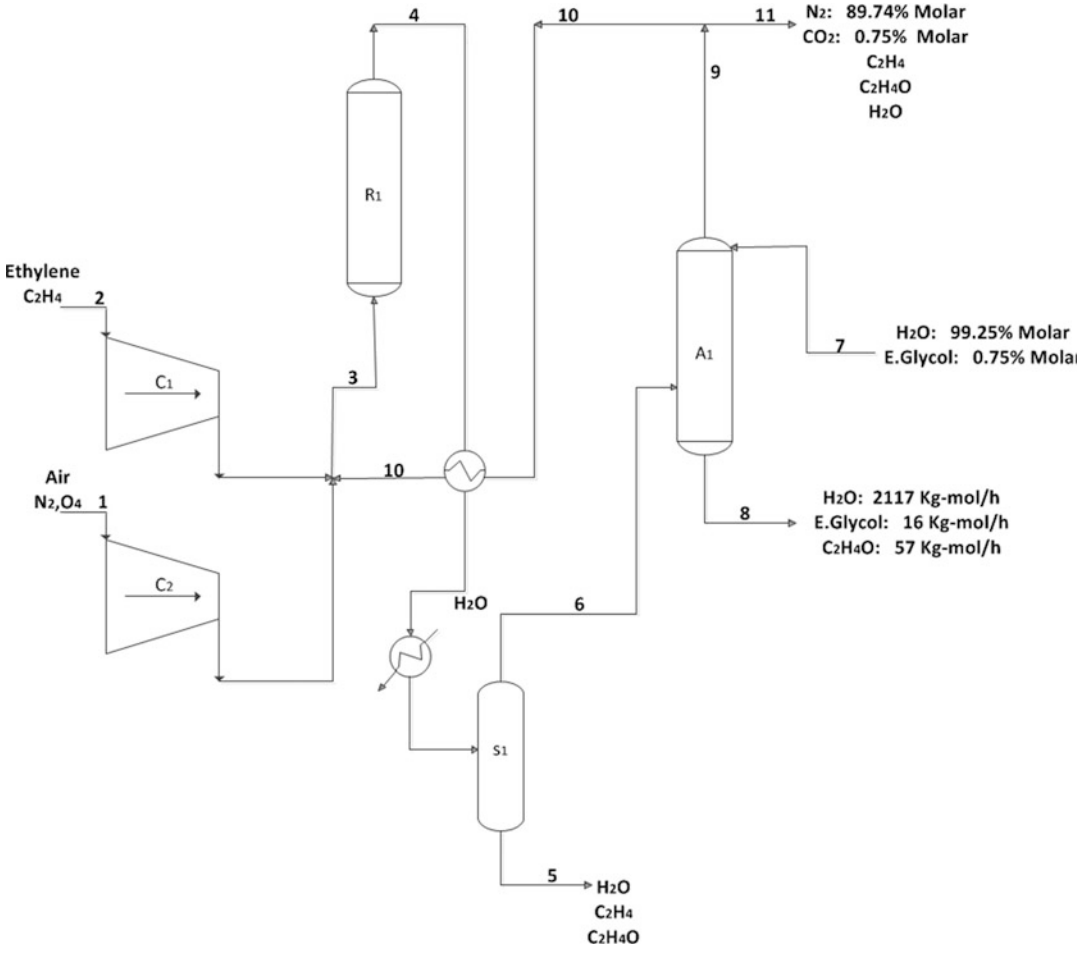


Fig. 8.15 Process for ethylene oxide production

Ethylene (C_2H_4) and air are compressed in compressors C-1 and C-2 and mixed with the recycle stream of the absorber (A-1). The resultant mixture contains 5.02 % ethylene and 12.41 % oxygen (molar basis) and is fed to reactor R-1. The main reactions in R-1 are

$$C_2H_4 + 0.5O_2 \rightarrow C_2H_4O,$$
$$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O.$$

The output stream of reactor R-1 is cooled down with the recycle stream from the absorber and then with water. Then the resultant liquid–vapor mixture is separated in the separator drum S-1. The vapor from S-1 feeds the absorber, where 97.85 % of the ethylene oxide is absorbed using an aqueous solution (0.75 % molar) of ethylene glycol ($C_2H_6O_2$).

The gases that are not absorbed leave through the top of the column. 60 % of this stream is recycled to reactor R-1 and the rest is purged.

Separator drum: $C_2H_4O/C_2H_4 = 2$ (molar basis) (this stream also contains H_2O).

Fed stream: $O_2/C_2H_4 = 2.033$ (molar basis).

In addition, the output liquid stream from the absorber contains 57 [kg mol] C_2H_4O, 16 [kg mol] ethylene glycol, and 2,117 [kg mol] H_2O. The purge contains 82.74 % N_2 and 8.13 % CO_2 and O_2, C_2H_4, C_2H_4, and H_2O.
(a) Quantify the output and input streams of the process. (b) What is the degree of conversion of the reactions?
A: (a)

Stream	Molar content (kg mol)
1 (air)	N_2: 955.25 O_2: 254.12
2 (ethylene)	C_2H_4: 125.00
5	H_2O: 92.77 C_2H_4O: 0.35 C_2H_4: 0.175
7	$C_2H_6O_2$: 16 H_2O: 2,117
11 (purge)	N_2: 955.25 O_2: 84.34 CO_2: 94.00 C_2H_4O: 0.5 H_2O: 1.14 C_2H4: 19.29

(b) First reaction: 37.6 %; second reaction: 31.5 %

Additional Web References

Balances on Reactive Systems https://www.youtube.com/watch?v=jyso8NSytWw
Single Reaction with Recycle https://www.youtube.com/watch?v=bF19fAoRpVo
Balancing Chemical Equations https://www.youtube.com/watch?v=UGf60kq_ZDI
Balancing Combustion Reactions http://www.youtube.com/watch?v=ed0h11IfEPU
Stoichiometry - Conversions Using Balanced Equations http://www.youtube.com/watch?v=wySZDEbqbnM
Two Continuous Stirred Tank Reactors in Series http://www.youtube.com/watch?v=7RLQ9sHkdkk
Balances on Multiple Units with Reaction https://www.youtube.com/watch?v=tQyrSvll_nc
Extent of Reaction for Material Balances https://www.youtube.com/watch?v=YusSU0jlOUk
Three Methods for Solving Reactive Material Balances https://www.youtube.com/watch?v=MSzTIRAv5io

9[2] Fundamentals of Mathematical Modeling, Simulation, and Process Control

If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.

John von Neumann

Reverend Thomas Bayes (was) a brilliant mathematician who devised a complex equation known as the Bayes theorem, which can be used to work out probability distributions. It had no practical application in his lifetime, but today, thanks to computers, is routinely used in the modeling of climate change, astrophysics and stock market analysis.

Bill Bryson

Remember that all models are wrong; the practical question is how wrong do they have to be to not be useful.

George Edward Pelham Box

9.1 Chapter Purpose

Believe it or not, we have extensively, unconsciously, exposed you to, practiced, and developed mathematical models throughout this book. The first exposure to mathematical modeling in this book was perhaps in Chap. 2 (Sect. 2.4.1), where we introduced the ideal gas law. The ideal gas law is a mathematical representation that relates the pressure (P), volume (V), moles (n), and absolute temperature (T) of an ideal gas with a simple, but powerful, mathematical equation. Then, in the same chapter, we deduced and practiced the use of simple mathematical models like the one to convert temperatures from the Celsius scale to the Fahrenheit scale. In addition, in Sect. 2.7, you were exposed to several mathematical models that are common in chemical and bioprocess engineering. Then, in several of the following chapters, we utilized mathematical models. For example, in Chap. 6, we showed and employed different mathematical models that are frequently utilized in, for example, thermodynamics, heat transfer, and mass transfer. At this stage, a good exercise for you would be to identify throughout the book all mathematical models and to begin to understand their importance in chemical and bioprocess engineering.

The main purpose of this chapter is to understand and discover, through examples, the importance of mathematical modeling and to start constructing simple mathematical models. The second purpose is introduce you to process and bioprocess simulation. Again, in the case of simulation, you have already been exposed to and practiced simulation through some exercises in previous chapters of the

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book. For example, in Chap. 8 (Sect. 8.6, problem 3), it was proved that two continuous bioreactors in series (total volume V) were more effective than one continuous bioreactor of volume V. Then the question was to determine how many reactors in series were needed to reach a specific concentration of the substrate at the output stream. To answer the question, we first developed a general mathematical model that was capable of predicting the output substrate concentration (S) as a function of the number of bioreactors (n), and with this model and with the help of Excel, we constructed a table that *simulated* the output substrate concentration S for different values of n. Once more, we invite you to discover other examples and problems presented in the book that bear a direct relation to process and bioprocess simulation. Third, this chapter will introduce you to process and bioprocess control. Controlling a process means mastering it and keeping it running in peak condition, safe for people and equipment. How do I maintain a controlled process? How do I detect that a process is out of control? What is a smart way to take control? These are some of the questions that will be addressed in this chapter.

Finally, our main target is to continuously show you, through examples, the beauty and broad application of chemical and bioprocess engineering.

9.2 What Is Mathematical Modeling?

A mathematical model is an abstract representation of reality, in mathematical language, used to find solutions to different types of problems.

For example, we would like to determine the number of poles (Fig. 9.1) needed to separate two adjacent sites. According to experts, we should put the poles $\boldsymbol{d}$ meters apart (including poles at both extremes). Further, the distance between poles could be less than $\boldsymbol{d}$ meters but no greater. The experts also suggest that this fence utilize poles that are $\boldsymbol{e}$ meters in diameter ($\boldsymbol{e} \lll \boldsymbol{d}$). Then, if the fence has a total length of $\boldsymbol{a}$ meters ($\boldsymbol{a} \ggg \boldsymbol{d}$), how many poles are needed?

The variables in the problem are as follows:

a: Total length of fence in meters
n: Number of poles
d: Maximum distance between two poles in meters
e: Diameter of each pole in meters

Then, analyzing Fig. 9.1, we can write the following equation:

$$a = ne + (n-1)d. \tag{9.1}$$

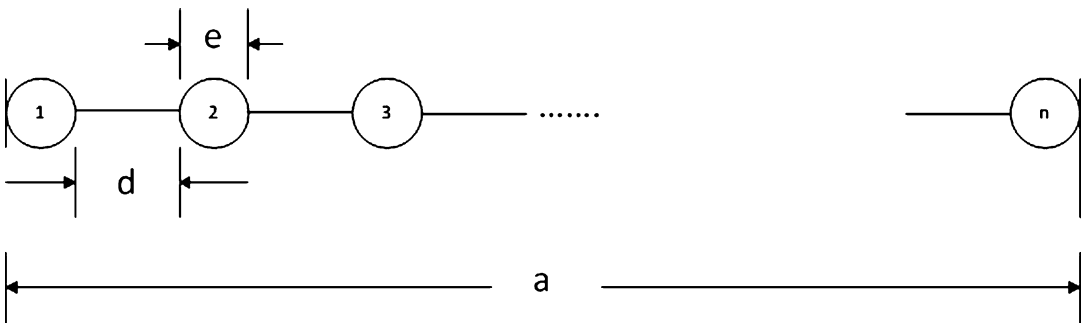


Fig. 9.1 Schematic representation of a fence

Solving for n we get

$$n = \frac{a+d}{d+e}. \tag{9.2}$$

If, for example, $a = 100$ m, $d = 4$ m, and $e = 0.2$ m, then $n = 24.76$ poles. Of course, we cannot employ 24.76 poles, but if we arrange (9.2) as follows:

$$d = \frac{a-ne}{n-1}, \tag{9.3}$$

then, from (9.3), we can check that, by using 25 poles, the distance between poles will be $d = 3.96$ [m], which is in agreement with the expert recommendation ($d \leq 4$ m).

Equation (9.2) is a very simple but practical mathematical model that allows us to calculate the number of poles as a function of the fence length, the distance between poles, and the pole diameter. For example, if you decide that 25 poles is too many, and your budget will allow for just 20 poles, then, using (9.2), you can simulate different scenarios and determine the distance between poles if you are employing 20 poles or any other number of poles on this fence.

Mathematical models and simulation are powerful tools for engineers. In addition, with the help of computers, we can solve and simulate extremely complex mathematical models. Throughout your career, you will be learning, step by step, how to build interesting mathematical models for chemical and bioprocess engineering. In the next section, we will start building simple models, and in doing so, we will show you the potential and broad applications of this engineering tool.

9.3 Importance of Building Mathematical Models and Constructing Simple Models for Chemical and Bioprocess Engineering

9.3.1 What Have We learned?

One interesting lesson that we can learn from having solved problem 3 in Sect. 8.6 is that the larger the number of bioreactors (n) in series (for the same total volume V) was, the lower the output substrate concentration (S) was. But, possibly, of much greater interest was the discovery that the output substrate concentration was not very sensitive to an increasing number of bioreactors (Table 9.1).

From the table you will notice that the output substrate concentration decreases with the number of bioreactors, but this is barely noticeable. For example, we increased the number of reactors from 1 to

Table 9.1 Oulet concentration of substrate as a function to the number of bioreactors

Number of bioreactors	Output substrate concentration (g/L)
1	33.3
2	32.0
3	31.5
4	31.2
5	31.04
6	30.93
7	30.85
8	30.78

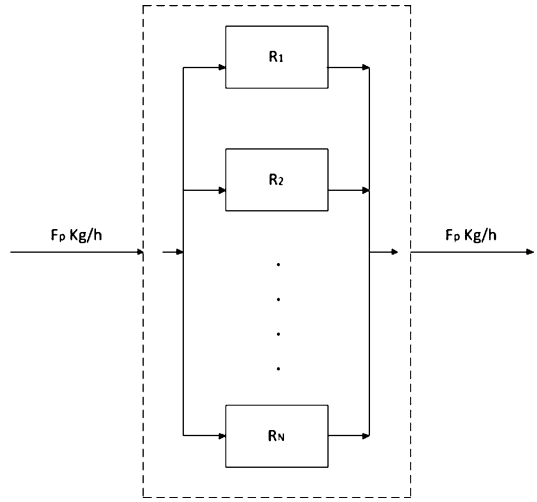


Fig. 9.2 *N* batch reactors operated in a continuous mode as a whole battery

8 (700 %), the substrate concentration just decreased from 33.3 to 30.78 g/L (7.6 %). In this case, intuitively, for several practical and operational reasons, we can infer that it is better to have just one bioreactor instead of several bioreactors in series.

Mathematical models will help us improve our decisions. In this example, we have developed a tool that allows us to further analyze the system and then find the optimum number of bioreactors. With a good model, we will be able to run simulations of different scenarios, improve our understanding and knowledge of the system, and design better control systems, as we will see in Sect. 9.4. Moreover, with an adequate mathematical model, we can drastically reduce the required number of experiments.

9.3.2 Building Simple Mathematical Models

In this section, we will attempt to develop mathematical models for different situations that you will face as a process or bioprocess engineer. Although these examples are limited and tailored to your mathematical background, it will be interesting and rewarding for you to discover that you can be involved with fascinating and ever-challenging examples.

Adapting a reactor batch-operation stage to a continuous processing line (can also be applied to, for example, bioreactors, autoclaves, and dryers).
In contrast to normal true batch processes, some processing plants are operated with just one stage functioning in a batch mode (e.g., winemaking, canned food production, dehydration processes) (Simpson et al. 2002). Moreover, most of the time it could be advantageous to achieve a fully continuous processing line, as shown in Fig. 9.2. As depicted in Fig. 9.2, although each reactor (R_1, R_2, …, R_N) is operated in a batch mode, the whole processing line operates in a continuous mode.

A Gantt chart showing the temporal programming schedule of a battery reactor system (Fig. 9.3) can be used as a first step in determining the number of reactors as a function of its effective volume (V_R) and volumetric flow rate (F_P) from the processing line. The continuous operation of the reactor

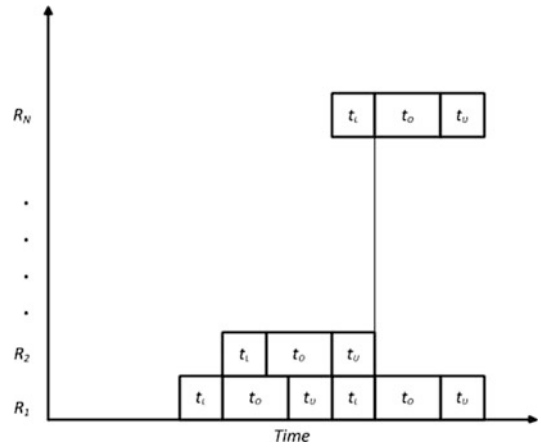


Fig. 9.3 Gantt chart of the reactor-operation against time

battery can be achieved if the loading step of the last reactor (R_N) finishes at the same time as the first reactor finishes its complete cycle and is ready for loading. As depicted in Fig. 9.3, this means that the loading time (t_L) multiplied by the number of reactors (R_N) must be equal to the total cycle time of one reactor ($t_L + t_O + t_U$). This relationship can be expressed mathematically as follows:

$$t_L R_N = t_L + t_O + t_U, \tag{9.4}$$

where R_N is the number of reactors and t_L, t_O, and t_U are the loading time, processing time, and unloading time, respectively. If we assume equal loading and unloading times ($t_L = t_U = t_X$), then substituting t_L and t_U by t_X in (9.4) and solving for N_R we get

$$R_N = 2 + \frac{t_O}{t_X}. \tag{9.5}$$

According to (9.5), the minimum number of reactors for a continuous processing line operation is three ($R_N \geq 3$). In addition, if we take into account that the effective volume of a reactor is the loading time (t_X) multiplied by the volumetric flow rate of the processing line (F_P), then

$$V_R = t_X F_P. \tag{9.6}$$

Then, solving (9.6) for t_X and replacing it in (9.5) we get

$$R_N = 2 + \frac{F_P t_O}{V_R}; \quad \text{therefore} \quad R_N = f(F_P, t_O, V_R). \tag{9.7}$$

Carrying out some calculations, assuming $F_P = 1{,}000$ L/h and $t_O = 10$ h, and then substituting F_p and t_O in (9.7), we can determine the number of reactors (R_N) according to its volume (V_R), as shown in Table 9.2.

This procedure, developed for chemical reactors, can be easily adapted and extended to other processes, like winemaking, canned food production, and batch dehydration.

Table 9.2 Reactors volume as a function of reactors number

Reactor volume, V_R (L)	Number of reactors, R_N
10,000	3
5,000	4
2,500	6
2,000	7
1,000	10

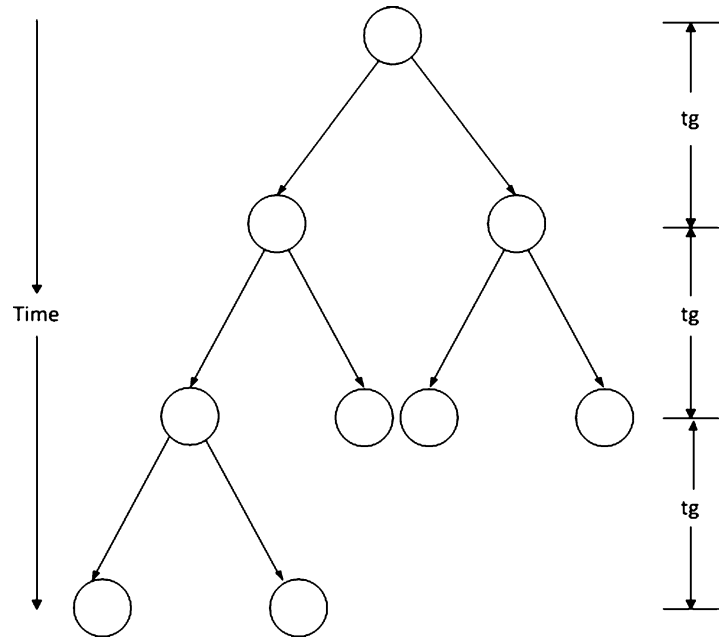


Fig. 9.4 Schematic representation of cellular division

Table 9.3 Bacteria population against time

Time	Bacteria population
0	$N_0 = 2^0 N_0$
$1t_g$	$2N_0 = 2^1 N_0$
$2t_g$	$4N_0 = 2^2 N_0$
$3t_g$	$8N_0 = 2^3 N_0$
...	...
...	...
nt_g	$N = 2^n N_0$

Modeling cell growth

Unicellular organisms (e.g., bacteria) duplicate in a process called cell division. Each cell divides into two new cells within a certain time, called the generation time (t_g). Schematically, we can represent this division process as depicted in Fig. 9.4.

Therefore, if we have N_0 bacteria at time = 0, then after one generation ($1t_g$) we will have $2N_0$ bacteria, and after two generations ($2t_g$) we will have $4N_0$ bacteria, and so on, implying that after each generation time, the bacterial population is doubling. Putting this information in tabular form, we get Table 9.3.

In addition, the number of generations (n) is related to time (t) and generation time (t_g) as follows:

$$n = \frac{t}{t_g}\left(\text{for } t = 0, \quad n = 0; \quad \text{for } t = 1t_g, \quad n = 1, \text{and so on}\right). \tag{9.8}$$

Then substituting n from (9.8) in the expression $N = 2^n N_0$, we get

$$N = 2^{\frac{t}{t_g}} N_0. \tag{9.9}$$

Now arranging (9.9) we obtain

$$\ln\left(\frac{N}{N_0}\right) = \frac{t}{t_g} \ln 2. \tag{9.10}$$

By definition, $ln2/t_g = \mu$ (specific growth rate); thus the number of bacteria after time t can be expressed by an exponential equation as follows:

$$N = N_0 \, e^{\mu t}. \tag{9.11}$$

According to (9.11), bacterial cell division obeys an exponential growth curve. As you will see in future courses, this simple mathematical model will be very useful when designing batch bioreactors and also in multiple practical applications in food science, environmental engineering, biological engineering, and biotechnology.

9.4 The Importance of Simulations in Chemical and Bioprocess Engineering

So far, we have provided examples of the importance of mathematical modeling, and although the examples were simple, we have been able, with rudimentary mathematical tools, to construct some mathematical models that are useful in chemical and bioprocess engineering. In addition, as mentioned and explained in Sect. 9.1, simulation can help us gain a better understanding of our system and determine which variables are most important within the mathematical model. Simulations also help us to analyze different scenarios and in this way get experience. Normally, it is thought that experience is only gained by practice. However, with a good mathematical model, through simulation we can learn a lot about our system and gain experience. And as you know, experience is the mother of the sciences.

As has been common throughout the book, we will illustrate the usefulness of simulation through examples.

Energy conservation

As a new engineer in a processing plant, you are assigned to investigate the benefits of adding insulation to a wall (stainless steel, 0.02 [m] thick, 8 m^2 of area) of a piece of equipment that is operating at 100 °C (ambient temperature is 20 °C). According to your boss, the company is wasting too much energy, and it might be profitable and environmentally friendly if the wall were insulated

Table 9.4 Heat losses against fiberglass thickness

Fiberglass thickness, L (m)	Heat losses, Q (W)
0	480,000
0.01	2,420
0.02	1,213
0.03	810
0.04	607
0.05	486
0.06	405

with some good insulator material (e.g., fiberglass). First, you take a look at your notes (Chap. 6, Table 6.2 and equations (6.16) and (6.17)) and decide to calculate the actual losses (without insulation) and then run a simulation to evaluate the heat losses under different fiberglass thicknesses. According to your notes, you estimate the actual heat losses as follows.

The heat losses (Q) for a wall under steady-state conditions can be estimated using the following equation:

$$Q = A\frac{k\Delta T}{L}, \tag{9.12}$$

where, in this case,
$A = 8\ \text{m}^2$, $k_{\text{STAINLESS STEEL}} = 15\ \text{w/m}\ ^\circ\text{C}$, $\Delta T = 80\ ^\circ\text{C}$, and $L = 0.02\ \text{m}^2$; then
$Q = 480{,}000\ \text{W}$.

Now, from equation (6.17), we can write an expression for heat losses of a composite wall including fiberglass as

$$Q = A\frac{\Delta T}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}. \tag{9.13}$$

$k_{\text{FIBERGLASS}} = 0.038\ \text{W/m}\ ^\circ\text{C}$, then substituting ΔT, L_1, k_1, and k_2, we get

$$Q = 8\frac{80}{\frac{0.02}{15} + \frac{L}{0.038}}\ \text{W/m}\ ^\circ\text{C}.$$

Of course, if $L = 0$ (no fiberglass), then $Q = 480{,}000$ W. Table 9.4 shows the heat losses (Q) for different fiberglass thicknesses (L).

From Table 9.4 we can reach at least two important conclusions. First, the addition of an insulator is extremely effective in reducing heat loss. For example, the addition of 1 cm (0.01 m) of fiberglass reduced the heat losses almost 200 times (from 480,000 to 2,420 W). Second, although the addition of more fiberglass reduces heat loss, its effectiveness decreases as the thickness increases. From these statements we can infer that in similar processing situations, it is highly convenient to add an insulator, but the remaining question is: what is the optimum thickness? In Sect. 11.8, in problem 31, you will be challenged to find an optimum insulator thickness.

Simulation is the expression of mathematical modeling. As mentioned at the beginning of this section, simulation helps us to gain experience and allows us to optimize.

9.5 Why Do Automatic Process Control?

9.5.1 Management of Disturbances

Using the example of the shower from Chap. 3 (Sect. 3.2), we can ask ourselves what would happen if, after we got the water temperature to just the right setting, someone turned on cold water elsewhere in the house. The flow of cold water in the shower would probably decrease. Because the hot water comes through a different line, it would not decrease, and so the shower would get hotter. In this case, at home, the problem is not as critical because we can just step away from the hot water and adjust the temperature. However, in a process plant, emptying a pond might not be possible, or overheating could pose a fire hazard. Formally speaking, events that change features in materials entering a process are called perturbations. All processes are affected by them, whether the changes involve water flows, gases, vapors, fuel, coolant, temperatures, compositions, or what have you.

To take it to the industrial level, imagine a liquid that needs to be received and stored at 20 °C, but the actual temperature in the tank is 30 °C. We might think of passing the liquid through a radiator to cool down the temperature by 10 °C; this would solve the problem of the storage temperature. Something closer to reality is that the liquid would leave the tank at a temperature ranging between 25 and 32 °C, and because the temperature varies during the day, or because water is being added at a different temperature in winter than in summer, or because there are other water consumption needs at the plant, we would receive less water flow, for example. Then, when the water temperature is cooled by 10 °C, we store it at a temperature ranging between 15 and 22 °C. There might even be an extreme situation where the liquid leaves the tank at a temperature between 10 and 30 °C, when it sometimes needs to be cooled and sometimes requires heating. The treatment of disturbance is a major issue in the automatic control of processes. The control task is to ensure that, regardless of the inlet temperature of the liquid into the tank, it comes out at the desired temperature, i.e., 20 °C.

9.5.2 Maintaining Optimum Operating Conditions

Another critical aspect is to keep the process running under favorable conditions, which might mean maximizing production, minimizing energy consumption, or attaining some other desired outcome. Suppose there is a chemical reactor where a reagent A becomes a valuable product B. Furthermore, it is known that the higher the temperature of the reactor, the larger the amount of B will be produced. In the first instance, you could say that it is best to keep the temperature as high as possible. However, the more you want to heat, the greater will be the cost of heating. So there is a tradeoff; perhaps the most preferable situation would be to have a temperature that ensures a good supply of B, but without excessive consumption of heating oil.

Figure 9.5a shows that the higher the temperature, the greater will be the revenue generated by the increased production of B, but higher temperatures will bring higher heating costs. Figure 9.5b shows the difference between the revenue and the costs, i.e., the profits.

As depicted in Fig. 9.5b, at a temperature of 10 °C, there are no heating costs, but also not much production of B. At temperatures above 103 °C, the cost of heating exceeds the revenue received from the production of B and then it is no longer profitable to produce. As depicted in Fig. 9.5b, the optimum temperature is 70 °C. These three conditions are marked by an *X* in both graphs. Maintaining the temperature as close as possible to 70 °C will be the task of automatic control, i.e., any time the reactor temperature deviates from 70 °C, the system will be brought back to continuously operate at 70 °C.

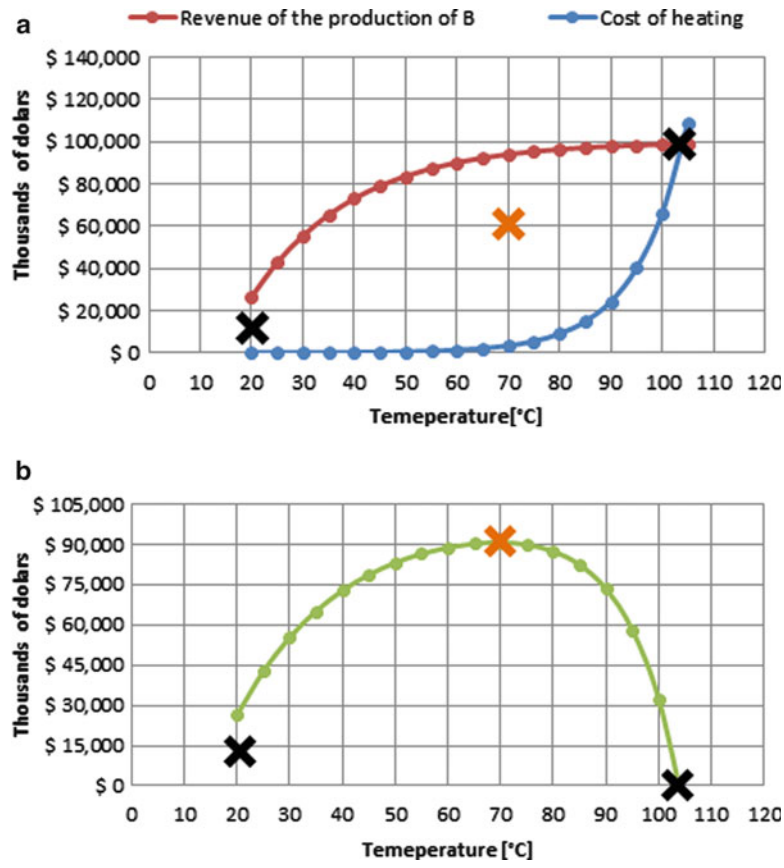


Fig. 9.5 (**a**) Revenues and costs of production of B. (**b**) Profits in production of B

9.5.3 Decreased Variability in Bottlenecks

There are cases where a suitable process control can even increase the overall performance of the process, especially in production processes working with bottlenecks. Such is the case of a factory that makes cereal bars where the product labels indicate a fiber content of 4 %. Given that the machines at the factory are not well controlled, the addition of fiber has a variability of ±2 %. In fact, the machines, programmed to add 4 % fiber, deliver a product that has between 2 and 6 % fiber. Therefore, to ensure that the products comply with the information on the label, 6 % fiber must be added and all bars are expected to have between 4 and 8 % fiber.

Suppose that, due to supplier constraints, the plant only has 100 kg of fiber a day and each cereal bar weight 100 g. Whereupon, under current conditions, the processing plant will be able to produce 16,667 cereal bars day. The situation would be very different if we could improve the control of the machines so that the error in the added amount of fiber were ±1 % instead of ±2 %, as shown in the center of Fig. 9.6. In this case, you could add 5 % fiber and expect the bars to have between 4 and 6 % fiber. It is easy to calculate that in this new scenario, the plant could produce 20,000 cereal bars daily. Although not conclusive, it seems attractive to invest in improving the control system.

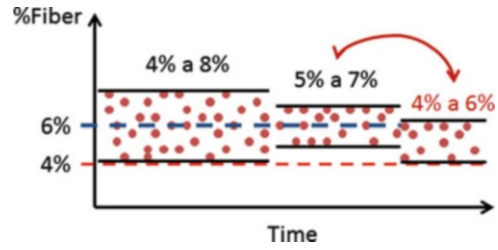


Fig. 9.6 Error in the amount of fiber against time

9.6 Control Strategies

9.6.1 PID Controller

The simplest control algorithm to calculate the compensating action of the variable is called proportional control. It is the first component of a more complex algorithm called a proportional integral derivative. As its name suggests, the command to the actuator is calculated in proportion to the error, as shown in the following equation:

$$\Delta c(t) = \mathrm{Kc} \times e(t), \tag{9.14}$$

where Kc is the proportional constant of the controller expressed as a %/bar and $e(t)$ is the negative feedback error (bar).

Thus, substituting (9.14) in (3.4) gives

$$c(t) = \mathrm{ce} + \mathrm{Kc} \times (\mathrm{PSP} - \mathrm{Pm}). \tag{9.15}$$

To have a better appreciation of the controller behavior, imagine that an operator that uses compressed air suddenly decreases its consumption. At steady state, the input mass flow rate of air equals the output mass flow rate of air, and there is no mass change in the storage tank. However, a decrease in the output mass flow rate of air will cause a rise in the tank pressure (input air $>$ output air). To do some simple calculations, assume the following information: Kc $= 0.5$ %/bar, ce $= 50$ %, and a set point of 80 bar. Then, using (9.15), we can estimate the progress of the process as follows:

Time 0: the system is in steady state, and the measured pressure is equal to the desired state. Therefore,

$$c(0) = 50 + 0.5 \times (80 - 80) = 50 + 0 = 50. \tag{9.16}$$

Time 1: a perturbation is introduced raising the pressure to 85 bar, and the controller responds by closing the inlet valve 2.5 %, as shown by (9.17):

$$c(1) = 50 + 0.5 \times (80 - 85) = 50 - 2.5 = 47.5. \tag{9.17}$$

Time 2: the pressure is lowered to 82 bar but still exceeds the desired pressure. So the controller closes the valve in 1.0 % increments, as shown by (9.18):

$$c(2) = 47.5 + 0.5 \times (80 - 82) = 47.5 - 1.0 = 46.5. \tag{9.18}$$

Table 9.5 Summary of the data of the estimated progress of the process

t (s)	P_m (bar)	e (bar)	Δc (%)	c (%)
0	80	0.0	0.0	50.0
1	85	−5.0	−2.5	47.5
2	82	−2.0	−1.0	46.5
3	79	−1.0	+0.5	47.0
4	81	0.5	−0.5	46.5
5	80	−0.5	0.0	46.5
6	80	0.0	0.0	46.5
7	80	0.0	0.0	46.5

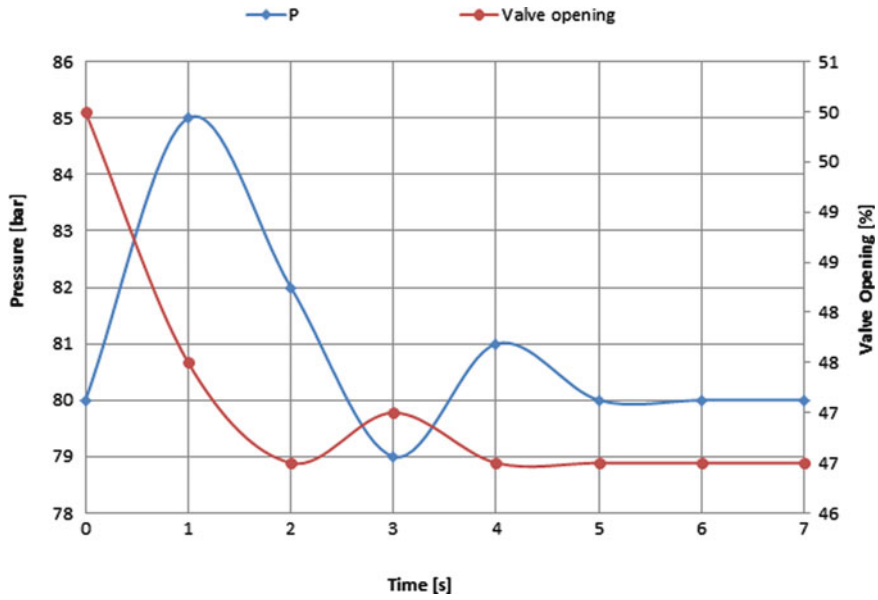


Fig. 9.7 Behavior of binary signal to control valve

Time 3: after closing the valve twice in quick succession, the pressure is now at 79 bar. As the pressure is now less than 80 bar, the controller responds by closing the outlet valve by 0.5 %, as shown by (9.19):

$$c(3) = 46.5 + 0.5 \times (80 - 79) = 46.5 + 0.5 = 47.0. \tag{9.19}$$

Time 4: with the outlet valve closed, the pressure has risen to 81 bar, again over the desired pressure, so the controller opens the valve 0.5 %:

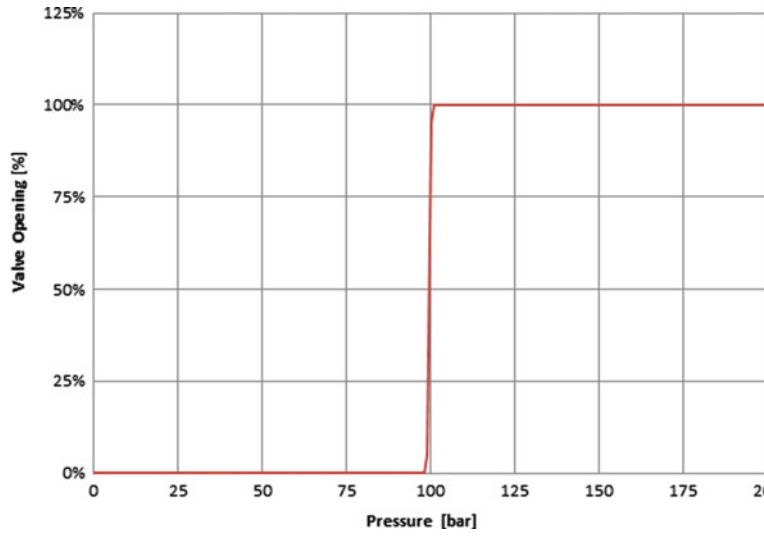
$$c(4) = 47.0 + 0.5 \times (80 - 81) = 47.0 - 0.5 = 46.5. \tag{9.20}$$

Time 5: since the last adjustment, the pressure is balanced at 80 bar, so the controller does not perform any more corrections:

$$c(5) = 46.5 + 0.5 \times (80 - 80) = 46.5 + 0.0 = 46.5. \tag{9.21}$$

Table 9.5 summarizes the data of the estimated progress of the process. The same data are depicted in Fig. 9.7.

Fig. 9.8



Unlike what is shown in Table 9.5, strictly speaking, we should mention that a pure proportional control cannot reach the set point with an accuracy of 100 %; it only reaches a value close to the set point, where the remaining difference is known as the offset.

In the search for greater efficiency in the ability of controllers to control processes, the proportional term is complemented by an integral expression and another, derivative, expression. In future courses, you will learn about this improvement and about controller tuning.

9.6.2 On/Off Controllers

The valve actuation PAHv-102 is governed by a binary control logic PAH programmed in controller-102. Binary means that there are only two possible states of the actuator, open or closed. As mentioned previously, the valve will open to 100 % if the pressure inside the tank E-102 becomes equal to or greater than 100 [bar]; otherwise, the valve will remain open 0 % (or 100 % closed). This can be represented as a piecewise function, composed as follows:

$$\text{if} \begin{cases} \text{Pressure} < 100 \text{ bar then valve is closed} \\ \text{Pressure} \geq 100 \text{ bar then valve is open} \end{cases}$$

In graphical terms, the behavior of the valve is depicted in Fig. 9.8.

9.7 Multivariable and Supervisory Control

In the examples of controlled processes that we have been discussing, we have worked with systems handling one resource and with one target variable, i.e., single-input/single-output (SISO) systems. However, in actual practice, the process is affected by multiple input variables and has multiple target variables, i.e., a multiple-input/multiple-output (MIMO) system.

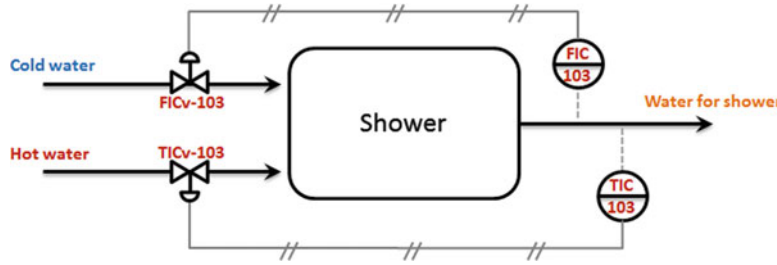


Fig. 9.9 Control loop for the shower system

9.7.1 Multivariable Control

Recall the example of the shower in Sect. 3.2. The action was the ability to change the cold water flow and the target the shower temperature (more specifically the pleasant feeling on the skin). But there may be situations in which, although the temperature is right, one cannot take a shower because the water flow is too low or too high. Implicitly there appears a second objective, which is adequate water flow. Intuitively, you can put together two control loops as indicated in Fig. 9.9.

Suppose that the temperature is fine, but the water flow is less than desired. If FIC-103 increases the cold water flow, it will increase the total water flow; however, the water temperature will go down. Consequently, TIC-103 will detect that the temperature has decreased and will try to compensate by increasing the flow of hot water. Of course, adding more hot water will lead to an increase in the total flow, but if both drivers (FIC-103 and TIC-103) are well tuned, the process will end with both converging to the desired values.

The problem that occurs when a control loop disturbs the stability of the other control loop is called coupling, and it can be a difficult problem to address.

Alternatively, one might try controlling the total flow by manipulating the hot water flow and the temperature by manipulating the cold water flow. Given multiple options for designing the control loops, the choice is usually based on specialized techniques. When variables cannot be linked, to avoid important couplings, uncouplers can be used. Uncouplers are elements that compensate actions on the loops in which no action has been taken.

9.7.2 Supervisory Control

Over time, new ways of managing more complex processes have been developed. Attempts have been made to go beyond the limitations of a PID controller, including staff experience in supervising process control systems. This does not necessarily involve a phenomenological model of processes because what is sought is a set of rules based on experience.

A supervisory control system could collect information on the flow and water temperature of the shower and, in concert, change the amount of cold water and hot water to meet both objectives simultaneously. The system is described in Fig. 9.10.

TT-106 and FT-106 transmitters report and record the state of target variables and instruct the system to define the desired flow of hot and cold water. Several variants of supervisory controls exist, but it is commonplace to look for a system that uses fuzzy logic to determine the changes in the set point of the process loops. An example of a fuzzy classification of the shower case is as follows:

In the case of temperature:

Less than 35 °C is a low temperature.

Between 35 and 45 °C is a pleasant temperature.

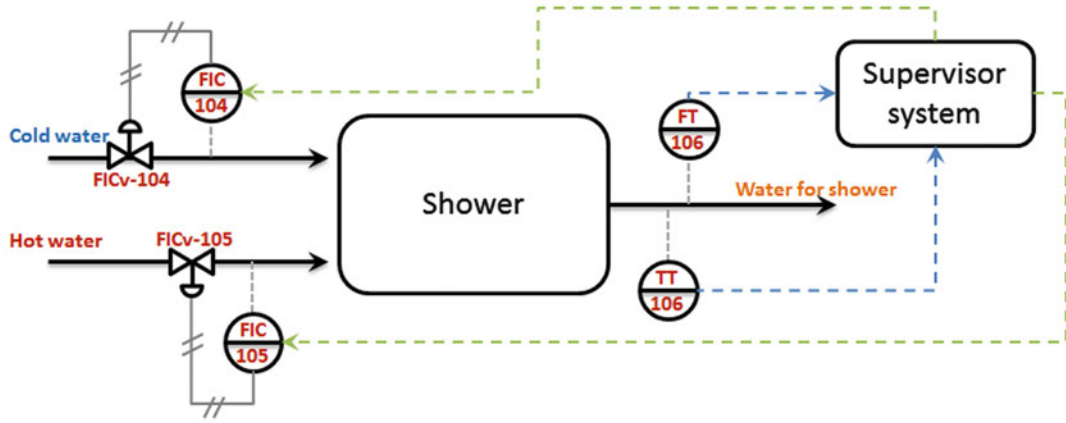


Fig. 9.10 P&ID system with supervisory control

Table 9.6 Rules for supervisory system

		Temperature		
		Low	Desired	High
Flow	Low	H: Increase C: Maintain	H: Increase C: Increase	H: Maintain C: Increase
	Desired	H: Increase C: Decrease	H: Maintain C: Maintain	H: Decrease C: Increase
	High	H: Maintain C: Decrease	H: Decrease C: Decrease	H: Decrease C: Maintain

H: action to be taken on hot water flow
C: action to be taken on cold water flow

Greater than 45 °C is a high temperature.
In the case of water flow:
Below 8 L/min indicates low flow.
Between 8 and 12 L/min indicates desired flow.
Greater than 12 L/min indicates high flow.

Once the status of the process performance is defined, the actuator may be defined as described in Table 9.6.

The supervisory system can achieve a better response in terms of speed and stability. As an example, if the temperature is acceptable but the flow rate is very low, the supervisory system simultaneously increases the flow of hot and cold water, achieving a greater flow but without changing the temperature. Another example is where the temperature is too high and the total flow too low. In that case, the system will maintain the hot water flow and increase the cold water flow; this would result in a lower temperature and a higher flow, favoring both goals at once.

9.8 Proposed Questions

1. Give one good reason why you should install an automatic control system in a process.
   A: The equipment is designed to operate effectively under certain specific conditions, such as at a certain temperature or concentration in the flows. However, a raw material might come to the equipment at a different temperature or a different concentration than those assumed in the

design. Therefore, to meet the required specifications, it is essential to fit automatic controls that are capable of minimizing such disturbances and obtain a regular product, independent of the input variations.

2. Why is it important to include the integral term in a PID controller?
   A: Because each time the process deviates from the desired set point, this is the only way to effectively reach steady state with zero error between measurement and control reference values.
3. How poorly will a nontuned PID controller function?
   A: Tuning is key for determining how well and robustly (meaning the ability to operate under a wide variety of conditions and disturbances) a controller is performing. Tuning determines the controller response time, or how fast it can reach steady state when activated or following disturbances in the system, and the controller's ability to minimize steady-state error.

   Visit the following Web site for more information on different types of fedback control. https://newton.ex.ac.uk/teaching/resources/CDHW/Feedback/ControlTypes.html
4. What is the basic idea behind supervisory control and what kind of advantage does it have over traditional PID controllers?
   A: Supervisory controls seek to mimic the behavior of human supervisors who have had a lot of experience with the process, translating their knowledge into a set of logical rules to be performed each time the process requires it. The advantage of supervisory controls over traditional PIDs is that they allow for coordinated use of more than one resource at a time, bringing a process to a desired condition rapidly and efficiently.
5. What is an ON/OFF controller and what is its primary use?
   A: In a given scenario, a resource will be fully closed, but in all other scenarios the resource will be fully open (or vice versa). For example, in the case of a pressure relief valve on a pressurized tank, in normal conditions, the relief valve operates fully closed. But if for some unforeseen reason the pressure exceeds the safety limit, then the controller will open the valve completely, releasing the pressure inside the tank. One of the main uses of this type of control is in its security mechanisms due to its discrete nature.

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PID Control - A brief introduction https://www.youtube.com/watch?v=UR0hOmjaHp0

Scale-Up in Chemical and Bioprocess Engineering

10[2]

Scientists dream about doing great things. Engineers do them.

James A. Michener

Engineering is a great profession. There is the satisfaction of watching a figment of the imagination emerge through the aid of science to a plan on paper. Then it moves to realization in stone or metal or energy. Then it brings homes to men or women. Then it elevates the standard of living and adds to the comforts of life. This is the engineer's high privilege.

Herbert Hoover

10.1 Introduction

Scale-up in the chemical and bioprocessing industries is the process of applying knowledge acquired at the laboratory scale, e.g., in test tubes, Petri dishes, flasks, small reactors, and laboratory-scale fermenters, to a large scale, e.g., in pipes, filters, reactors, and fermenters, in an efficient and economical way. Scale-up is an economic, engineering, and scientific discipline (Heitmann and Rhees 1984). Although the practical application of scaling up emerged only after the First World War, in 1912, a prominent chemical engineer, J. Whiting, defined the scaling-up process as follows: (1) laboratory stage, (2) small-scale model, (3) large-scale or development unit, (4) semicommercial plant, and (5) commercial plant (Heitmann and Rhees 1984). Since Whiting's definition first came out, the field has seen many advances, and the scaling-up process has been systematized. Even the field of chemical engineering has evolved and today includes bioprocess engineering. Advances in scale-up transformed the chemical industry of the twentieth century and, in addition, led to the creation and development of the bioprocess industry of our era.

Penicillin production

Perhaps the history of the development of penicillin symbolizes a big change in the chemical engineering profession in many ways, including the scaling-up process. It is a gripping story and marks the birth of biochemical engineering, perhaps better called bioprocess engineering. According to the historical background, the person who first noticed the presence of penicillin was not Alexander Fleming but the French medical student Ernest Duchesne, in 1896. Alexander Fleming rediscovered penicillin in St. Mary's Hospital in London in 1928. Fleming noted that a culture of *Staphylococcus aureus* in a Petri dish was contaminated with a blue-green mold (*Penicillium notatum*), and

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staphylococcus stopped growing in the vicinity of the fungus. Alexander Fleming published his findings in 1929 and called the substance *penicillin*. Although Fleming was the first to observe and speculate on the pharmaceutical potential of penicillin, it would take 14 years to finally translate into a product on an industrial scale. Why did it take so much time? Partly because it was a biochemical process, and, although Fleming understood clearly how to produce small amounts of penicillin on a laboratory scale, it was unclear how to achieve industrial-scale production and, in addition, make it economically profitable. Another situation that complicated scaling up was that it took years to be absolutely certain that penicillin was an effective drug. Its effectiveness began to be clarified mainly by the work of Howard W. Florey and his research group in 1939. In these studies, Florey and his group were able to produce sufficient amounts of penicillin to be tested in mice. Then Florey and a colleague, Norman G. Heatley, traveled to the USA to raise funds and pursue industrial-scale production. Although it took years, a number of companies with strong government support began producing penicillin in 1943 and on a larger-scale in 1945. In fact, penicillin was used as a medicine on soldiers who participated in D-Day (Heitmann and Rhees 1984).

Among other achievements, the extremely successful history of the scaling up of the production of penicillin resulted in a big scaling down of its price from $20 per dose in 1943 to just $0.55 per dose in 1946, an almost 40-fold reduction. Today, the price of a pill is in the range of US$0.10–$0.20. In addition, in the early 1940s, the term *biochemical engineering* was coined. Since then, bioprocess engineering has emerged as a potent field.

In 1945, Sir Alexander Fleming, Ernst B. Chain, and Sir Howard W. Florey were awarded the Nobel Prize in Physiology or Medicine *"for the discovery of penicillin and its curative effect in various infectious diseases."*

10.2 Understanding Size Change

We are used to watching, in movies, superheroes, giants that seem like us (in terms of their physical proportions), and enormous animals (e.g., King Kong, Godzilla), which often are extremely large—many times their real size in nature. To take a concrete example, most of us are probably familiar with *Gulliver's Travels* of Jonathan Swift (1667–1745). In the first part of the novel, the narrator describes the life of Lemuel Gulliver and how he enjoys traveling. Lemuel Gulliver takes his first trip to the island of Lilliput (1699–1702), where the inhabitants, the Lilliputians, measure just 6 [in.] tall, approximately 12 times smaller than us. Initially, Gulliver is welcome on the island, but with the passage of time, he is accused of treason for not obeying the orders of the King and is sentenced to be blinded. Fortunately, with the help of a friend he manages to escape the island in a boat, is later rescued by a ship that sails these waters, and, finally, returns home safely. After returning home, he embarks on a new trip, now on the *Adventure* ship. Later, the ship is caught in a terrible storm and runs aground, where his companions abandon him in Brobdingnag. There he is rescued by a peasant. In Brobdingnag, the people are 12 times bigger than us. By Gulliver's estimates, the inhabitants are about 18.1 m. The farmer takes him home and leaves him in the care of his daughter. Of course, the peasant put him on exhibit as a curiosity and charges viewers for it. Although the novel continues and is enjoyable and full of irony, we will stop here and analyze mathematically the short paragraph that we just read and also ask ourselves some questions.

Does size matter?

Galileo Galilei (1564–1642) was an Italian astronomer, mathematician, philosopher, and physicist. In 1638, he wrote a book entitled *Dialogues Concerning Two New Sciences,* wherein he explains and analyzes why objects cannot be of just any arbitrary size and clarifies why there are no giants like in

Gulliver's Travels. Evidently, there is no gorilla like King Kong towering 18.5 m tall (like the inhabitants of Brobdingnag). Galileo might have been the first person to analyze the topic of scaling in a scientific way. Nowadays, scaling is utilized in many branches of science and engineering, from anatomy to process and bioprocess engineering.

But why are there no giants like King Kong or Godzilla?
For simplicity, if some animal were shaped like a regular cylinder with a height of H meters and a diameter of D meters, then its volume would be

$$V = \frac{\pi D^2}{4} H. \tag{10.1}$$

If we increase the size of our animal ten times, from H to $10H$ and from D to $10D$, then the volume of this little giant (V_G) would be

$$V_G = \frac{\pi (10D)^2}{4} 10H = 1,000 \frac{\pi D^2}{4} H = 1,000V. \tag{10.2}$$

That is, the linear dimensions, height and diameter, increased ten times (the animal is ten times bigger) but the volume (weight) increased 1,000 times. Before doing any anatomical analysis, we can speculate that the bones of this little giant would break under the weight of the body because the new animal would not be 10 times heavier, as it is 10 times bigger, but 1,000 times heavier. Doing some simple calculations and assuming that the bone resistance (strength) is approximately proportional to the area of the bone's cross section, and choosing a diameter d for the bone, we have

$$\text{Original cross section } (A) : A = \frac{\pi d^2}{4}, \tag{10.3}$$

$$\text{Final cross section } (A_G) : A_G = \frac{\pi (10d)^2}{4} = 100 \frac{\pi d^2}{4} = 100A. \tag{10.4}$$

As mentioned, the little giant is 10 times bigger than the original one, but its volume increased 1,000 times and the bone strength increased just 100 times. This little giant would almost surely collapse. The volume increased very rapidly (by L^3)—ten times more than the area (which increased by L^2).

Gazelles are antelopes of the genus *Gazella*, and they are graceful and beautiful animals. One of their main characteristics is their speed; they attain maximum speeds of almost 100 [km/h]. But, as was mentioned and explained earlier, they could not be much bigger than they already are unless (a) they had thicker legs like those of a rhinoceros or (b) they had a compressed body with oblique legs to gain stability like giraffes. The interesting thing is that rhinoceroses and giraffes are of the same order as gazelles. In addition, rhinoceroses and giraffes also attain high running speeds (50–70 [km/h]), much quicker than the world's fastest human, Usain Bolt (approximately 37.6 [km/h]).

Why can a wet fly not fly?
In summer, many people like to go swimming in a pool and go off the diving board. Once out of the water, you might shake some of the excess water off your body before using your towel. All this comes naturally to us but can be a nightmare for a fly. As you may know, flies cannot fly when they are wet; they have to crawl. Why?

An interesting concept in scaling (comparing different sizes) is to take into account the surface-area-to-volume ratio (A/V). As we will show later in this chapter, this concept is very important when, for example, scaling heat transfer processes, reactors, and fermenters. In the specific case of a fly, we will estimate the amount of water that a human being and a fly carry with them out of a pool as a proportion of body weight. Again, we will use a regular cylinder as the reference shape, in this case for both fly and human. For the human being we will use a regular cylinder of 0.250 [m] diameter and a height of 1.60 [m] (approximately 80.0 [L]) and the fly will be 0.0200 [m] (2.00 [cm]) long with a diameter of 0.00500 [m] (5.00 [mm]). Now we are ready to calculate the volume of the human being and of the fly before going to the pool, but to calculate the water they carry, we need to define the thickness of the layer of water for both human and fly. Let us assume a value of the thickness layer, as a rough calculation, and then estimate what percentage of the body weight is carried out when both human and fly get out of the pool. Let us assume that the thickness of the layer is d.

Human being

$$\text{Before} : V = \frac{\pi(0.25)^2}{4} 1.6 = 0.0785 \left[\text{m}^3\right] = 78.5 \text{ [L]}; \tag{10.5}$$

$$\text{After} : V_{\text{W}} = \frac{\pi(0.25 + 2d)^2}{4} (1.6 + 2d). \tag{10.6}$$

If $d = 1$ [mm] $= 0.001$ [m], then the amount water that the human being will carry with him as a percentage of his volume is

$$100 \frac{V_{\text{W}} - V}{V} = 100 \frac{79.9 - 78.54}{78.54} \cong 1.73 \text{ \%}. \tag{10.7}$$

Clearly, the human being can easily shake off all the water. He just carries a small amount of water in relation to his volume (proportional to weight).

Fly

$$\text{Before} : V = \frac{\pi(0.005)^2}{4} 0.02 = 0.000393 \text{ [L]}; \tag{10.8}$$

$$\text{After} : V_{\text{W}} = \frac{\pi(0.005 + 2(0.001))^2}{4} (0.02 + 2(0.001)) = 0.000847 \text{ [L]}. \tag{10.9}$$

Thus, the amount water the fly will carry as a percentage of its volume is

$$100 \frac{V_{\text{W}} - V}{V} = 100 \frac{0.000847 - 0.000393}{0.000393} \cong 115.5 \text{ \%}. \tag{10.10}$$

Obviously, the human being will have a much easier time getting out of the pool. The fly, on the other hand, will have a very hard time getting out. Can you imagine carrying 80–90 [kg] of water?

Some may argue that the assumption of the thickness of the water layer is not accurate, but before considering this, let us try to understand is happening in this "scaling" situation (fly to human). As mentioned, the critical concept here is the surface-area-to-volume ratio. Let us calculate this parameter for both the human being and the fly:

$$\text{Human being}: \left(\frac{A}{V}\right)_{\text{Human}} = \frac{\pi DH + 2\left(\frac{\pi D^2}{4}\right)}{\frac{\pi D^2}{4}H} = \frac{4}{D} + \frac{2}{H} = \frac{4}{0.25} + \frac{2}{1.6} = 17.3, \tag{10.11}$$

$$\text{Fly}: \left(\frac{A}{V}\right)_{\text{Fly}} = \frac{4}{D} + \frac{2}{H} = \frac{4}{0.005} + \frac{2}{0.02} = 900. \tag{10.12}$$

As suspected, the surface-area-to-volume ratio of the fly is much greater than the surface-area-to-volume ratio of the human being, in this example, approximately 52 times greater. You could argue about whether the data and the shape used as reference are strictly representative, but clearly, the smaller the object, the bigger the surface-area-to-volume ratio. Here are some examples of the surface-area-to-volume ratio of various three-dimensional bodies:

$$\text{Sphere}: \left(\frac{A}{V}\right)_{\text{Esphere}} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r}; \tag{10.13}$$

$$\text{Cube}: \left(\frac{A}{V}\right)_{\text{Cube}} = \frac{6a^2}{a^3} = \frac{6}{a}; \tag{10.14}$$

$$\text{Tetrahedron}: \left(\frac{A}{V}\right)_{\text{Tetrahedrum}} = \frac{\sqrt{3}a^2}{\frac{\sqrt{2}a^3}{12}} = \frac{6\sqrt{6}}{a}. \tag{10.15}$$

In all cases, as is normal, the smaller the object (a or r), the bigger the surface-area-to-volume ratio. An extreme example is the surface-area-to-volume ratio of a bacterium. Assuming a bacterium with a spherical shape with a diameter of 2.0 [μm], then

$$\text{Bacterium}: \left(\frac{A}{V}\right)_{\text{Bacterium}} = \frac{3}{r} = \frac{3}{2 \times 10^{-6}} = 1.5 \times 10^6. \tag{10.16}$$

It has a surface-area-to-volume ratio that is almost 1,700 times bigger than that of the fly!

The extremely high surface-area-to-volume ratio is one of the main reasons why microorganisms like bacteria are able to reproduce in just 20–30 min. Although two objects can be geometrically similar in terms of linear dimensions L, the area changes by L^2 and volume changes by L^3. A strong message here is that size really does matter. Several aspects of life are strongly affected by size, for example, metabolism rate and temperature regulation. As we will discuss later, in process engineering, you normally start working and doing research studies at the laboratory scale with the aim of having an end product at the industrial level. For example, if you are testing and experimenting with different types of wines, your laboratory-scale fermenter will have a volume of 1–5 [L]. Meanwhile, an industrial fermenter could have a volume of 50,000–100,000 [L], i.e., 10,000–100,000 times bigger than the laboratory-scale fermenter.

Does size confer advantages?
Like us, insects require oxygen to live and produce CO_2 as a waste product. For insects, the oxygen transport mechanism (respiration) is by diffusion through a series of tubes called the tracheal system, and it is clear that if the distances are short, the mechanism is efficient. But, interestingly, this mechanism significantly limits the size of insects, and it is hard to imagine that they could measure more than 2 [cm]. Returning to the subject of giant animals like King Kong but with a different mechanism, giant insects cannot exist, as is commonly portrayed in horror movies around the world.

One of the most obvious advantages of being human is that we can control and regulate temperature efficiently. In fact, the amount of food required is proportional to the transfer area of the body (remember, the bigger the body, the lesser surface-area-to-volume ratio). For example, 5,000 mice weigh around the same as one human (200 [lb]), but their demand for oxygen and food is approximately 70 times our demand. A mouse, for the sole purpose of controlling temperature, needs to consume in food one-fourth its body weight per day (which in our case would be about 15–20 [kg]). This is one reason why most small animals cannot live in icy environments or at low temperatures.

Socialization (management). Of all animals, humans have a capacity to socialize in larger groups, which might be explained by the ability of our brains to handle greater complexity. Dunbar's number (Robin Dunbar 1947–) is a theoretical cognitive limit on the number of people with whom humans can maintain stable social relationships. Although not necessarily an exact number, the number that has received the widest acceptance is 150. According to Dunbar's research, in examining 21 hunter-gatherer societies for which we have reliable historical information, the average size of the communities was 148.4 (approximately 150). This number appears in different settings of human groups—e.g., companies, combat battalions—and it seems that higher numbers require imposing a set of rules intended to manage the administration and organization of the group. Dunbar argues that management and organization in groups with fewer than 150 persons is optimal for the management and maintenance of a certain level of informality and familiarity.

10.3 Principle of Similarity

One of the important concepts for scaling up and down a piece of equipment and its operation from the laboratory to industrial scale is *the principle of similarity*. If we can find and optimize a design and operating conditions at the laboratory scale (model), it would be highly desirable to reproduce it on an industrial production scale. According to Johnstone and Thring (1957), "*Process similarity is achieved between two processes when they accomplish the same process objectives by the same mechanisms and produce the same product to the required specifications*." As we have previously mentioned, in the wine-making example, this involves increasing the size of our experiment at the laboratory scale by approximately 10,000–100,000 times or more. On the other hand, we found that when changing scale, it is not possible to retain all the features and characteristics of the object at one time. As an example, in our previous analysis, we discovered that when increasing size, it was not possible to maintain a constant surface-area-to-volume ratio, and the implications were substantial (the wet fly).

The most important stages of similarity in process and bioprocess engineering are (a) geometric similarity, (b) mechanical similarity, (c) thermal similarity, and (d) chemical and biochemical similarity.

Geometric similarity
Figure 10.1 depicts reactors/fermenters at the laboratory (model) and industrial scales.

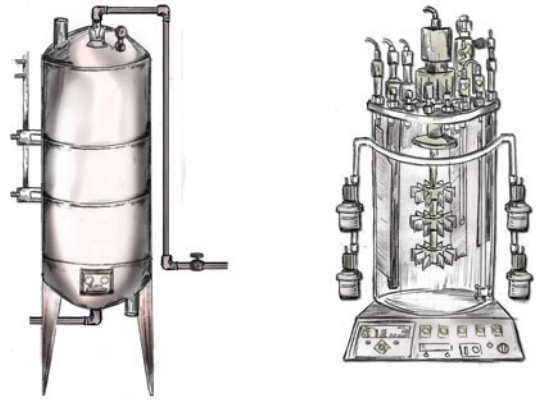

Fig. 10.1 Laboratory and Industrial fermenters

These reactors/fermenters will be geometrically similar if for example:

$$\left(\frac{H}{D}\right)_{\text{Model}} = \left(\frac{H}{D}\right)_{\text{Industrial}}. \tag{10.17}$$

Where H is the height and D the diameter of the reactor/fermenter. As expressed by Johnstone and Thrings (1957), "*two bodies are geometrically similar when to every point in the one body there exists a corresponding point in the other*."

Mechanical similarity

Mechanical similarity includes static, kinematic, and dynamic similarities. In addition, mechanical similarity is an extension of geometric similarity.

(a) **Static similarity.** Two geometrically similar bodies are considered statically similar when, subject to a constant stress, they undergo a deformation in such a way that they remain geometrically similar.

(b) **Kinematic similarity**. Kinematic similarity requires that both the length and time scales be similar. That is, the prototype and the model should have the same speeds at corresponding points. Therefore, the flow patterns between the model and the prototype will be similar. As mentioned with respect to static similarity, kinematic similarity also includes geometric similarity.

(c) **Dynamic similarity**. Dynamic similarity requires that the forces on the model and the prototype be similar at corresponding points. Again, dynamic similarity also includes geometric similarity.

Thermal similarity

Thermal similarity is related to heat flows. Temperature differences between two points at the same instant in time in the model must be equal to the temperature differences at the corresponding points at the same instant in the prototype.

Chemical and biochemical similarity

Chemical and biochemical similarities are associated with transformations due to chemical or biochemical reactions within the system. According to Johnstone and Thrings (1957), "*Geometrically and thermally similar systems are chemically similar when corresponding concentration differences bear a constant ratio to one another and when the systems, if moving, are kinematically similar*."

10.4 A Glimpse of Dimensional Analysis

Dimensional analysis is a method that facilitates the study of systems by reducing the number of independent variables in such a way that it is not affected by changes in scale. According to Bridgman (1969): "*The principal use of dimensional analysis is to deduce from a study of the dimensions of the variables in any physical system certain limitations on the form of any possible relationship between those variables. The method is of great generality and mathematical simplicity*." Dimensional analysis attracted serious attention in the late nineteenth century mainly through the work of Lord Rayleigh, Reynolds, Maxwell, and Froude in England and Carvallo, Vaschy, and other scientists in France (Sonin 2001).

There are two classical methods in dimensional analysis, *Buckingham's pi theorem* and the *method of indices* by Lord Rayleigh. Here we will briefly explain the more common of the two: *Buckingham's theorem*.

Buckingham's theorem. This theorem can be divided into three steps as shown below (Sonin 2001).

Step 1: define the dependent variable and find all the relevant independent variables. This is a critical and, normally, difficult task. A simple example will be to relate the distance covered (dependent variable) by a body in free fall before hitting the ground. Assuming that the air resistance is negligible, the two independent variables are time (t) and acceleration of gravity (g):

$$d = f(t, g), \tag{10.18}$$

where d is distance, t is time, and g is acceleration of gravity.

Step 2: identify the dimensions of the dependent and independent variables. In our example:

d: L,
t: t,
g: Lt^{-2}.

Step 3: "If an equation contains n separate variables and dimensional constants and these are given dimensionless formulas in terms of m fundamental dimensions, then the number of dimensionless groups in a complete set is n − m" (Johnstone and Thrings 1957). In our example, we have three variables, including g, a dimensional constant (d, t, and g), and we have two fundamental dimensions (L and t). Thus, we can form just one dimensionless group.

In our example:

$$d[g]^a[t]^b = 1. \tag{10.19}$$

Thus, $L\left[\frac{L}{t^2}\right]^a[t]^b = 1$; $L^{1+a}[t]^{b-2a} = L^0t^0$.

Therefore:

$$1 + a = 0,$$

$$b - 2a = 0.$$

Therefore, $a = -1$ and $b = -2$.

Finally, there is one dimensionless group:

$$\left[\frac{d}{gt^2}\right],$$

and (10.19) takes the form

$$d = f\left(g, t^2\right) = kgt^2. \tag{10.20}$$

From our basic knowledge of high school physics we know that

$$d = \frac{1}{2}gt^2. \tag{10.21}$$

10.5 Understanding Scale-Up (-Down) in Chemical and Bioprocess Engineering

We will show the relevance of scaling up/down with a vivid example. For this purpose and due to the importance that bioethanol has acquired, we will analyze, in some detail, so-called alcoholic fermentation. In addition, we will try to understand and quantify how the cooking time of a turkey is related to its size.

Alcoholic fermentation.
Ethanol is an oxygen containing organic compound, due to its characteristics as a solvent, germicide, antifreeze, depressant and a fuel (Asli 2009). In addition, since the energy crisis of the 1970s, and now with global warming concerns, alcohol fermentation has been the subject of the most important scientific efforts to obtain a sustainable and renewable energy source (Pramanik 2003, 2005). Alcoholic fermentation is the conversion of sugar into ethanol. Normally, alcoholic fermentation is carried out by yeast, *Saccharomyces cerevisiae,* in controlled operating conditions. As depicted in Fig. 10.2, *S. cerevisiae* reproduces in a similar way to bacteria, but with a significant difference: although from one cell a second cell emerges, they are not two new cells, as in bacteria, but mother and daughter cells. When the newborn cell emerges, it leaves a scar on the mother cell. The number of scars indicates the "age" of the mother cell.

Fermentation is a biochemical reaction in which the substrate (sugar) is converted into alcohol (ethanol), as presented in the following stoichiometry, in this case, the conversion of glucose into ethanol plus carbon dioxide:

$$C_6H_{12}O_6 \rightarrow 2C_2H_5OH + 2CO_2.$$

Strictly speaking, the reaction in a commercial fermenter is better expressed as:

$$\begin{aligned} &\text{Glucose (substrate)} + \text{Nutrients} + \text{Cells (catalyst)} \\ &\quad \rightarrow \text{Ethanol (product)} + CO_2 + \text{Cells} + \text{byproducts}. \end{aligned} \tag{10.22}$$

Alcoholic fermentation is an exothermic reaction (generating heat), and its kinetics (velocity of the reaction) depends strongly on temperature. Several factors affect the course of fermentation (e.g., pH, alcohol concentration, high salt concentration), but temperature has the greatest effect. Several research studies have shown that the optimum temperature for *S. cerevisiae* in alcoholic fermentation is close to 32 °C (Costa et al. 2009).

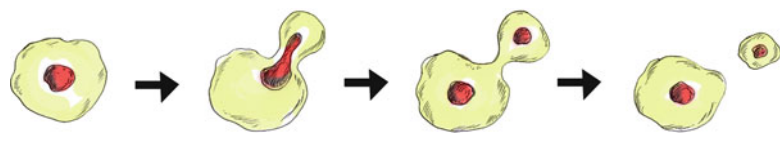

Fig. 10.2 Reproduction of *Saccharomyces cerevisiae*

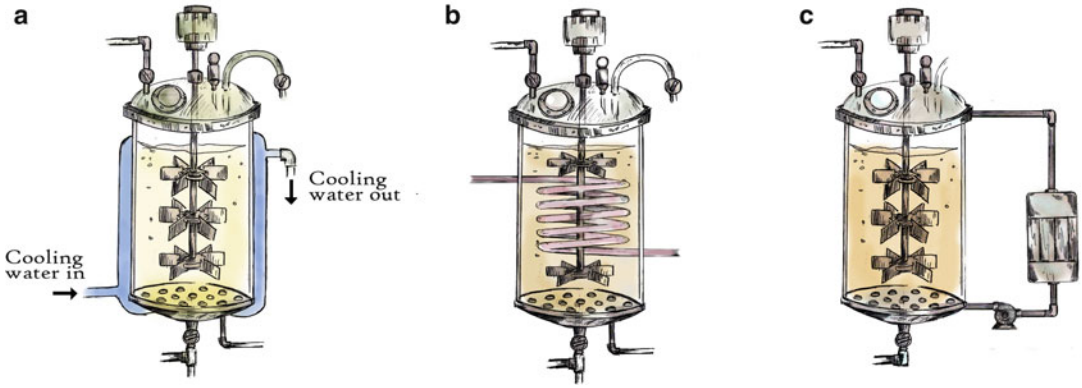

Fig. 10.3 Three different temperature-control systems: Jacket, Serpentine and External Heat Exchanger

Table 10.1 Comparison between a laboratory and industrial scale fermenter: heat transfer

	Laboratory fermenter	Industrial fermenter	Ratio industrial/ laboratory
Heat generation [energy units/h]	Q_F	100,000 Q_F	100,000
Heat transfer area [m^2]	0.016875π	36.5403π	~2,165
Heat generation per m^2 [energy units/h m^2]	~18.9 Q_F	~871.1 Q_F	~46.1

Laboratory and commercial fermenters. In industrial operations (fermenters from 50,000 to 100,000 [L]), to maintain a constant temperature during this exothermic process, heat must be removed. Normally, this can be done using (a) a jacket surrounding the vessel, (b) internal coils, or (c) an external heat exchanger. Figure 10.3 presents the schematic of a fermenter showing all three alternatives to control temperature during fermentation. In addition, to achieve a homogeneous culture—to maximize mass transfer—it is not necessary to implement mechanical agitation due to the strong agitation resulting from CO_2 release during fermentation.

On the other hand, at the laboratory scale (1–20 [L]), to maintain a constant temperature, instead of using a cooling device, as is done at the industrial scale, heat must be added. Also, to maintain the required homogeneity of the culture, it is necessary to perform mechanical agitation. The question is why it is necessary at the laboratory scale to add heat and perform mechanical agitation? Since we already know that alcoholic fermentation is exothermic and releases significant amount of CO_2, resulting in a strong agitation of the culture at the industrial level.

Some simple calculations

Heat transfer. To understand what is happening, we will consider for our analysis a laboratory reactor/fermenter with a volume of 1 [L] and an industrial fermenter with a volume of 100,000 [L]; the two fermenters are geometrically similar, i.e., $(H/D)_{\text{Laboratory}} = (H/D)_{\text{Industrial}} = 3$. In addition, the exothermic reaction will liberate Q_F (energy units/hL), where Q_F is independent of fermenter size and depends only on the type of microorganism. To estimate the heat transfer area for calculation purposes, we will consider in both laboratory and industrial scale cases, where the area is given by the mantle of the vessel (πDH). Based on the aforementioned data and our assumptions, we can generate the following table (Table 10.1).

Table 10.2 Comparison of laboratory and industrial scale fermenters: power of agitation

	Laboratory fermenter	Industrial fermenter	Ratio industrial/laboratory
Power of agitation [J/m^{2}s]	0.226 kF_G	10.45 kF_G	~64

As depicted in Table 10.1, while the amount of heat generated increases 100,000 times between the industrial to laboratory scale, the heat transfer area increases just 2,165 times. As mentioned, at the laboratory scale, the heat generated is not enough to maintain or increase the temperature of the culture, meaning that the heat released to the ambient environment is greater than the heat generated by the culture. On the other hand, at the industrial scale, the heat transferred to the ambient environment is lower than that generated during fermentation. The volume increased (proportional to heat generation) 100,000 times and the area just 2,165 times. Returning to the wet fly example at the beginning of this chapter, the key to understanding this unexpected situation is that the smaller the size, the bigger the surface-area-to-volume ratio. Interestingly, based on the aforementioned results, there should be a reactor/fermenter size (volume) where the heat generated is equal to the heat released to the ambient environment. In such a case, it is not necessary to remove or add heat. We encourage students to revisit Chap. 6 and, with the help of heat transfer equations, develop an expression for the volume of a reactor/fermenter that could maintain a constant temperature without cooling or heating the vessel.

Agitation. When a culture of *S. cerevisiae* transforms glucose into ethanol, it also generates CO_2 as a gas in the operating conditions of fermentation (approximately 30 °C and 1 atm). As mentioned, the gas flow generated by *S. cerevisiae* at the industrial scale is sufficient to violently agitate the culture, but, in contrast, at the laboratory scale it is not enough to achieve homogenization of the culture. Again, the question is why? Using the same fermenters as in the previous example, we will estimate the power of agitation due to CO_2 generation in each case: laboratory and industrial scale. We will call F_G [m^3_g/m^3_cs] the amount of gas [m^3] generated per cubic meter of culture per second, where F_G is independent of the size of the fermenter and depends mainly on the type of cells (*S. cerevisiae*) and the operating conditions of the fermentation (e.g., temperature). The power of agitation can be expressed as

$$P_g = k\frac{F_G V_F}{A_C}, \tag{10.23}$$

where P_g is the power of agitation [J/m^{2}s], k is constant [J/m^3_g], F_G is the flow of CO_2 per cubic meter of culture [m^3_g/m^3_cs], V_F is the volume of the fermenter [m^3_c], and A_C is the area of the cross section of the fermenter [m^2].

Therefore, we obtain the following table.

As shown in Table 10.2, the power of agitation at the industrial level is approximately 64 times greater when compared with the laboratory scale. This is why we observe severe agitation at the industrial level, and, on the other hand, if we do not perform mechanical agitation in the laboratory, cells will precipitate and the fermentation process will not progress accordingly.

Cooking a turkey.

Naturally, if we are to prepare a good and tasty turkey, it is logical to turn to the experts, or rather follow the advice and recipe of our charming grandmother. Looking at her old notes, we find the following recipe for stuffed and unstuffed turkey.

Of course, we are not in a position to question our grandmother's recipe, which for years has resulted in excellent turkeys that we have been sharing at family dinners on both Thanksgiving and Christmas. But Table 10.3 has called to our attention that when the turkey's weight doubles from 14

Table 10.3 Cooking times for turkey as reported in the old notes of our grandmother

Turkey mass [lb]	Cooking time (unstuffed) [h]	Cooking time (stuffed) [h]
14	3.2	4
20	3.8	4.8
28	4.8	6

to 28 [lb] the cooking time does not increase in the same way. Let us revisit the principle of similarity and dimensional analysis and make a robust determination about the great recipe of our dearly beloved grandmother.

Turkey cooking time analysis. First, we will focus on identifying the relevant variables that have the greatest impact on the cooking time of the turkey. Clearly, the first is its size, but how do we define size here? It could be the mass of the turkey (volume), but clearly more representative is the area exposed by the turkey to the environment in the oven. Why? Remembering our heat transfer equations (Chap. 6), the heat transfer rate (Q) is directly proportional to the area exposed by the body, its thermal properties (in this case thermal diffusivity), and the temperature difference between the oven and the turkey. Thus, we can identify five variables: cooking time (t), heat transfer area (A), thermal diffusivity (α), oven temperature (T_a), and the temperature at the turkey's center (T_T). As the last variable, it seems reasonable to assume that the turkey is cooked properly when the center temperature reaches some desired value (e.g., 70 °C). Following *Buckingham's pi* theorem—or *Rayleigh's method of indices*—we can reduce these five variables to two dimensionless groups:

$$\frac{T_T}{T_a} \text{ and } \frac{\alpha t}{A_T}. \tag{10.24}$$

Why? First, we invite students to confirm that these two groups are dimensionless and, in addition, to do the analysis to find them. As expected, because the dimensions in this system were completed by three primary quantities, L, T, and t, there should be two dimensionless groups (five variables—three primary quantities). It is worth mentioning that the dimensionless number $\alpha t/A_T$ is known as the Fourier number in heat transfer theory.

Therefore,

$$\frac{T_T}{T_a} = f\left(\frac{\alpha t}{A_T}\right). \tag{10.25}$$

Although we do not know what the function is that relates these two dimensionless groups, we know that to achieve the same thermal effect in two different bodies, the dimensionless group $\alpha t/A$ must be the same in two bodies. Therefore,

$$\left(\frac{\alpha t}{A_T}\right)_1 = \left(\frac{\alpha t}{A_T}\right)_2 = k, \tag{10.26}$$

where k is a constant and 1 and 2 denote two different body sizes. Given that the thermal diffusivity does not change with the size of the turkey, we can rewrite (10.26) as follows:

$$\left(\frac{t}{A_T}\right)_1 = \left(\frac{t}{A_T}\right)_2 = k. \tag{10.27}$$

Table 10.4 Comparison of cooking time as calculated by dimensional analysis and grandmother recipe

Turkey mass ([lb]	Cooking time (unstuffed) [h]	Predicted cooking time (unstuffed) [h]	Cooking time (stuffed) [h]	Predicted cooking time (stuffed) [h]
14	3.2	3.0	4.0	3.8
20	**3.8**	**3.8 (reference)**	**4.8**	**4.8 (reference)**
28	4.8	4.8	6.0	6.0

Before passing judgment on our grandmother's recipe, we need to relate the area of the turkey to its mass. Assuming that the turkey is roughly a sphere, then

$$m_T = \rho V = \rho \frac{4\pi r^3}{3} \text{ and } A_T = 4\pi r^2, \tag{10.28}$$

where m_T is the turkey's mass, ρ its density, V its volume, and r its radius.

Therefore,

$$m_T = \rho \frac{4\pi}{3} \left(\frac{A_T}{4\pi}\right)^{3/2}. \tag{10.29}$$

Then

$$A_T = k_0 {m_T}^{2/3}. \tag{10.30}$$

Finally, replacing A_T in (10.27) we obtain

$$\left(\frac{t}{{m_T}^{2/3}}\right)_1 = \left(\frac{t}{{m_T}^{2/3}}\right)_2. \tag{10.31}$$

Although with this equation we are unable to predict the required cooking time for our turkey, we can use (10.31) to extrapolate the cooking time from one size to a larger or a smaller size. Let us use as a reference the cooking time for a 20 [lb] turkey (Table 10.3). Then we can calculate the cooking time for 14 and 28 [lb] and compare with the data reported in our grandmother's table (Table 10.4).

Again, we invite students to verify these results using (10.31). Although not perfect, the extrapolations are a very good estimate, and we can say confidently that grandma's recipe enjoys strong scientific support. Finally, although the results are very close, I prefer to use my grandmother's recipe instead of our scientific calculations. Why?

10.6 Scale-Up in Nature

What do we see? Today, scientists and engineers have better tools: not only powerful computing capabilities but also much better microscopes or, more properly, nanoscopes (e.g., to capture 3D movies of living cells). Initially, we used our eyes, then we constructed lenses, and today, we have passed from electron microscopes (scanning electron microscopes) to nanoscopes [scanning probe microscopes (SPMs)]. Although with an electron microscope we can see, in optimal conditions, at the scale of the diameter of an atom, the problems or limitations lie in the characteristics of the sample. Electron microscopes require a vacuum setting, and the biological material should be dried in a

specific manner to avoid shrinkage. According to Rachel Harrison, "*live objects could not withstand the vacuum, which limited actual observations of the functions of the objects*." In the 1980s, SPMs were developed and until today they provide the highest resolution. Since then, other microscopes have been invented: atomic force microscopy (AFM) and magnetic force microscopy (MFM). In 1989, scientists at IBM Laboratories (Almaden, San Jose, CA) using a scanning tunneling microscope (STM) were able to manipulate and spell "IBM" with 35 xenon atoms. With these kinds of microscopes, we can now manipulate and observe objects at the nanometer and atomic levels. Therefore, it is time to turn our eyes to look and learn from Mother Nature.

Multiscale. As presented in Chap. 4, the lotus plant has inspired the development of materials that are repellent to water, and likewise the structure of honeycombs has inspired structural and mechanical engineers, architects, and artists. As expressed by Coppens (2003), in the chemical and bioprocess engineering fields, there are big differences between engineers' solutions and nature's solutions to similar problems. One aspect of nature that could benefit our field is the way it stretches the gap at large scale, which is a big problem for chemical and bioprocess engineers (Coppens 2005).

Fractals. As described in Chap. 4, fractals are self-similar patterns that constantly repeat from small to large scales. Two interesting examples of fractals in nature that can help process engineers to think and rethink processes in terms of respiration (gas transport) are the structure of lungs and trees. One of the characteristics of fractal objects is that they are almost purely surface. A clear example of this is the lungs. For humans, the area of the lungs is roughly the area of a tennis court (Havlin et al. 1995). Obviously, this feature is critical to increasing the efficiency of the transport of gases in the breathing process. This situation is analogous to trees that, through their thousands of leaves, possess a huge transfer area that facilitates respiration.

10.7 Project Homework

As stated at the beginning of this chapter, an important activity that can serve to familiarize students (in terms of Bloom's taxonomy to reach an adequate level of understanding) with this crucial topic will be to assign team homework because to fully understand and apply scale-up in process engineering requires almost complete mastery of unit operations. We suggest assigning simple and well-defined topics, not necessarily directly related to process engineering, to familiarize students with the concept of scaling up. What we suggest is that the course be subdivided into several groups, with each group being assigned a specific topic, possibly including some modest calculations. The homework could be developed in 2–3 weeks and then shared with the entire class in 10–15-min presentations. Thus, with little effort, the entire class will have a broad perspective on scaling up and discover the importance and difficulties of scaling up in process and bioprocess engineering. To facilitate the work of the professor/instructor, we have included a list of potential topics and several references to guide and lead the effort.

10.7.1 Potential Topics

General questions

1. Why are ants capable of supporting several times their own weight?
2. Why can water striders (family Gerridae) walk across the surface of water?
3. What is the maximum size of birds?

Engineering situations

4. Penicillin production
5. Single-cell proteins
6. Bioethanol
7. Copper bioleaching
8. Wine production
9. Manufacture of yeast
10. Scale-up of chemical reactors

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Alexander Fleming http://www.acs.org/content/acs/en/education/whatischemistry/landmarks/flemingpenicillin.html

Scale Up Using Penicillin as an Example http://www.youtube.com/watch?v=n5-TZYZJ51w

Scaling Analysis http://www.youtube.com/watch?v=wOfmvoBNHZU

The Importance of Mixing: Comparing Large Scale and Lab Scale https://mt-emea.adobeconnect.com/_a55714086/p39323957/

Penicillin documentary http://www.nobelprize.org/mediaplayer/index.php?id=343

Dimensional Analysis http://www.youtube.com/watch?v=YFu6pidMbBk

Characterization and scale-up considerations of single-use bioreactors http://www.youtube.com/watch?v=v5DZpFwkHLM

Optimization and Chemical/ Bioprocess Optimization

11 [3]

Do not worry about your difficulties in mathematics. I can assure you mine are still greater.

Albert Einstein

Pleasure in the job puts perfection in the work.

Aristotle

An investment in knowledge always pays the best interest.

Benjamin Franklin

11.1 Chapter Purpose

This chapter is about understanding an important and fundamental topic for all engineers: optimization. The importance of this chapter lies in its two objectives: to familiarize and excite you with the applications of optimization you will encounter in your career in chemical and bioprocess engineering and provide you with some elementary tools (using graphics and spreadsheets) to solve interesting and challenging optimization problems. We will introduce you to this captivating topic through examples, and you will be presented with some classic problems in chemical engineering, environmental engineering, food engineering, biochemical engineering, biotechnology, and others, i.e., classic chemical engineering problems including most of the branches of bioprocess engineering.

Paraphrasing Aristotle, we would like to engage and fascinate you with the topic of optimization and enable you to reach your highest potential. Oh, and remember what Einstein told us about our difficulties in mathematics.

11.2 What Is Optimization?

Recall from Chap. 4 that understanding the logic behind the construction of a honeycomb can help us improve and optimize the design and construction of various pieces of equipment and structures. Honeycomb construction can help us understand the concept of optimization while at the same time, as mentioned in Chap. 4, helping us to apply this logic and improve the design of equipment and processes. But the question persists: what is optimization? Considering the honeycomb construction we can infer that this storage device is optimum because it is structurally adequate and able to hold a

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specific amount of honey utilizing a minimum amount of wax. As depicted in Chap. 4 the amount of wax utilized is vital because honeybees need approximately 4–6 [g] of honey to produce 1 [g] of wax. Taking a dictionary definition we can say that optimization is to achieve the best outcome. In the honeycomb construction the "best" means to construct a reliable storage device utilizing the minimum amount of wax.

As we will see in this chapter, including in the solved and proposed problems (Sects. 11.7 and 11.8), optimization has a wide array of applications that range far beyond the field of chemical and bioprocess engineering. A rereading of Chap. 4 would reveal that chapter was almost entirely, though indirectly, devoted to optimization. The beauty of Chap. 4 is that it invites us to discover the wonders of nature but also subtly teaches us, through examples, the concept of optimization. In addition, optimization is a daily occurrence in all our lives. For example, when you were in high school, you could have taken several routes to get to school, but you probably chose the route that was the best for you, the optimum, optimum in the sense that you probably based your decision on the distance traveled (minimizing time), but perhaps you considered other variables as well, such as personal safety (maximizing safety). All the time and every time (more than we think) we, perhaps unwittingly, make decisions making systematic use of the concept of optimization (maximization and minimization). A little reflection would show us that we apply the concept of optimization all the time, or at least maximize or minimize the variables that are of interest to us. Examples of optimization abound in our everyday lives, but a few will suffice: (a) choosing a restaurant, (b) picking a vacation spot, (c) choosing a girlfriend/boyfriend, (d) choosing a college or university, and (e) buying a car. Try to come up with five more examples of optimization like those in items (a)–(e). All these examples have in common at least two main features: (a) the optimum is not absolute (universal) in the sense that, for example, the best restaurant for me is not necessarily the best restaurant for you, and (b) the optimum is not decided by maximizing or minimizing just one variable; there are several variables and some of them might be in conflict. For example, what is the optimum route to go to school? If you choose a route so as to minimize the time to get there, it might put you in harm's way; on the other hand, if your aim is to maximize safety, then it might take too much time. So what is the best route? One way to handle the situation is, for example, to have as your primary consideration minimizing the time it takes to get there but given a certain minimum level of safety. So your problem first had two variables, but you have reduced it to a problem with one variable and one constraint. But now how do you decide on the minimum level of safety? Is there a way to quantify the level of safety depending on the route? As we will see in our engineering problems, most of the time, the final decision will be based on our engineering criterion, which represents our cumulative experience.

11.3 Do We Have the Required Knowledge to Deal with Problems of Optimization and Process Optimization?

Unfortunately not. In fact, as you will discover and learn as an engineer, most practical and real optimization problems require an advanced level of mathematics. But, as stated at the beginning, our goal is to familiarize and engage you with this absorbing, fascinating, and important topic in chemical and bioprocess engineering. Fortunately, with very basic mathematical tools (high-school level) not only will you be fully familiarized with the concept and potential applications of optimization, but you will also be able to solve some interesting process engineering problems.

Paraphrasing Einstein, do not be worried about your knowledge of mathematics: just with your current preparation in mathematics and some understanding of spreadsheets, we will be able to guide you and tackle exciting problems while at the same time fulfilling one of our main goals, which is to familiarize and captivate you with chemical and bioprocess engineering. Because not every freshman

engineering student has received solid training in spreadsheets, in Sects. 11.5 and 11.6 we will give you, through warm-up examples, the required tools to solve the optimization problems presented in this book.

Of course, you do not have the knowledge required to solve these interesting problems, but you already have the capability to at least approach them. With the help of spreadsheets you will be able to attempt and solve problems that are stimulating and that will give you a broad awareness of chemical and bioprocess engineering. In addition, with confidence we can state that the most important part of dealing with engineering problems is to have the capability to correctly formulate them, and this capability was developed and heavily stressed in previous chapters.

11.4 Understanding Optimization in Chemical and Bioprocess Engineering

Most, if not all, engineering problems are subject to optimization. We cannot mention all the possibilities of optimization that you will encounter in chemical and bioprocess engineering, but in the sections on solved and proposed problems (Sects. 11.7 and 11.8), you will be exposed to a wide variety of optimization situations. In this section, we present several real optimization situations in different applications in chemical engineering, food processing, biochemical and bioprocess engineering, and environmental engineering.

Multieffect evaporation

Multieffect evaporators are a sequence of heat exchangers connected in series where a liquid is boiled to produce vapor and a concentrated liquid solution. A classic example in the chemical industry is the determination of the optimal number of effects in an evaporation system, were the optimum is found when there is an economic balance between energy saving and added investment, this is, a minimization of the total cost. Normally multieffect evaporators, in various applications, have three to six stages, although in some specific applications they have much more than that. As an example, in the tomato concentration industry multieffect evaporators are, most of the time, manufactured with five effects. To understand the optimization problem, we should mention that evaporators can be constructed from one, two, three, or sometimes over ten effects. The tradeoff is that adding an effect to the equipment will lead to energy savings during the operation (making it more efficient), but on the other hand, the equipment cost will be higher. Then, to determine the optimum number of effects, it is necessary to carry out an economic analysis to establish the number of effects that minimizes the total cost, taking into account additional effects means greater energy savings (decreased variable costs) higher equipment cost (increased fixed costs).

Number of fermenters

Industrial fermentation plants consist of three main sections: preparation, fermentation, and product recovery. The preparation section usually contains operations such us medium preparation and sterilization and inoculum propagation. The fermentation section is the heart of the plant where the transformation of raw material into products takes place. The product recovery section encompasses the downstream operations needed to obtain the product of interest with the required purity (Reisman 1988). In the design of fed-batch and batch fermentation plants, one faces the problem of figuring out the adequate combination of the number and size of fermenters to be used to meet the desired production schedule. In principle, the problem has an infinite number of solutions because for any given fermenter size, a number of units of that size will do the work. Nevertheless, not all solutions are equal from an economic standpoint (Simpson et al. 2005).

Anaerobic digestion

Pig waste slurry is normally revalued by obtaining methane gas (biogas). At present, the production of biogas is a well-known technique, but it can be improved, economically speaking, by the addition of organic waste from the food industry. However, this may lead to problems of organic overload and inhibition, among others. Prediction and optimization of these input parameters, and the corresponding operating conditions (e.g., pH, temperature, microorganism concentration), will contribute to determining the optimum operation and performance of the digestion plant.

Optimum number of retorts in canned-food plants

Batch processing with a battery of individual retorts (autoclaves) is a common mode of operation in many food-canning plants (canneries). Although high-speed processing with continuous rotary or hydrostatic retort systems can be found in very large canning factories (where they are cost-justified by high-volume throughput), such systems are not economically feasible in the majority of small to medium-sized canneries (Norback and Rattunde 1991). In such smaller canneries, retort operations are carried out as batch processes in a cook room where the battery of retorts is located. Although the unloading and reloading operations for each retort are labor intensive, a well-designed and managed cook room can operate with surprising efficiency if it has the optimum number of retorts and the optimum schedule of retort operation. Using just one retort entails a long downtime; adding more retorts can make the operation more efficient but will raise investment costs. As in the previous example (fermenters) there exist literally an infinite number of alternatives, but only one is optimum in economic terms (Simpson et al. 2002).

Optimum temperature to dry food products

Dehydration is a common process used to reduce the water activity of foods, thereby preventing the proliferation of deteriorative microorganisms and extending the product shelf life. Usually the drier uses hot air (70–90 °C) to remove water (lowering water activity) from the food product, and the dehydration process takes from 30 [min] to hours, and the food product will be dehydrated but also exhibit quality losses. So the question is: what is the best temperature at which to dehydrate food products? The higher the temperature, the shorter the process (maximizing production output); on the other hand, the product will be much more affected in terms of quality (vitamin loss, color change, and shrinkage). Clearly, there is a tradeoff, and there exists an optimum strategy to carry out the dehydration process.

Manufacturing and processing plant location

Real-world engineering problems, including chemical and bioprocess engineering problems, are generally characterized by the presence of many conflicting and nonmeasurable objectives or particular objective functions. These types of optimization problem are called multiobjective (multipurpose, multicriterion) problems. Taking into account the nature of real-life problems, it is better to look at these problems as multiobjective optimization problems. One interesting multiobjective optimization problem is to find the best location for a manufacturing or processing plant. Why? Because there are several variables that affect the best location, for example, distance from the market, quality, availability and cost of raw materials, availability of skilled and unskilled labor, availability and cost of water and energy, and environmental regulations.

Optimizing industrial water utilization and management

Nowadays chemical, food, and bioprocessing companies are major consumers of water, and the water sources that are available have great relevance. Many studies indicate that water resources are often used inefficiently. In addition, at this time the demand for potable water, environmental regulations, increasing costs of supply and wastewater treatments, and other factors require that industries find efficient solutions, i.e., optimizing the use and management of water for processing.

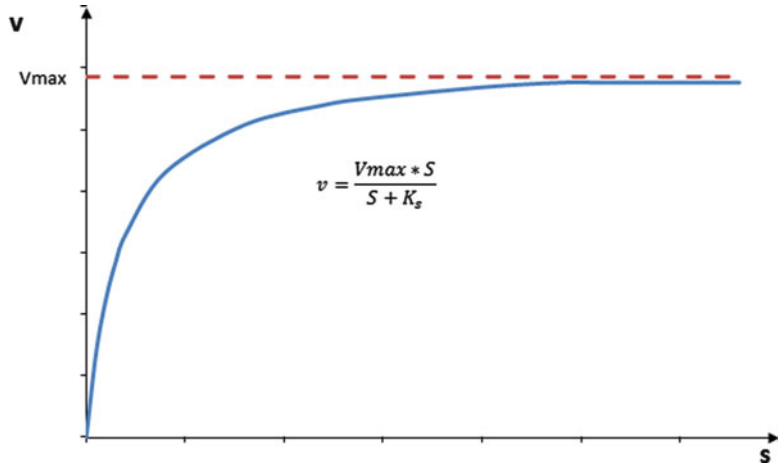

Fig. 11.1 Michaelis-Menten model

Determining the optimal number of experiments

Experimentation is normally expensive, and sometimes very tedious, and difficult. As an example, advances in nanotechnology make it easier to examine nature at a deep level because today we have advanced microscopes and tools that allow us to better observe nature and that have improved our understanding of the micro and nano worlds, but such equipment is very expensive and the experiments rather difficult. When planning new experiments it is important to determine, first, what experiments to do. Several mathematical techniques can help us, in specific cases, to determine what and how many experiments represent the optimal set. One of these techniques is the D-optimal design that normally significantly reduces the number of experiments and delivers good results. For example, if you have a mathematical model such as the one depicted in Fig. 11.1 (Michaelis-Menten model) and you want to fit the curve with experimental data, then to determine the parameters of the model ($V_{\max}$ and K_s), the question is how many experiments are required and for what values of the S-axis. D-optimal design helps to efficiently fix the set of experiments.

11.5 Maximum, Minimum, and a Warm-Up Example

As shown in Fig. 11.2, $x = a$ and $x = b$ represent a relative maximum and minimum for $f(x)$, respectively. We use the term *relative* because if we extend the amplitude of x values, we can probably find other maximum and minimum values. Why do we call $x = a$ the maximum? Because in the vicinity of $x = a, f(a)$ is the largest value for $f(x)$. In the same way, $x = b$ represents the minimum because $f(b)$ is the minimum value of $f(x)$ in the vicinity of $x = b$.

As indirectly mentioned in Sect. 11.3, you are probably not yet familiar with the concept of derivatives, which is essential in handling maximum and minimum problems. Although we will present and solve interesting and applicable maximum and minimum engineering problems, it is not our intention to familiarize you with the concept of derivatives. Soon in your career you will be introduced by experts to this important and key mathematical concept for all engineers. You are most probably familiar with the use of spreadsheets. In this section we will show you how to solve maximum and minimum problems using spreadsheets.

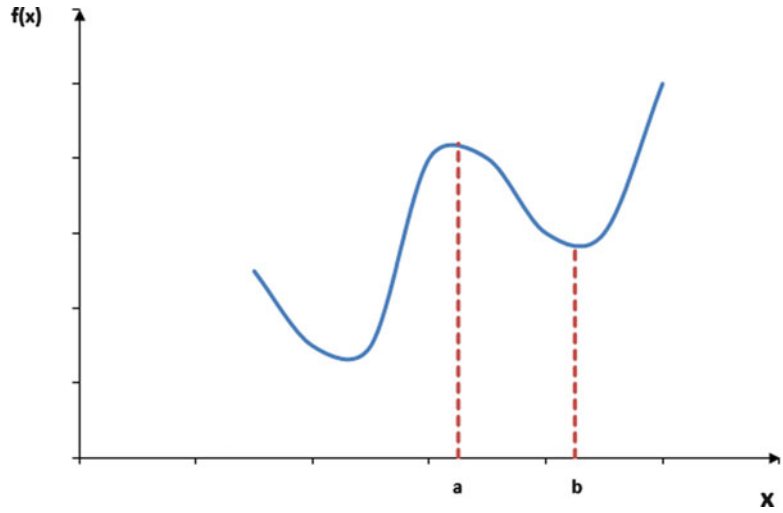

Fig. 11.2 Maximum and minimum in a $f(x)$ vs. x graph

11.5.1 Warm-Up Example: Minimization

Airplane and Cessna [6]. A commercial airliner is arriving in Columbus, Ohio, from the south at a speed of 800.0 [km/h]. When the airliner is 600.0 [km] from the Columbus airport, a Cessna aircraft leaves Columbus for the East Coast at a speed of 250.0 [km/h]. When will the minimum distance occur between the airliner and the Cessna? **Assumption**: The two aircraft are moving at an angle of 90° to each other.

Solution

Step I

Graphical representation and mathematical formulation

At the beginning ($t = 0$) the distance that separates the airliner and the Cessna is 600.0 [km] (Fig. 11.3a). Then t hours later the location of each plane will be as depicted in Fig. 11.3b.

As shown in Fig. 11.3b, D represents the distance between both planes at any instant t ($t \geq 0$). According to the Pythagorean theorem, we can write the following equation:

$$D^2 = (600 - 800t)^2 + (250t)^2.$$

Then

$$D = \sqrt{(600 - 800t)^2 + (250t)^2}. \tag{11.1}$$

One way to test the validity of (11.1) is replacing $t = 0$ and confirming that $D = 600.0$ [km].

Step II

Determine the right procedure to solve the problem. At your level, to obtain the value of t, to minimize the value of D in (11.1), we will use spreadsheets where we have two interesting alternatives to find the value of t to minimize D in (11.1).

1. Approximate graphical solution. Plot D (y-axis) against t (x-axis) and, first, visually determine if the function has a minimum or a maximum, whichever is applicable. If the function has a minimum or

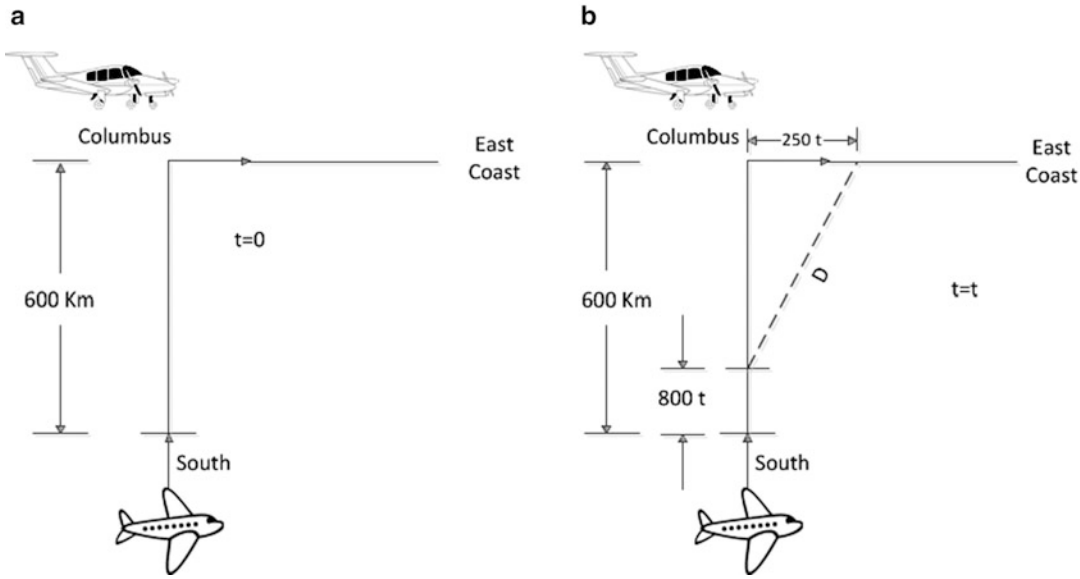

Fig. 11.3 (**a**) Initial position of the Cessna and the airplane (**b**) Position of the Cessna and the airplane after t hours

a maximum, construct a plot with sufficient data to approximately, from the graph, determine the maximum or minimum; in this specific case determine the value of t that minimizes D. We would like to emphasize that this procedure is just a rough approximate, not the true solution. It is not a rigorous procedure but will help us to have an approximate idea of the real solution.

2. Solver tool. Use the Solver tool (Excel spreadsheet) to find the maximum or the minimum of the function, in this case the value of t that minimizes D.

Although the Solver approach is more accurate than the graphical visualization, we recommend using the Solver tool but also, first, doing a graph to have a clear visualization of the problem and see if the function has a minimum or a maximum.

Now we will show you how to approach this warm-up example problem with both procedures as follows:

(a) Graphical solution.

To find the value of t that minimizes the distance D, we first construct a table using Excel.

Table 11.1 shows that the time (t) that minimizes the distance (D) between the planes is $t = 0.7$ [h]. Figure 11.4 shows the Excel graph of D against t ($0 < t < 1$) and with time increments of 0.1 [h].

According to Fig. 11.4, the minimum distance between the two planes is at $t = 0.7$ [h]. This solution is approximate because it depends on the time increment chosen and so should be refined. To get a more accurate solution, instead of dividing the values of t by 0.1 [h], we will divide by 0.01 [h] to get the results presented in Table 11.2.

According to Table 11.2, the time t that minimizes the distance between the planes is $t \sim 0.68$ [h] and the distance between the planes is $D \sim 179$ [km]. A more accurate solution can be obtained using the Solver tool of Excel as follows.

(b) Solver tool.

First, we assign t a cell (e.g., D9) with an initial value of 0. Then, in a different cell (e.g., G9), we write (11.1) (taking into account that our variable t is now represented by cell D9). Therefore, the Excel

Table 11.1

Time, t [h]	Distance, D [km]
0.0	600.0
0.1	520.6
0.2	442.8
0.3	367.7
0.4	297.3
0.5	235.8
0.6	192.1
0.7	**179.5**
0.8	204.0
0.9	255.0
1.0	320.2

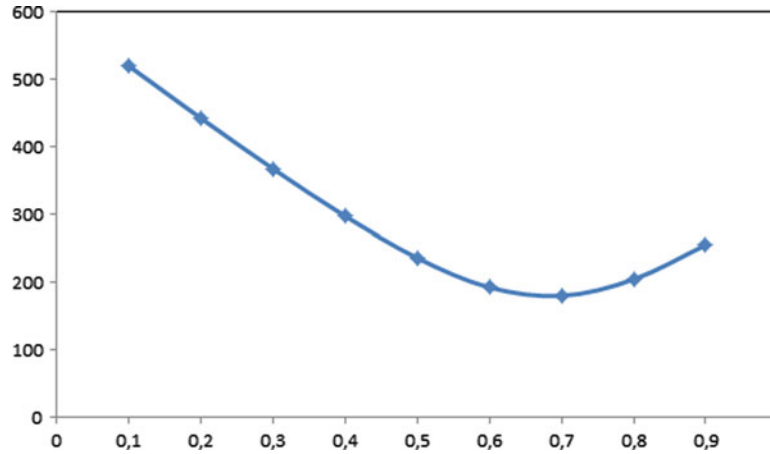

Fig. 11.4 Distance of the Cessna and the airplane against time

Table 11.2

Time, t [h]	Distance, D [km]
0.60	192.1
0.61	189.2
0.62	186.7
0.63	184.5
0.64	182.6
0.65	181.1
0.66	180.0
0.67	179.3
0.68	**179.0**
0.69	179.1
0.70	179.5

spreadsheet should look like this (cells D8 and G8 are just labels to indicate that cells D9 and G9 are time t and (11.1), respectively) (Fig. 11.5):

Cell G9 is equal to 600 because $t = 0$ (cell D9).

Second, click on Data on the toolbar. Solver is on the right-hand side (if Solver is not displayed, you should ask your tutor or professor for a detailed explanation on how to install Solver in an Excel spreadsheet).

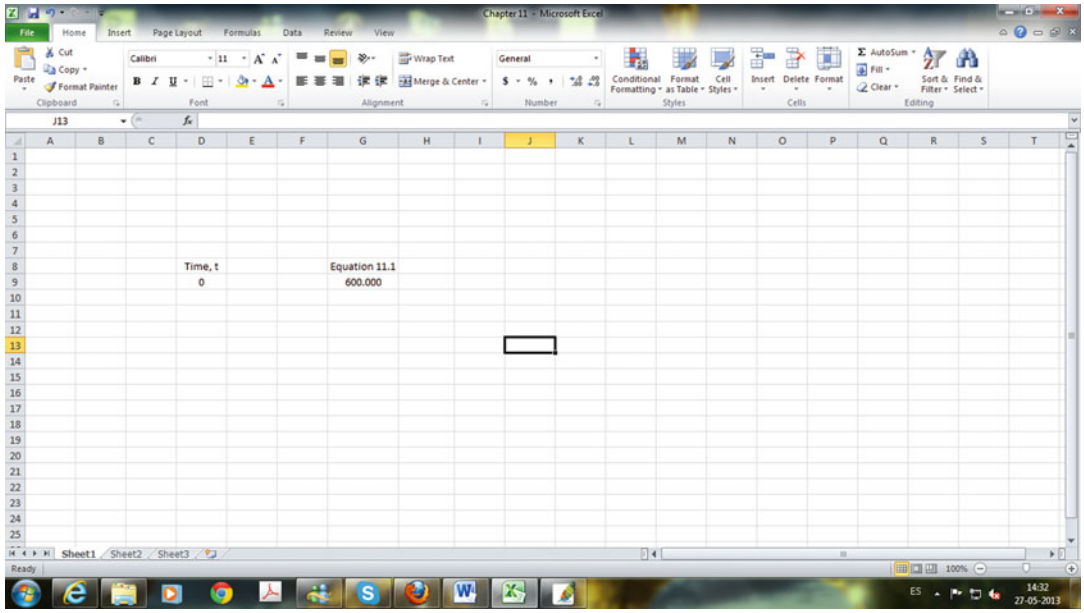

Fig. 11.5 Microsoft Excel screen to calculate distance against time (Eq. 11.1)

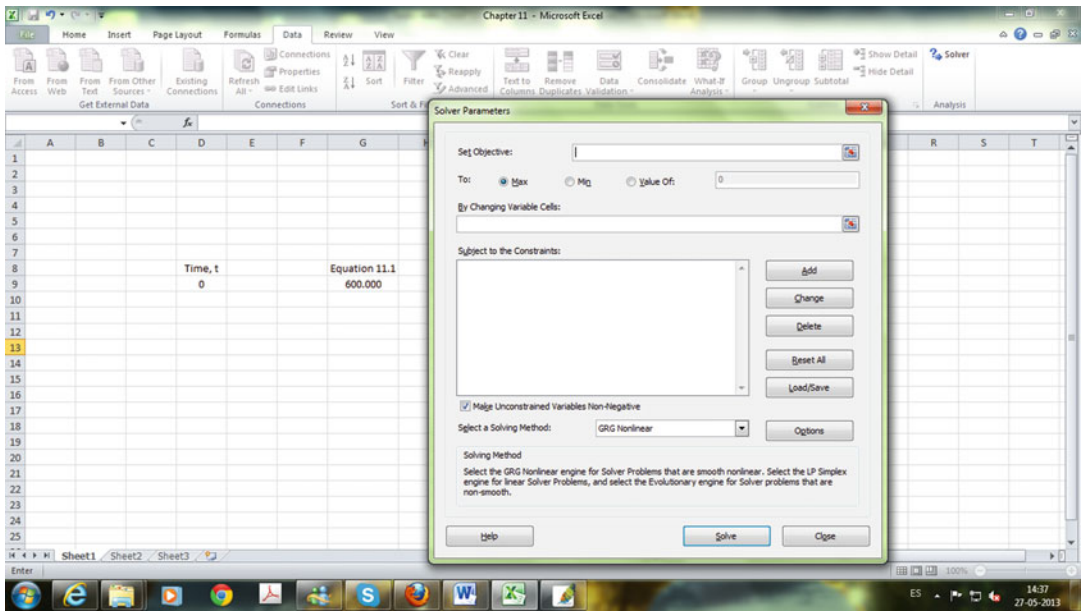

Fig. 11.6 Solver window in Microsoft Excel spread sheet

Now clicking on Solver you will see the following screen (Fig. 11.6):

Set cell G9 as the objective function (Set Objective), then choose Min (minimize), and select cell D9 in By Changing Variable Cells. In addition, click on the box to make D9 (unconstrained variables) nonnegative (remember $t \geq 0$) and the screen will now look like this (Fig. 11.7):

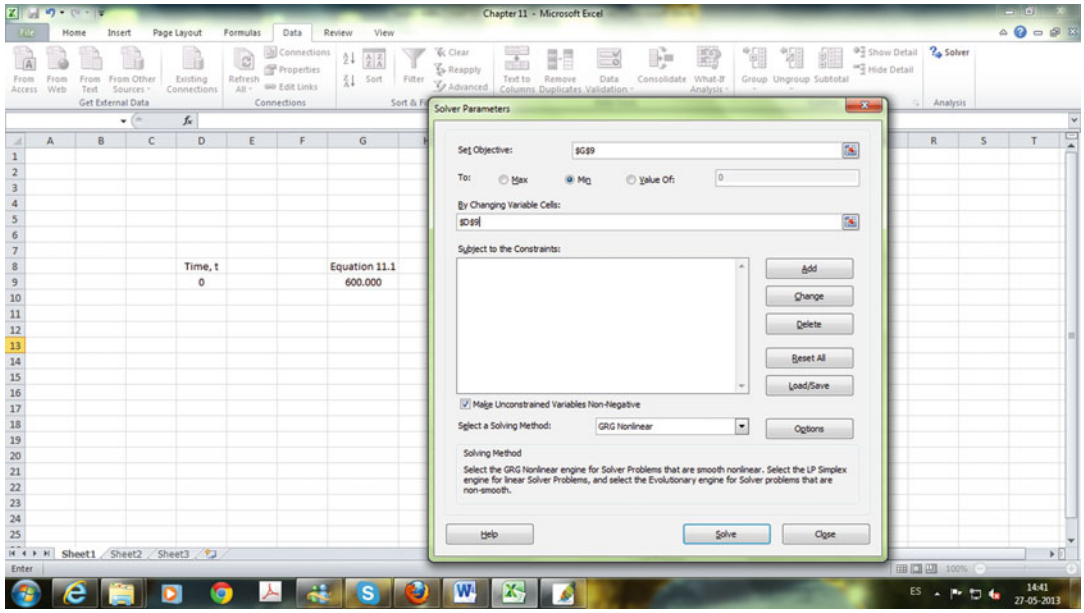

Fig. 11.7 Solver window in Microsoft Excel spread sheet showing the objective function (cell G9) and the changing variable (cell D9)

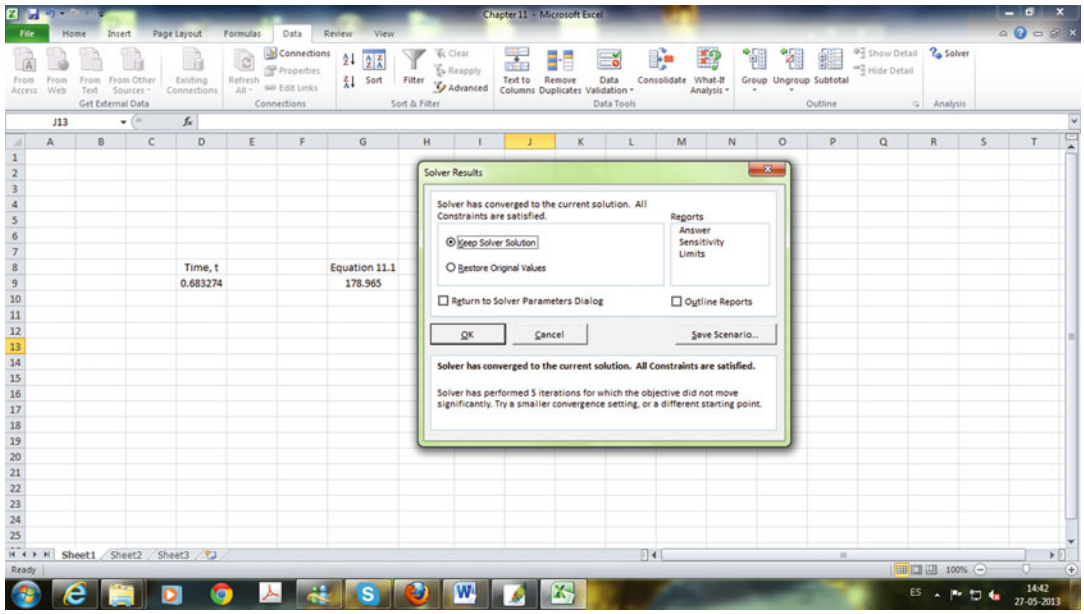
Chapter 11 - Microsoft Excel
Solver Results
Solver has converged to the current solution. All Constraints are satisfied.
Reports
Answer
Sensitivity
Limits
Keep Solver Solution
Restore Original Values
Return to Solver Parameters Dialog
Outline Reports
OK
Cancel
Save Scenario...
Solver has converged to the current solution. All Constraints are satisfied.
Solver has performed 5 iterations for which the objective did not move significantly. Try a smaller convergence setting, or a different starting point.
Time, t
0.683274
Equation 11.1
178.965

Fig. 11.8 Final screen depicting the solution found by Solver tool

Next, click on Solve to bring up the following screen (Fig. 11.8):

Indicating that Solver has found a solution, $t = 0.683274$ [h] and $D = 178.965$ [km]. Note that in an Excel sheet, you can define the number of decimal places. Also, although the Solver solution is more accurate than the graphical solution, the values are fairly close.

11.6 Origins of Operations Research and Process Optimization

11.6.1 Historical Background

History teaches us that until the mid-nineteenth century, the production of goods was mainly in workshops that operated in the traditional way, i.e., with low production volumes, starting the manufacture of a new product when the previous was finished (one to one), and without further specialization of labor, meaning a craftsman normally made the product in full. This form of craftsmanship in some cases led to high-quality products, of which those existing today are considered works of art.

From the time indicated, the Industrial Revolution brought about the birth of the first industries, the division of labor, interest in optimizing workers' time [the first operations research (OR) studies were devoted to this area], optimal placement of machines, specialization of individuals and industries, and the creation of various functional areas within companies, which sometimes had conflicting objectives. Then came the need to coordinate and define activity levels and better allocate and use resources (concept of efficiency) to achieve these levels of activity, with optimal results for the company (from a global point of view) and avoiding internal conflicts.

In the early twentieth century in the steel and coal industries, Frederic Taylor, a mechanical engineer and the father of scientific management, conducted the first studies OR in the USA on work organization, process improvement, optimization of workers' time, and machine location (layout). Henry Fayol, a mining engineer from France, made a contribution analogous to Taylor's but oriented toward administration, separating the functional areas of a company and stating the 14 general principles of management. At the same time, A. Erlang, working for the Danish telephone company in Copenhagen, studied timeouts of private branch exchange users, giving rise to the issue of waiting lines (generally applicable to any service) and making an important contribution to telephone engineering.

The Gantt chart was invented by Henry Gantt in the early part of the last century. Gantt charts coordinate various activities (with a common ultimate goal) of various people. Such charts are the predecessor of the most powerful methods of the Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT).

OR underwent tremendous development and use during World War II; it was applied to solve problems of logistics equipment and staff, as well as to plan military operations themselves. Consider the following examples:

- Choosing the type and quantity of materiel to manufacture with limited resources.
- Assigning materiel to different battlefronts and deciding how to transport materiel.
- Allocating human resources to the various fronts.
- Determining how to deploy patrol antisubmarine aircraft and ships.

After the war, these experiences were transferred to the commercial sector by people who were demobilized from the military and as a result of articles published in various journals. This happened mainly in England and the USA. The USA was in the position of global economic leader, given that European countries (particularly Germany) and Asia (especially Japan) saw their economies destroyed by the end of the war. The USA had the opportunity to export its products and services to almost everyone and, in turn, imported raw materials from the rest of the world, becoming an "economic engine," globally speaking.

US companies experienced strong growth in operational levels, and OR was used for various purposes, including determining the optimal levels of activity, planning, coordination, control, and allocation of resources.

During the postwar period, several tools were developed to assist in decision making to address various types of recurring industrial problems in the area of operations. The most applicable tools to today's world are linear programming and the CPM method. One of the first textbooks on OR appeared in 1957, *Introduction to Operations Research*, by Churchman, Ackoff, and Arnoff.

11.6.2 Operations Research and the Scientific Method

OR uses the scientific method, through experimentation or simulation, utilizing a general sequential procedure or methodology (which may be iterative) to determine the structure of a situation and find the cause and effect relationships between variables. The typical stages of a real OR application are as follows:

(a) Detect and identify the problem or situation on which you have to make a decision. (If there are alternatives, for example, in the area of resource distribution, which is the optimum?)
(b) Define the problem and its organizational environment, temporal, physical, and decisional context; raising of study objectives.
(c) Collect data and relevant parameters, relationships between variables of interest.
(d) Formulate a mathematical model to evaluate the alternatives available; this implies determining the factors, parameters, variables, and elements to be considered in a process of abstraction from reality.*
(e) Use an algorithm to find the best alternative.*
(f) Test the model: validate it using various parameters to determine the ability of the model to predict the behavior of the real system under various circumstances. Perform sensitivity analysis on different parameter values.
(g) Decide on the best alternative. Consider the key elements for a successful implementation, for example, the need for training, changes in organizational structure, and technological changes.
(h) Prepare a solution for implementation. Provide documentation and training.
(i) Implement the solution, establishing specific actions.
(j) Evaluate the results.

At this stage in your career, we will show you, in some detail, two of the ten stages (marked with *) of OR application: the formulation of a mathematical model and an application algorithm. In earlier chapters, you were continuously tested to formulate mathematical expressions to solve a wide variety of problems, so you are fully prepared to learn the right procedure to formulate and, with the help of an appropriate tool, solve these interesting and engaging problems.

11.6.3 Linear Programming

Linear programming (LP) is a mathematical model used initially in solving problems in the area of defense in order to optimize the effect of weapons systems in the event of war. It was later used in the private sector. In the latter, it has been used to help make the best decisions for distributing scarce resources among several potential products, i.e., situation planning and production scheduling to determine product types and quantities manufactured, so as to obtain optimal results for the company. In 1947, Dantzig developed a simplex algorithm that facilitates solving powerful LP models.

In the early 1950s, many US companies experienced strong growth in operational levels, and LP was used to determine activity levels and optimal resource allocation.

LP (understood as being synonymous with linear planning) is currently one of the most widely used tools in OR. Furthermore, LP has expanded its scope of application to optimize a variety of

situations, including investment portfolio problems, transportation and distribution of products, staff scheduling, inventory management, facility location (plants, warehouses), and combinations of these situations.

A decision-making situation using LP comprises three phases:

(a) Formulate the problem as a LP model.
(b) Resolve the LP model.
(c) Interpret the solution and perform a sensitivity analysis on it.

At this point, we would like to reemphasize that our goal is to familiarize you with phase (a), problem formulation. In addition, we will show you how to solve these problems utilizing the Solver tool of Microsoft Excel.

11.6.4 Modeling and Solving LP and IP Problems

The steps for formulating a LP model can be summarized as follows.

Step I

Define decision variables and codification. Correct definition and codification are very important in order to have a clear understanding of all variables involved in the problem. Variables should correspond to real-world variables, which make up the problem in a mathematical way; in LP models we attempt to determine the optimal values of these decision variables.

Step II

Objective function. Formulate the objective function (OF), which is a mathematical expression that must be expressed as a linear function of the decision variables. This expression should be optimized (maximize or minimize), subject to the constraints explained in the following step (e.g., limited resources).

Step III

Constraints. Formulate the constraints that limit or delimit the problem. These can derive from the availability of, for example, machinery, labor, raw materials, demand, and financial resources.

Step IV

Mathematical solution. In this aspect, we will not go into detail but just mention that the most common procedure for solving LP problems is the simplex method. In more advanced courses students will have the opportunity to learn this method and others that are used to solve integer programming (IP) problems, nonlinear problems, and others.

In this case, to solve these problems (LP and IP), we will detail the use of the Microsoft Excel Solver tool, which allows us to solve both LP and IP problems.

The fact that there are decision variables implies that there are several courses of action or alternatives from which to choose. For example, if a company has the option of producing three different products, the management team can utilize a LP model to decide which products to produce and at what level.

An important aspect in LP is the requirement that all mathematical expressions must be linear. Obviously, in practice, not all factors are linear, so nonlinear expressions are sometimes necessary to model specific situations, for example, in the case of economies of scale. Fortunately, it has been shown that many factors (work hours, use of machinery, benefits) are reasonably linear or can be approximated by expressions of this type. However, if there is found to be a factor that definitely cannot be expressed linearly, then the problem cannot be solved by LP. This is certainly is the case with nonlinear programming (NLP), which is beyond the scope of this textbook.

LP models can be used to model and solve various situations. The models must meet the following conditions:

(a) LP models must be able to express the situation by means of a mathematical model.
(b) All factors or terms considered in the objective function and constraints must be linear, i.e., they must be formed by the addition of one or more terms where each term must be the product of a coefficient and a decision variable, with an exponent having a value of one for the decision variable. If this condition is not met, then we are faced with a NLP problem.
(c) There should be one or more constraints on the decision variables of the problem, which determines if there will be different courses of action from which to choose, as established by the range of values that the decision variables can take.
(d) The decision variables can take continuous values (decimal). If this condition is not met, then we are faced with a problem of IP. IP is a special case of LP that is of great interest, and some examples and problems related to it will be presented in this textbook.

Integer programming (IP): corresponds to a special case of LP in which the decision variables can take only integer values. When values are also limited to 0 and 1, we talk of binary integer programming. Moreover, there are also mixed models in which some variables are integers and others are continuous.

11.6.5 Warm-Up Example

Below is an example where it is necessary to decide the level of weekly production of products 1, 2, 3, and 4 in order to maximize the benefits to the company.

A company can produce four products and wants to maximize its profits. The benefit of each of the products is 5, 4, 3, and 4 respectively in arbitrary units. In addition, the company has a staff of 200 skilled workers and 150 unskilled workers laboring 40 h/week. The time required to produce one unit of each product is presented in the following table. Formulate and solve the LP problem in order to determine the optimum production levels of each of the four products.

	Product 1	Product 2	Product 3	Product 4	Available hours per week
Required hours per skilled worker per unit of product	5	3	1	2	8,000 (200 × 40)
Required hours per unskilled worker per unit	5	7	4	8	6,000 (150 × 40)
Benefits per product	5	4	3	4	

Solution

As was mentioned in previous chapters, it is highly recommended to approach problems, in this case LP problems, with the right attitude and, more importantly, with a methodology. As discussed in Sect. 11.6.4, a right solution procedure includes four steps, as follows: **Step I**. Variable definition and codification. **Step II**. Formulation of objective function. **Step III**. Formulation of all constraints. **Step IV**. Implementation and solution with the Solver tool (Microsoft Excel).

Step I

Variable definition and codification

P_{1SW}: Units of product 1 manufactured by skilled workers
P_{1UW}: Units of product 1 manufactured by unskilled workers

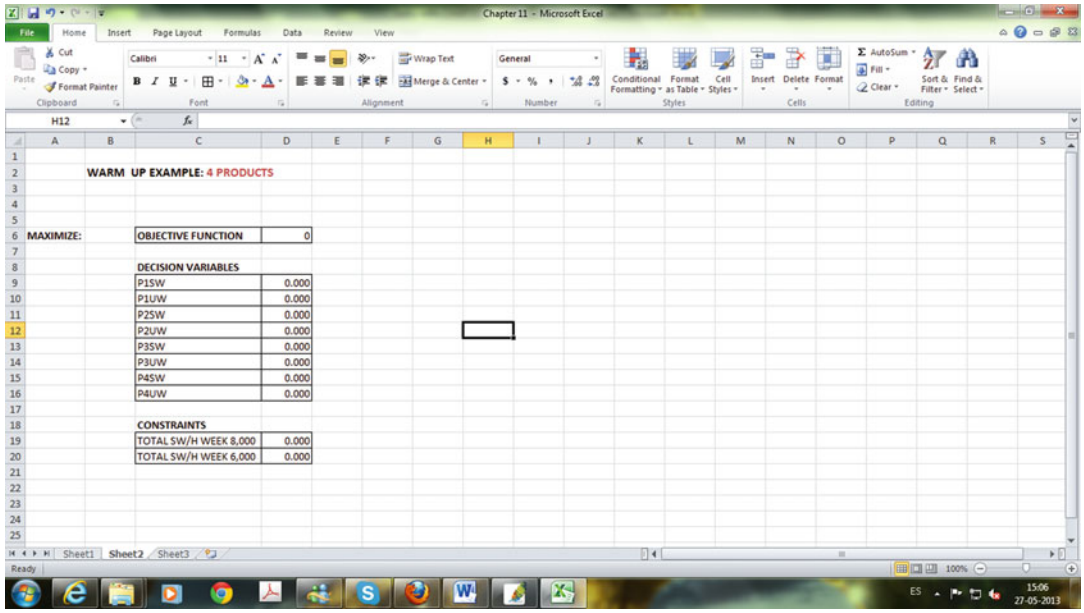

Fig. 11.9 Microsoft Excel screen with the assigned cells for the objective function, decision variables and constraints

P_{2SW}: Units of product 2 manufactured by skilled workers
P_{2UW}: Units of product 2 manufactured by unskilled workers
P_{3SW}: Units of product 3 manufactured by skilled workers
P_{3UW}: Units of product 3manufactured by unskilled workers
P_{4SW}: Units of product 4 manufactured by skilled workers
P_{4UW}: Units of product 4 manufactured by unskilled workers

Summarizing we have eight decision variables.

Step II

Formulation of objective function

The total benefits will be as follows:

$$\textbf{Benefits} = 5 \times P_{1SW} + 5 \times P_{1UW} + 4 \times P_{2SW} + 4 \times P_{2UW} + 3 \times P_{3SW} + 3 \times P_{3UW} + 4 \times P_{4SW} + 4 \times P_{4UW}.$$

Step III

Constraints

We have two constraints, 8,000 h/week of skilled workers and 6,000 h/week of unskilled workers. Therefore:

1. SKILLED WORKERS: $5 \times P_{1SW} + 3 \times P_{2SW} + 1 \times P_{3SW} + 2 \times P_{4SW} \leq 8{,}000$,
2. UNSKILLED WORKERS: $5 \times P_{1UW} + 7 \times P_{2UW} + 4 \times P_{3UW} + 8 \times P_{4UW} \leq 6{,}000$.

In addition, all decision variables should be equal to or greater than 0 ($P_{1SW} \geq 0, P_{1UW} \geq 0, \ldots, P_{4UW} \geq 0$).

Step IV

As shown in this example, we will use Solver from Microsoft Excel.

1. The following image shows the first step for the warm-up example (Fig. 11.9).

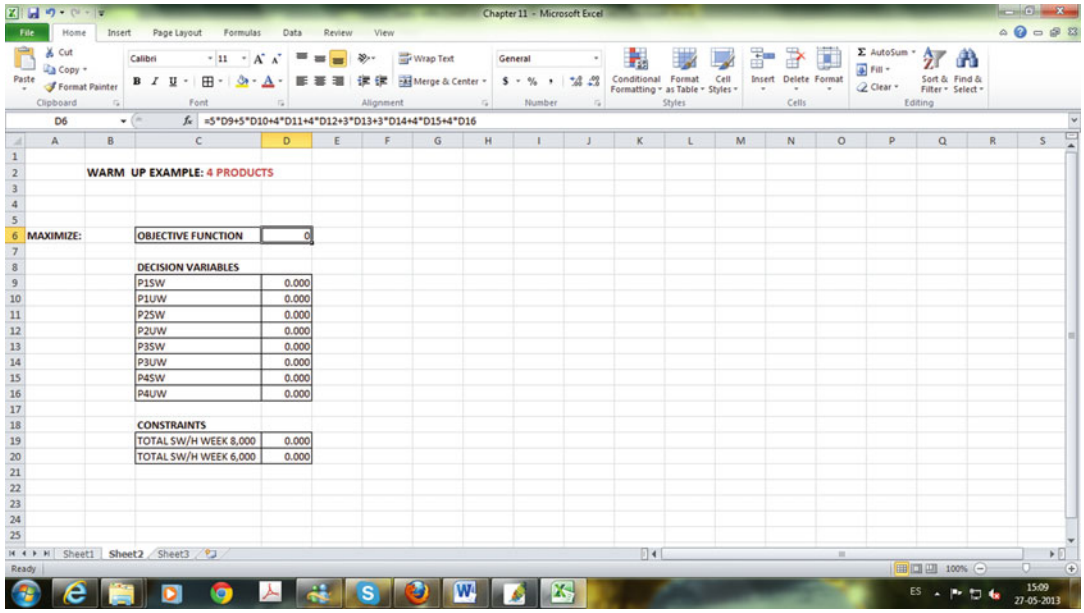

Fig. 11.10 Microsoft Excel screen with the objective function written in cell D6

(a) We have called this **WARM-UP EXAMPLE: 4 PRODUCTS** and, in addition, we have assigned cells for the objective function, decision variables and constraints. In the left as a reminder we included a cell indicating that it is a maximization problem.

(b) In cell D6 we need to write the objective function and then, according to the assigned cells for P_{1SW}, P_{1UW}, …, P_{4UW}. Clicking on D6, the screen should look like this (Fig. 11.10): where $f(x) = 5 \times D9 + 5 \times D10 + 4 \times D11 + 4 \times D12 + 3 \times D13 + 3 \times D14 + 4 \times D15 + 4 \times D16$.

This is equivalent to **Benefits** $= 5 \times P_{1SW} + 5 \times P_{1UW} + 4 \times P_{2SW} + 4 \times P_{2UW} + 3 \times P_{3SW} + 3 \times P_{3UW} + 4 \times P_{4SW} + 4 \times P_{4UW}$.

D9 represents the value assigned to P_{1SW} and is multiplied by 5 because the benefit of each unit of product 1 is 5, D10 corresponds to the value assigned to P_{1UW} and is also multiplied by 5 because the benefit of each unit of product 1 is 5, and so on.

(c) Writing the constraints: as shown in the previous screen, constraints are written in cells D19 and D20. Clicking on D19 we see (Fig. 11.11)

Where $f(x) = 5 \times D9 + 3 \times D11 + D13 + 2 \times D15$.

This corresponds to the available hours of SKILLED WORKERS: $5 \times P_{1SW} + 3 \times P_{2SW} + 1 \times P_{3SW} + 2 \times P_{4SW} \leq 8{,}000$.

As you see, we have written the equation but we have not included the constraint that limits us to 8,000 h. This will be considered in the Solver tool box that will be shown later.

In the same way, clicking on D20 we see (Fig. 11.12)

Where $f(x) = 5 \times D10 + 7 \times D12 + 4 \times D14 + 8 \times D16$.

This corresponds to the available hours of UNSKILLED WORKERS: $5 \times P_{1UW} + 7 \times P_{2UW} + 4 \times P_{3UW} + 8 \times P_{4UW} \leq 6{,}000$.

As you see, again we have written the equation but we have not included the constraint that limits us to 6,000 h. This will be considered in the Solver tool box that will be shown later.

Now that we have written the mathematical formulation of the problem in a Microsoft Excel spreadsheet, we can explain how to use the Solver tool box to obtain the optimum solution, in this case a maximum.

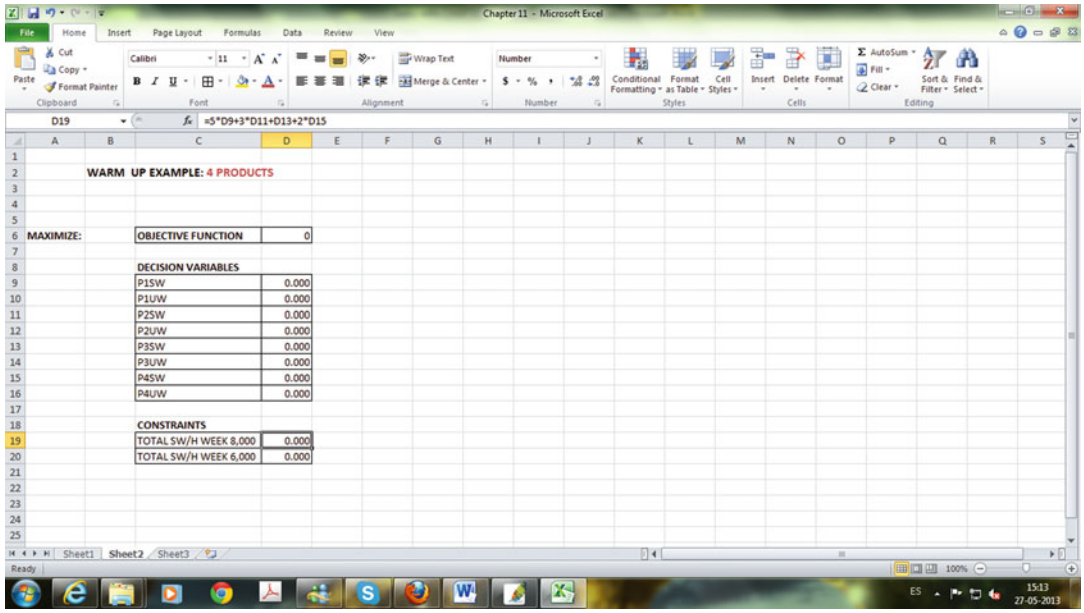

Fig. 11.11 Microsoft Excel screen with the first constraint written in cell D19

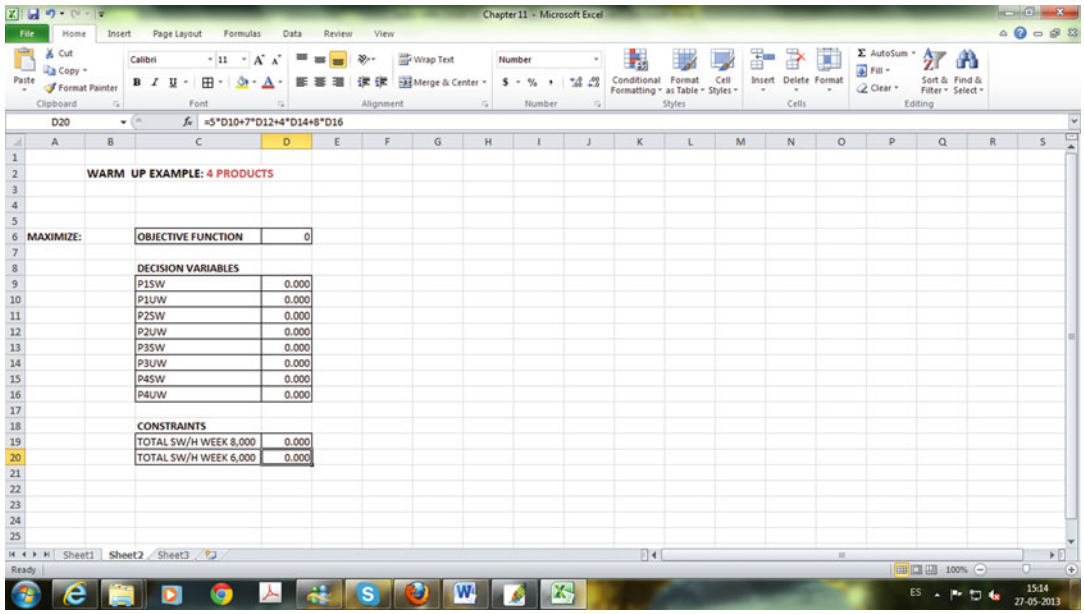

Fig. 11.12 Microsoft Excel screen with the second constraint written in cell D20

In Excel spreadsheet, click Data in the tool bar. On the right-hand side will appear Solver, as shown in the following image (Fig. 11.13):

If Solver does not appear, you need to install (see appendix 1 of this chapter).

Now, clicking in Solver, you will get (Fig. 11.14)

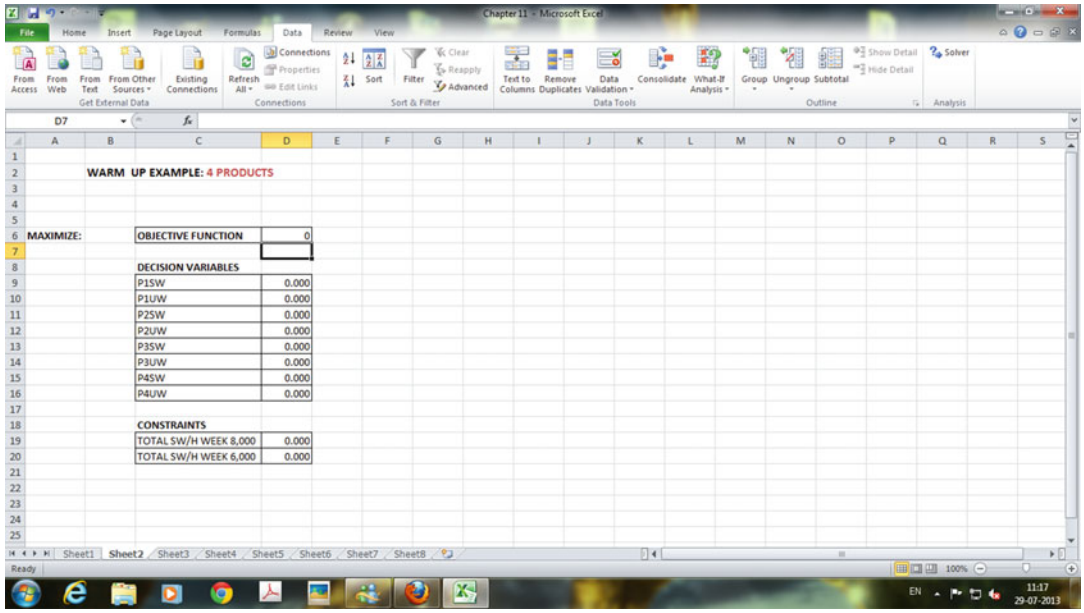

Fig. 11.13 Microsoft Excel screen showing Solver in the tool bar

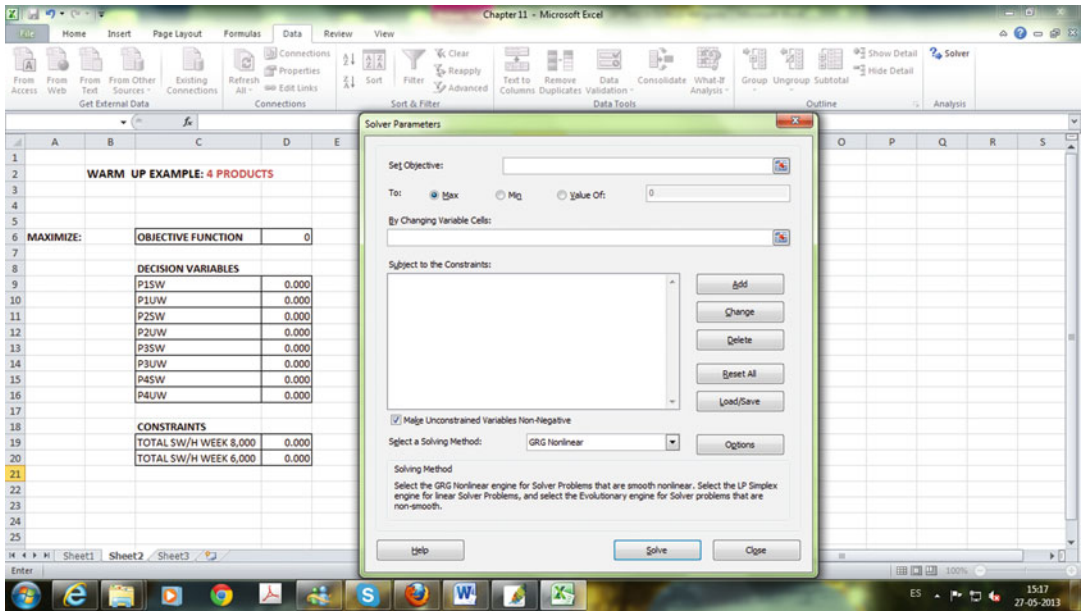

Fig. 11.14 Microsoft Excel screen showing Solver window

Let us see now what the Solver window will look like after the completion of all data (Fig. 11.15):

As you can appreciate, cell D6 has been set as the objective function, cells D9–D16 have been set as the decision variables, and we have included both constraints in cells D19 and D20. How do we get to this screen?

Initially, the Solver screen will look like this (Fig. 11.16):

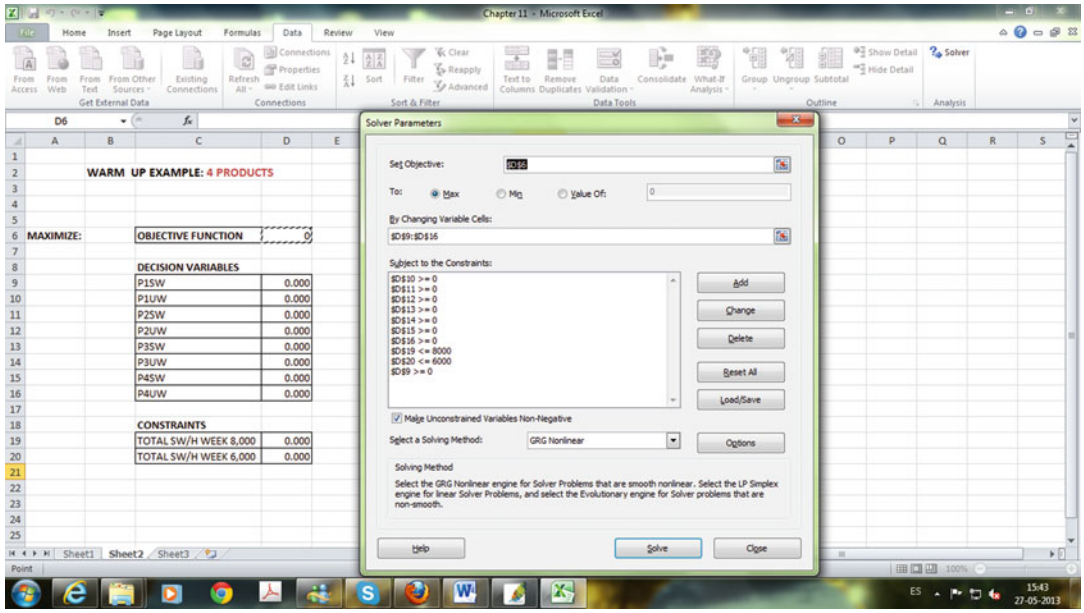

Fig. 11.15 Solver window showing cell D6 as the objective function, Cells D9 through D16 as the decision variables and constraints in Cells D19 and D20

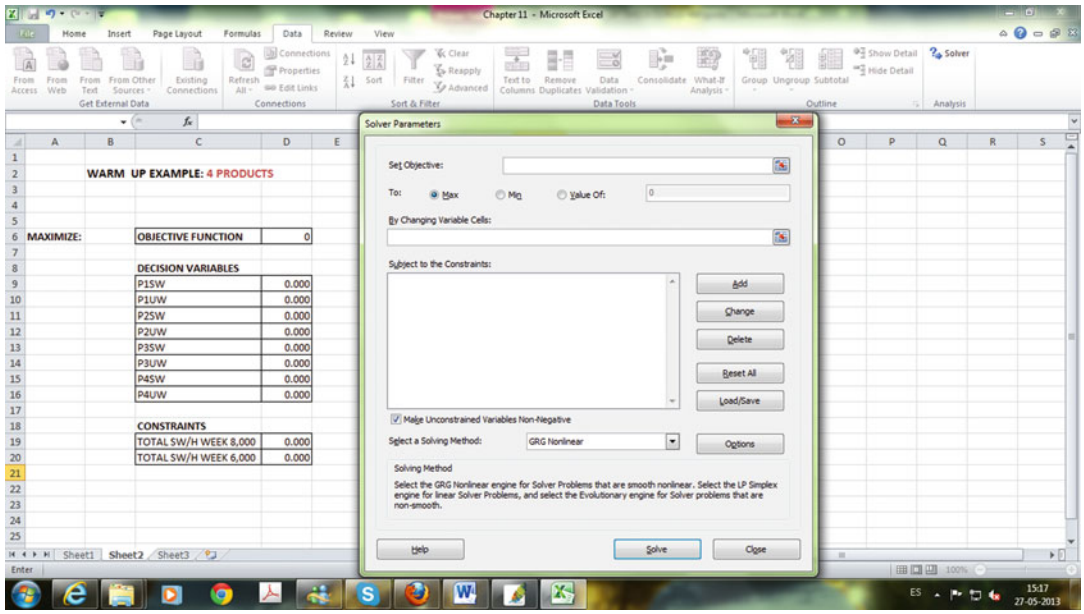

Fig. 11.16 Starting with the Solver window

The first step is to click on D6 to include the objective function in the Solver screen and, in addition, select **Max** (because we are maximizing). Then the screen will look like this (Fig. 11.17):

The second step is to include the decision variables (D9–D16). Therefore, you need to write D9: D16 below the statement that says **By Changing Variable Cells**, and the screen will look like this (Fig. 11.18):

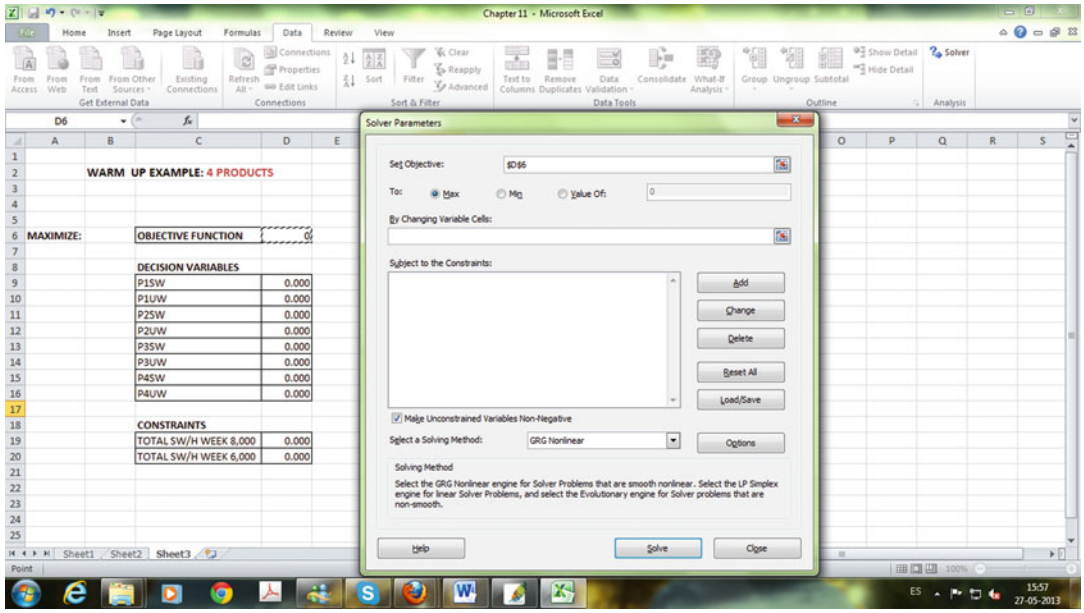

Fig. 11.17 Solver window with the addition of the objective function (D6)

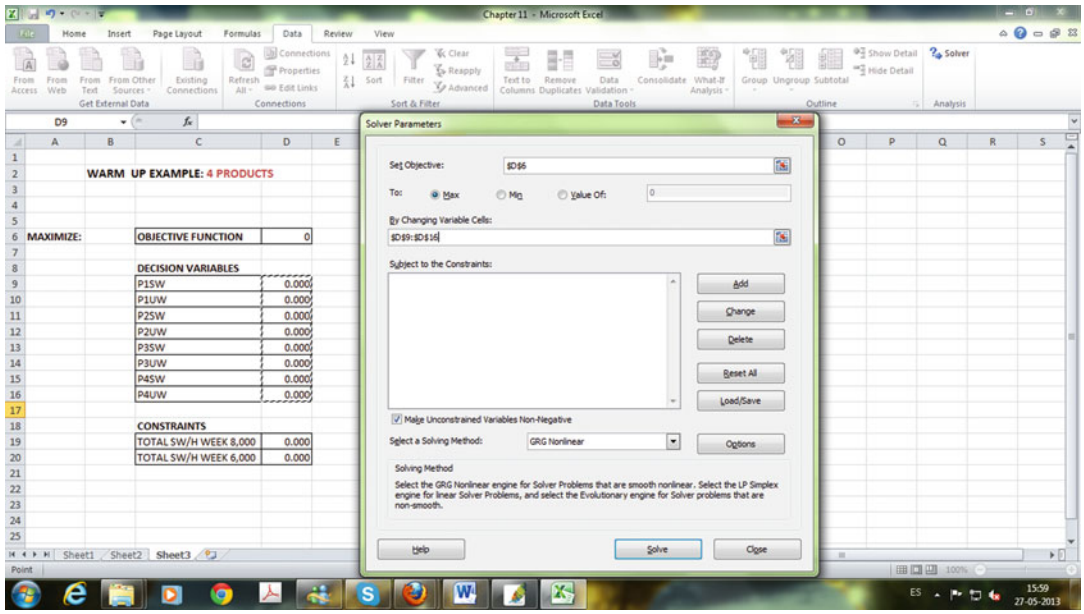

Fig. 11.18 Solver window with the addition of the changing variables (Cells D9 through D16)

The third step in completing the Solver window is to include all the constraints. First, we have the constraints that limit the total hours for skilled workers (8,000) and the total hours for unskilled workers (6,000). Then clicking Add in the Solver screen we get (Fig. 11.19)

Therefore, to include the first constraint, we need to click on D19 and select $\leq$ and 8,000 under the Constraint, and the screen should look like this (Fig. 11.20):

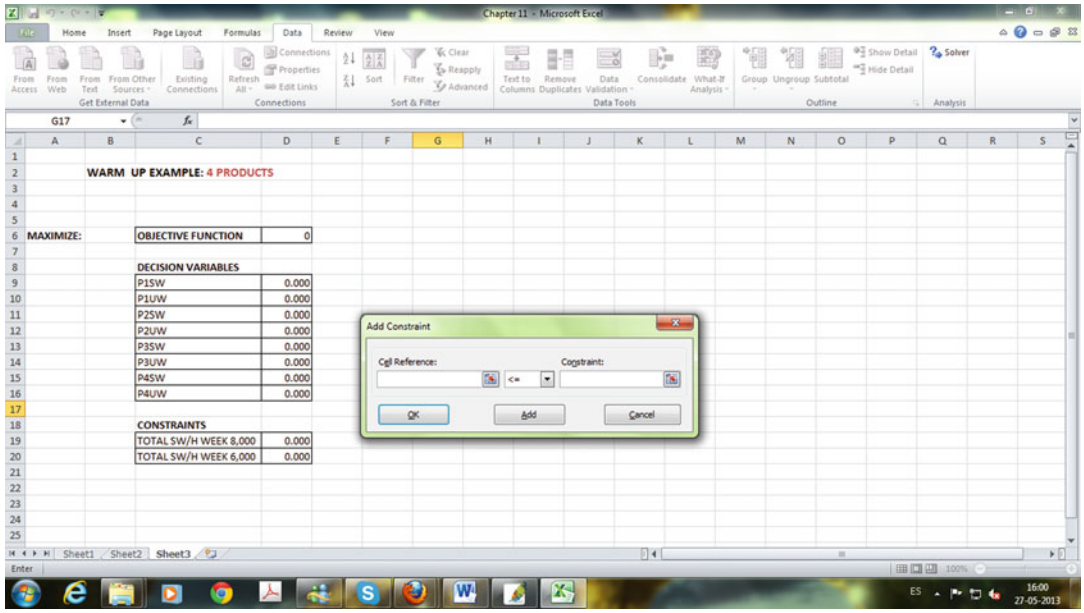

Fig. 11.19 Solver window to for the addition of all the constraints

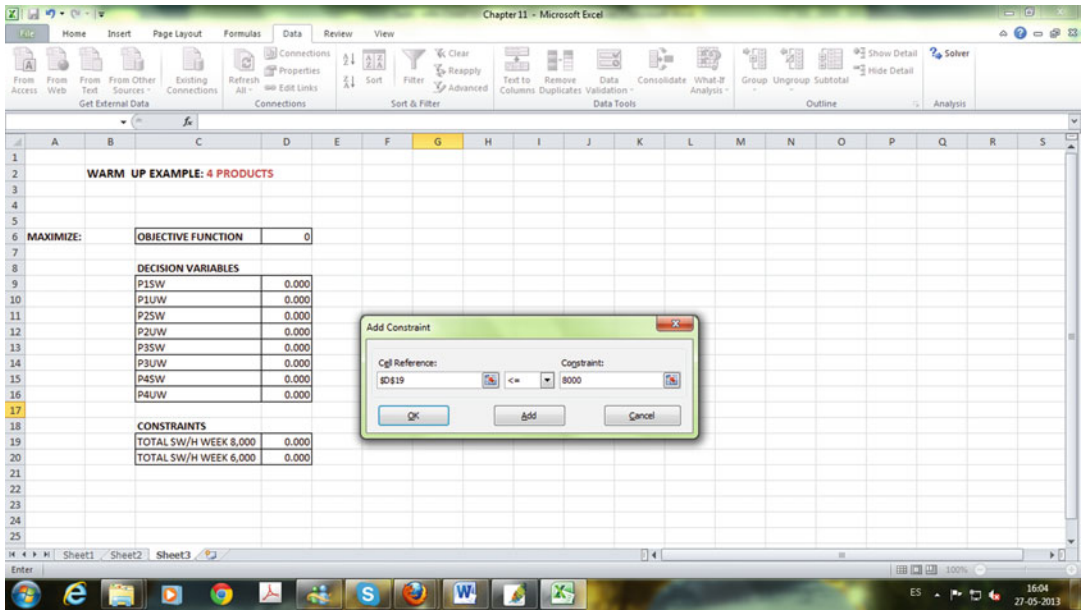

Fig. 11.20 Solver window showing the constraints that limit the total hours for skilled workers (8,000)

Finally, click on OK to get (Fig. 11.21)

In the same way you need to add all the constraints, including D20 $\leq$ 6,000 and the cells for P_{1SW}, P_{1UW}, . . ., P_{4UW} (D9–D16 should be $\geq$0). Finally, the Solver screen should look like the following image (Fig. 11.22):

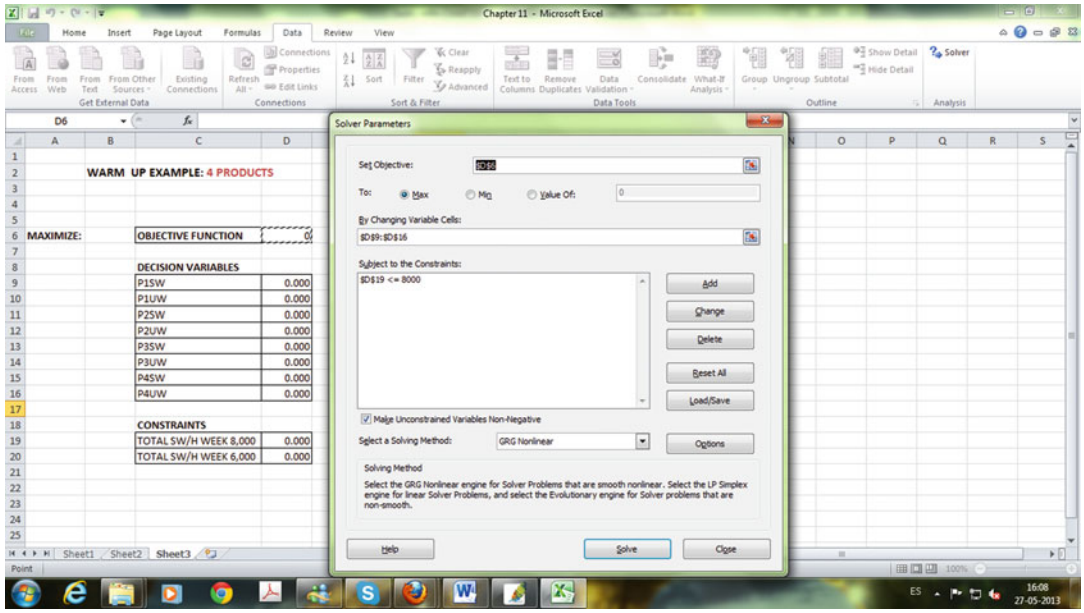

Fig. 11.21 Solver window with the addition of the first constraint

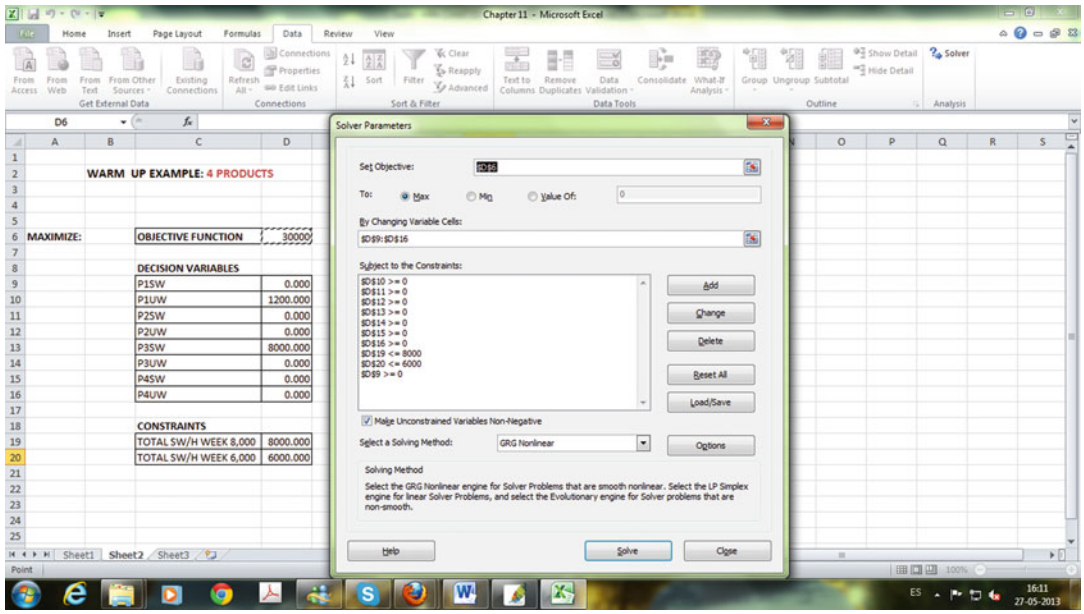

Fig. 11.22 Solver window with the addition of all the constraints, including that D20 $\leq$ 6,000 and the cells for P_{1SW}, P_{1UW}, …, P_{4UW} (D9 through D16 should be $\geq$ 0)

Now that we have included the objective function, selected Max (for maximization), included all the decision variables (D9–D16), and included all the constraints, we can solve the problem by clicking Solve to get the following screen (Fig. 11.23):

Indicating that Solver has found a solution that satisfies all the constraints and the optimality conditions. Therefore, we can click Ok to get (Fig. 11.24)

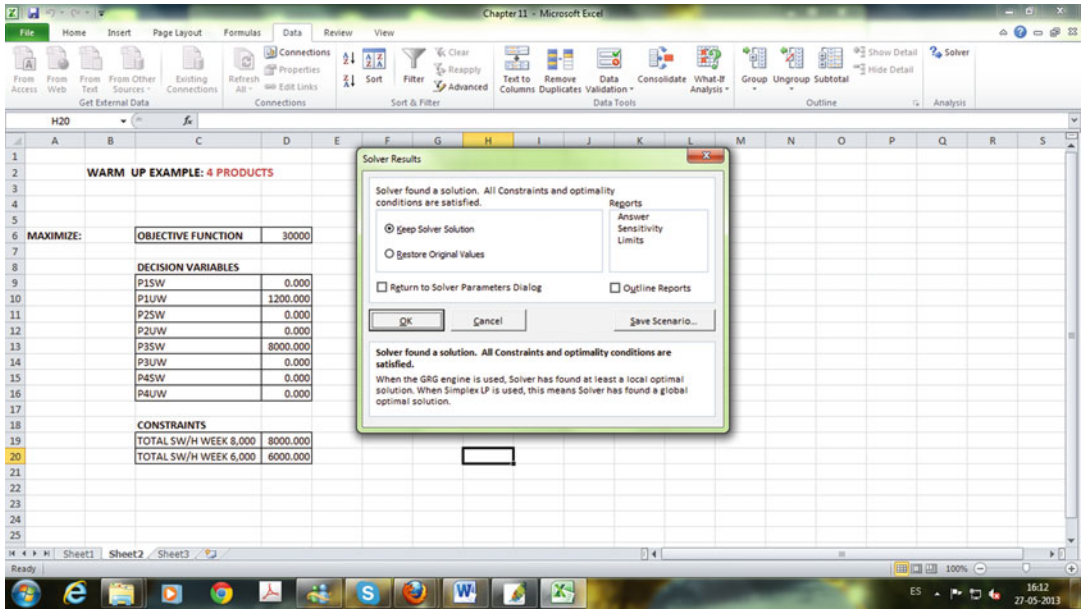

Fig. 11.23 Solver window showing that a solution has been found

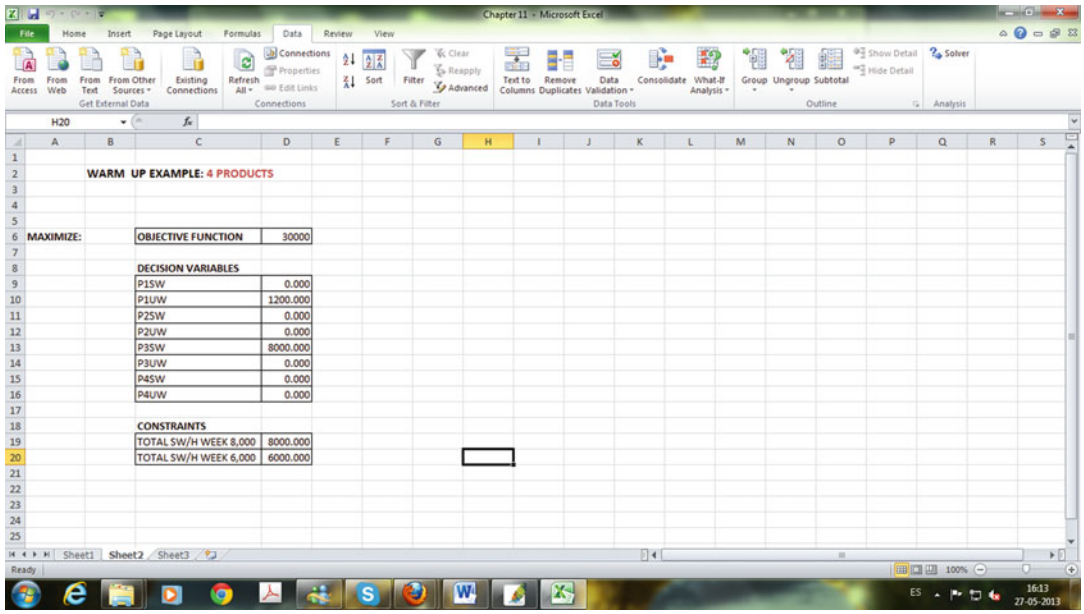

Fig. 11.24 Final screen depicting the solution found by Solver tool

According to this screen the optimum solution is

Objective function $= 30{,}000$ and
$P_{1SW} = 0$
$P_{1UW} = 1{,}200$ units of product 1 manufactured by unskilled workers
$P_{2SW} = 0$

$P_{2UW} = 0$
$P_{3SW} = 8{,}000$ units of product 3 manufactured by skilled workers
$P_{3UW} = 0$
$P_{4SW} = 0$
$P_{4UW} = 0$

Finally, we can check that both restrictions are satisfied and all the working hours are used; thus, we have no slack in labor resources. In the same way we can verify that with the number of units to be manufactured, the value of the objective function is 30,000.

11.7 Solved Problems

In this section we present and develop three maximum and minimum problems and two OR problems. In both cases the emphasis will be on problem formulation because in the warm-up examples we already detailed the solution procedure with the help of the Excel spreadsheet.

11.7.1 Maximum, Minimum, and Applications

1. Maximum or minimum? [3]. (a) Determine whether the following function [$f(x)$] presents a maximum or a minimum. (b) What is the value of x that maximizes or minimizes the $f(x)$ function?

$$f(x) = 2x^2 - 5x + 3.$$

Solution

(a) Graphical solution

As suggested in the warm-up example, we will first plot a graph to find the shape of the curve and a tentative solution. Then we will use the Solver tool to confirm the graphic solution or get a more accurate solution.

Plotting $f(x)$ for x values from 0 to 2 (Fig. 11.25).

According to the graphical solution, $f(x)$ presents a **minimum** value for $x = 1.25$.

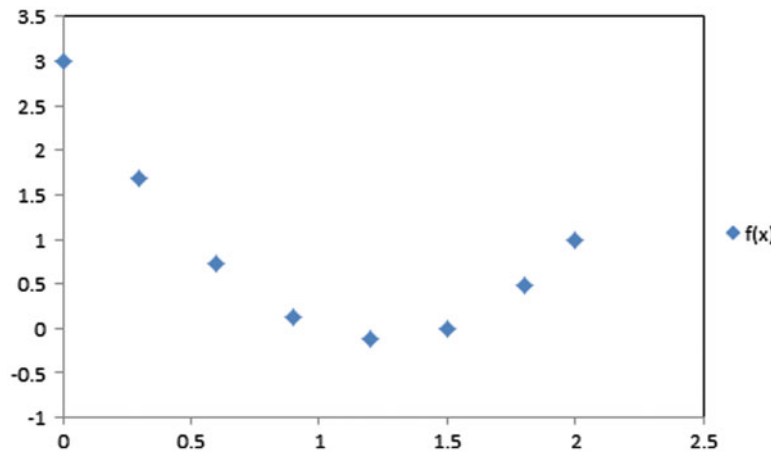

Fig. 11.25 Plotted values for the function $f(x) = 2x^2 - 5x + 3$

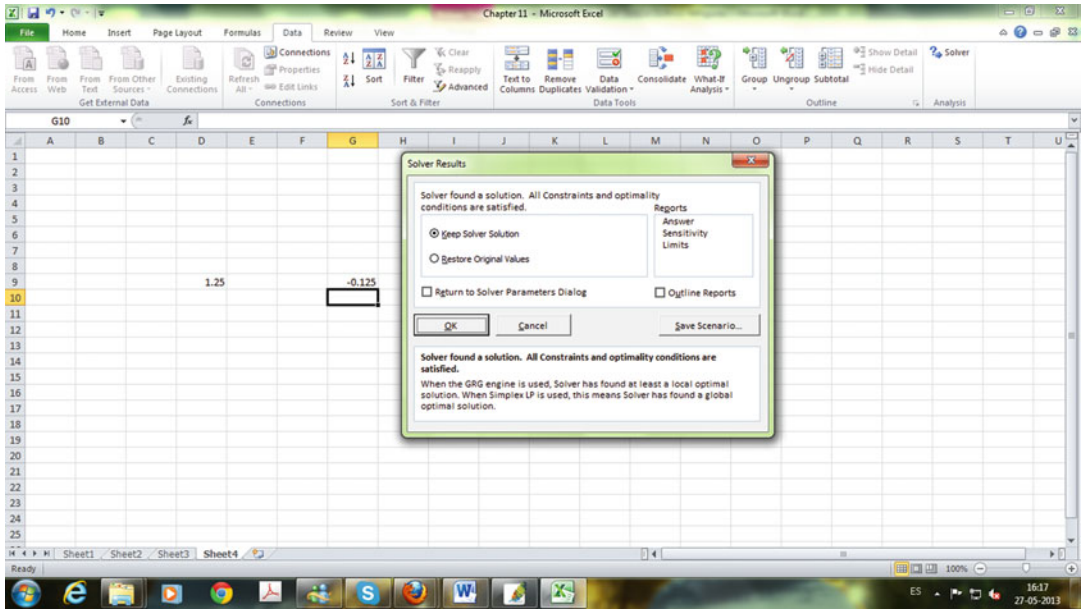
Chapter 11 - Microsoft Excel
Solver Results
Solver found a solution. All Constraints and optimality conditions are satisfied.
Reports
Answer
Sensitivity
Limits
Keep Solver Solution
Restore Original Values
Return to Solver Parameters Dialog
Outline Reports
OK
Cancel
Save Scenario...
Solver found a solution. All Constraints and optimality conditions are satisfied.
When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.
1.25
-0.125

Fig. 11.26 Solver window for the minimum value of x in function $f(x) = 2x^2 - 5x + 3$ ($x = 1.25$ and $f(x) = -0.25$)

(b) Solver

As explained in the warm-up example, we first choose a cell for x (D9) and assign it a value of 0. Then we write the function in cell G9. Now click on Data and then Solver. In the Solver tool box we set the objective function (G9), choose minimum, Min, then in By Changing Variable Cells, we select D9 and finally click on Solve to get (Fig. 11.26)

Showing the same result as was obtained with the graphical solution, x $= 1.25$ and $f(x) = -0.25$.

2. Book edition [5]. An important publisher is planning to edit a new and innovative book that combines chemical and bioprocess engineering. After a careful cost and marketing study the production manager reports to the general manager the following information:

$$Q = 3,000 - 20P, \tag{11.2}$$

$$C = 4,000/Q + 18. \tag{11.3}$$

Q: Quantity of books manufactured
P: Selling price per each book ($)
C: Cost per book ($)

Then the general manager decides to produce 1,500 books at a price of $75 and a cost of $20.67, giving the company profits of $81,500. What do you think? Is the general manager taking the optimum decision?

Solution

Step I

Graphical representation and mathematical formulation

In this case it is not necessary to do a graphical representation of the problem statement, but we need a mathematical formulation. The objective should be to maximize company profits, where we can define the profits (U) as follows: $Profits = input - output$. Therefore, we can write $U = PQ - CQ$.

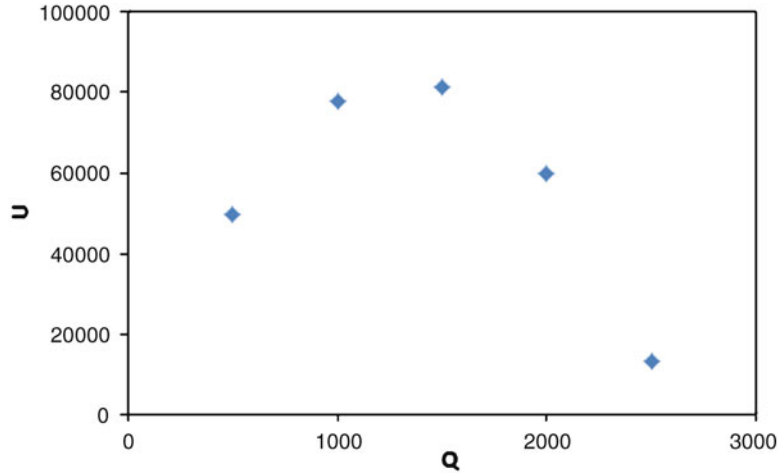

Fig. 11.27 Plotted values for U (profits) against Q (quantity of books manufactured)

Then from (11.2) and (11.3) and rearranging we get

$$U = \left(\frac{(3,000 - Q)}{20}\right)Q - \left(\frac{4,000}{Q} + 18\right)Q,$$

$$U = 132Q - \frac{Q^2}{20} - 4,000. \tag{11.4}$$

Step II

(a) Graphical solution

A plot of U (profits) against Q (quantity of books made) is depicted in the following graph (Fig. 11.27):

According to the graph the optimum value is at $Q = 1{,}500$. Refining the graph we get (Fig. 11.28)

Now we can see that the optimum is approximately $Q = 1{,}300$ and $U = 83{,}100$, suggesting that the general manager's decision was not optimal. Before coming to a final opinion and solution for the problem we will solve it with the Solver tool.

(b) **Solver**

We assign Q to cell F9 (with a starting value of 1,000) and write the objective function (U) in cell H9. Then in the Solver tool box we set H9 as the objective function, Max for maximization, and F9 as a changing variable cell. Then the screen of the spreadsheet will look like this (Fig. 11.29):

Clicking in Solve we get (Fig. 11.30)

Where the optimum value is $Q = 1{,}320$ and $U = 83{,}120$, which is better than our approximately graphical solution and confirming that the general manager's decision was not optimal.

3. **Box design [5]**. A fine chemical (4 [L]) should be shipped in a special box. The material to manufacture these special parallelepiped boxes comes in a sheet of the following dimensions: length 50 [cm], width 30 [cm] (Fig. 11.31). (a) What value of x maximizes the volume of the box? (b) What is the volume of the box? Use both graphical and Solver procedures to solve the problem.

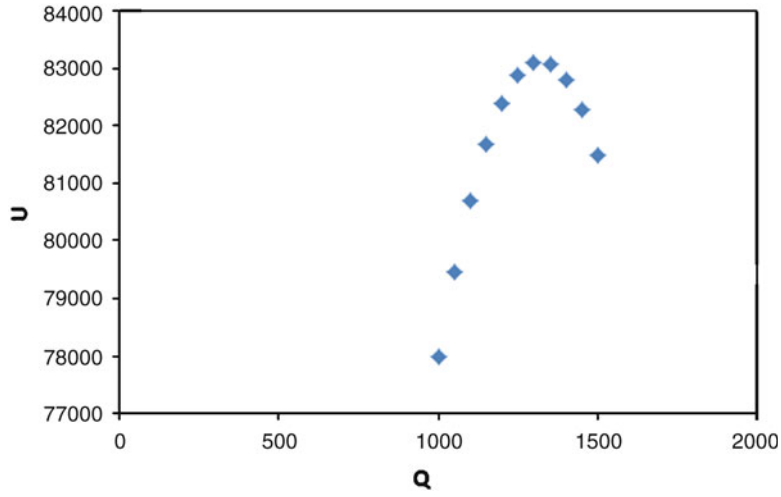

Fig. 11.28 Refined plotted values for U (profits) against Q (quantity of books manufactured)

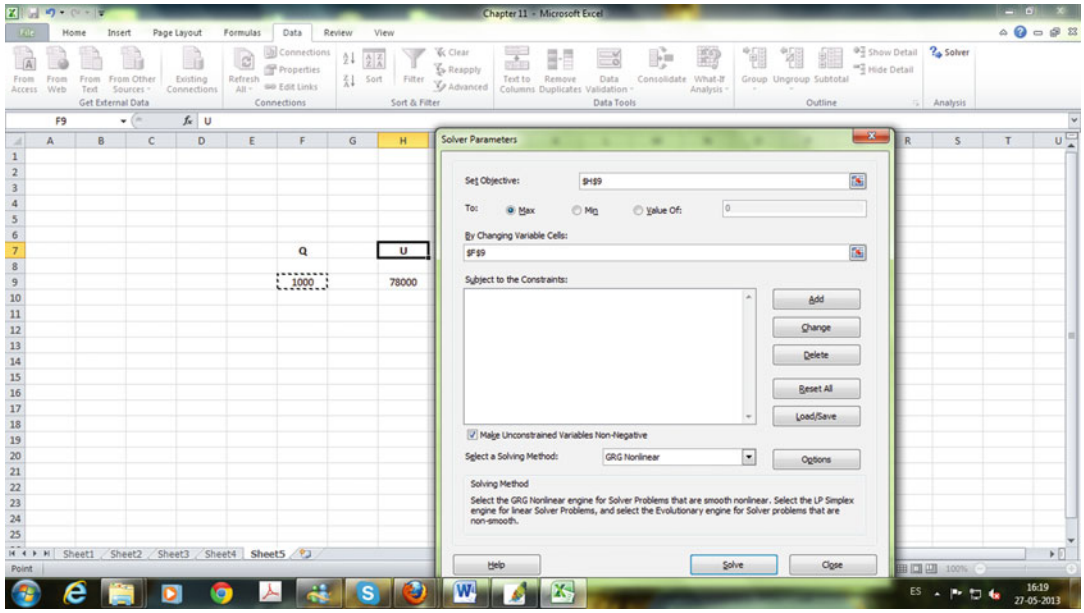

Fig. 11.29 Solver window where cell F9 has been assigned for Q (with a starting value of 1,000) and the objective function (U) in cell H9

Solution

Step I

Graphical representation and mathematical formulation

According to Fig. 11.32, the volume of the box as a function of x is equal to

$$V = (50 - 2x)(30 - 2x)x \quad \text{or} \quad V = 4x^3 - 160x^2 + 1,500x.$$

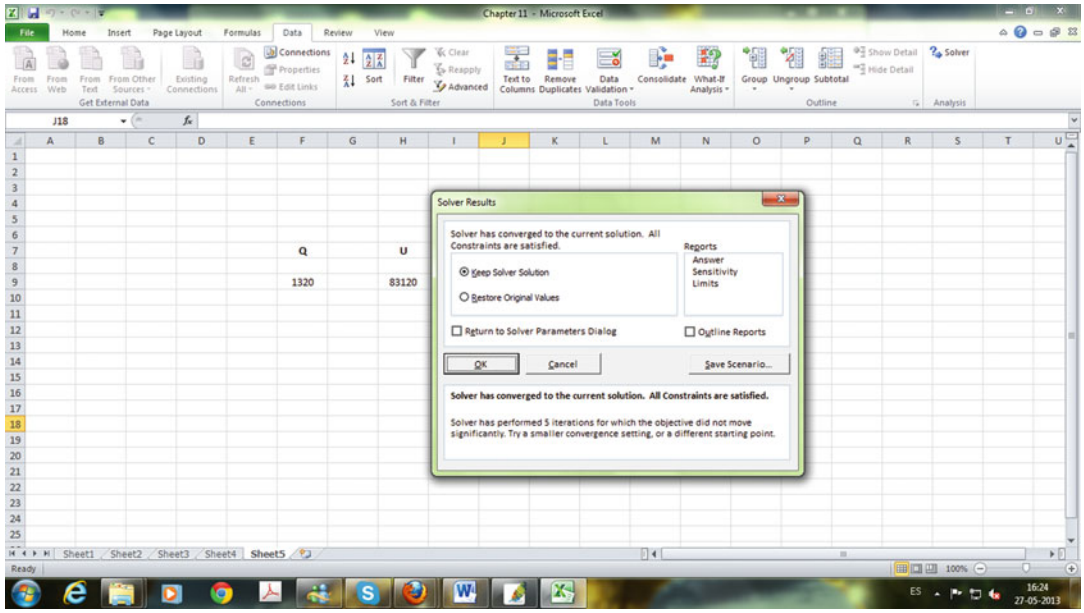

Fig. 11.30 Final screen depicting the solution found by Solver tool

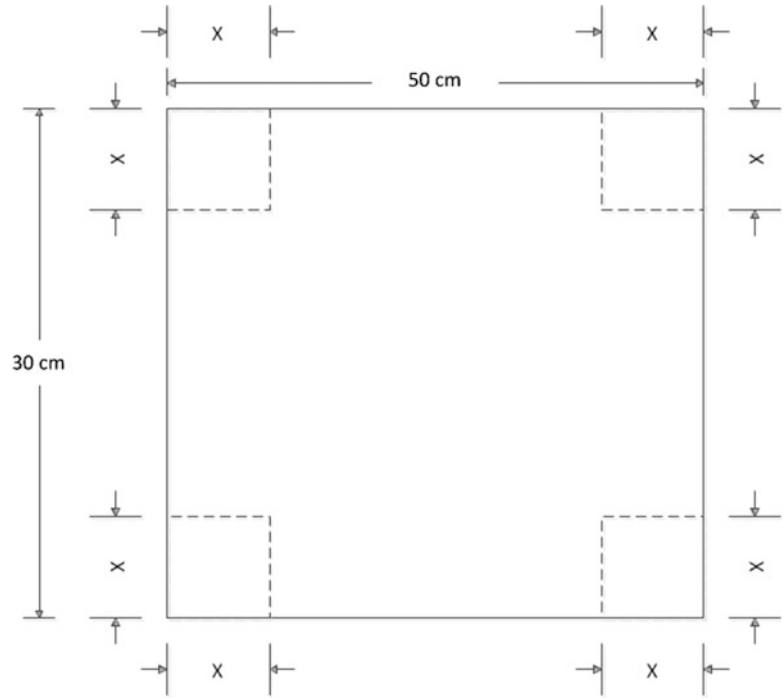

Fig. 11.31 Parallelepiped box from a sheet with the following dimensions: length 50 cm and width 30 cm

Step II

(a) Graphical solution

Before creating a graph, by inspection we can see that the value of x is in the range of 0–15. Then creating a table for x and V taking values of x from 0 to 15, we get

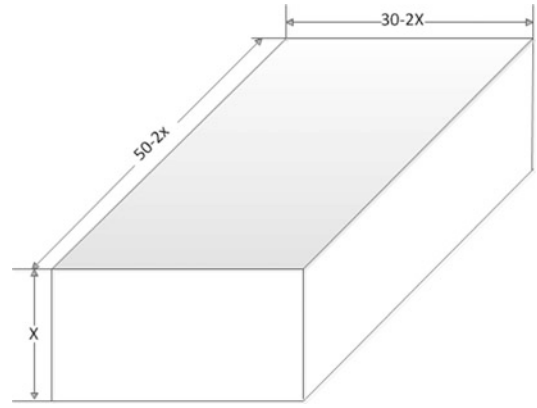

Fig. 11.32 Parallelepiped box

x [cm]	V [cm^3]
0	0.0
1	1,344
2	2,392
3	3,168
4	3,696
5	**4,000**
6	**4,104**
7	**4,032**
8	3,808
9	3,456
10	3,000
11	2,464
12	1,872
13	1,248
14	616.0
15	0.0

Then the maximum volume of the box is for 5 [cm] $< x <$ 7 [cm]. After several data refinements we get

X [cm]	V [cm^3]
6.00	4,104.0
6.02	4,104.2
6.04	4,104.3
6.06	**4,104.4**
6.08	4,104.4
6.10	4,104.3

Then, according to this approximate procedure, a value of $x = 6.06$ [cm] will maximize the volume of the box (approximately 4.1 [L]).

(b) Solver

We assign x to cell E5 [with a starting value of 6, following the result obtained in (a)], and we write the objective function (V) in cell G5. Then in the Solver tool box we set G5 as the objective function,

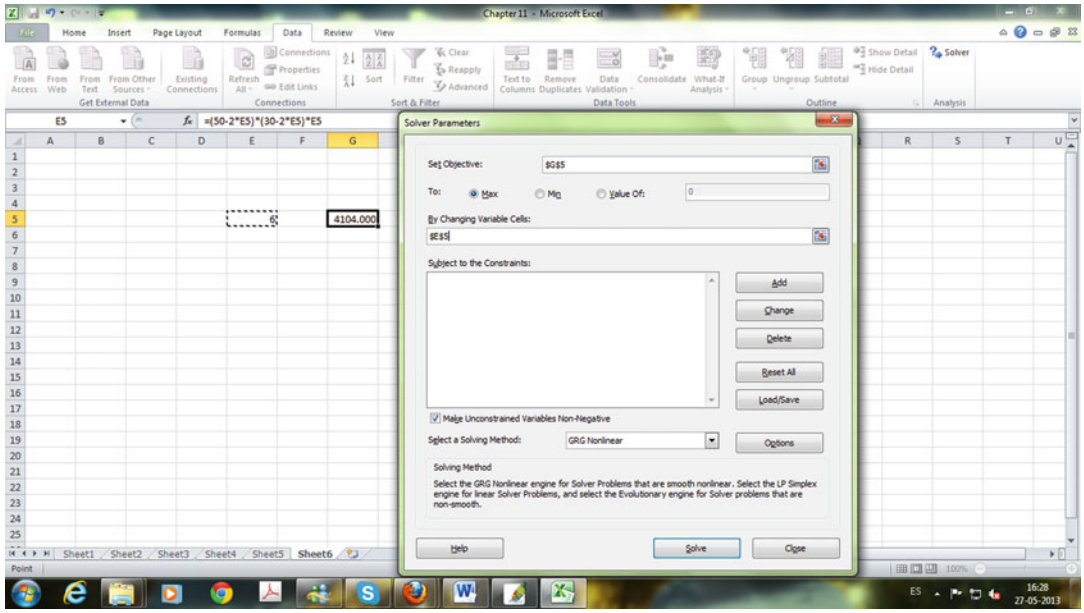

Fig. 11.33 Solver window where cell E5 has been assigned for x (with a starting value of 6)

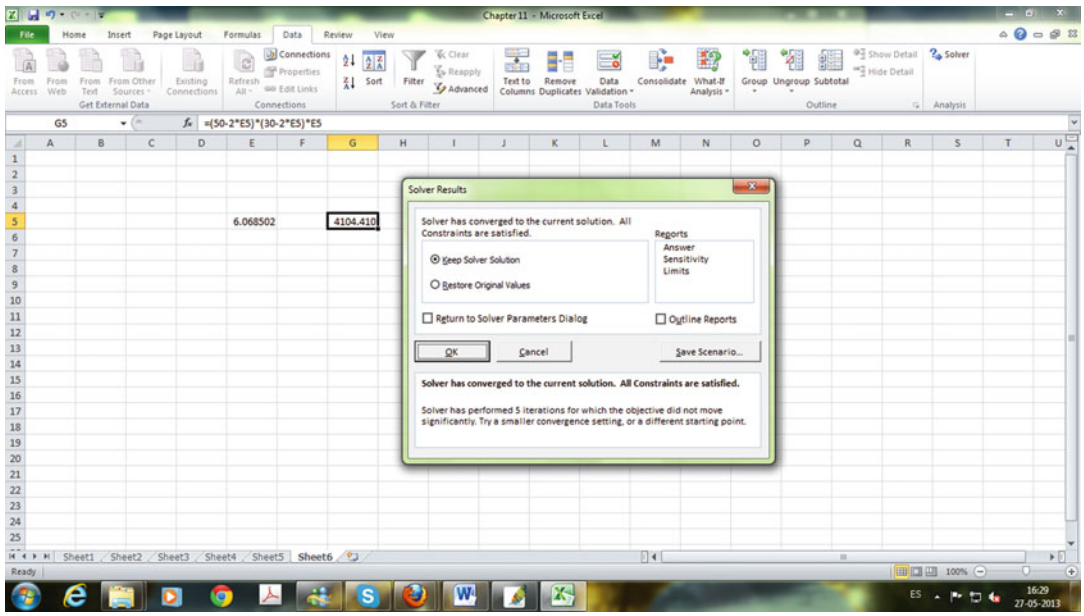
Chapter 11 - Microsoft Excel
Solver Results
Solver has converged to the current solution. All Constraints are satisfied.
Reports
Answer
Sensitivity
Limits
Keep Solver Solution
Restore Original Values
Return to Solver Parameters Dialog
Outline Reports
OK
Cancel
Save Scenario...
Solver has converged to the current solution. All Constraints are satisfied.
Solver has performed 5 iterations for which the objective did not move significantly. Try a smaller convergence setting, or a different starting point.
=(50-2*E5)*(30-2*E5)*E5
6.068502
4104.410

Fig. 11.34 Final screen depicting the solution found by Solver tool

Max, for maximization and E5 as a changing variable cell. Then the screen of the spreadsheet will look like this (Fig. 11.33):

Clicking on Solve we get (Fig. 11.34) where the optimum value for x is 6.0685 [cm] and $V = 4{,}104.4$ [cm^3]. Again the approximate solution given in (a) is close to the "exact" solution obtained with the Solver tool. Solver is not only more accurate but also simpler, although the graphical (or table) solution is much more visually friendly.

11.7.2 Operations Research Problems

4. Bakery [6]. A bakery has 140 hundredweight of flour to make two types of bread, premium and regular. Regular bread costs $35 per hundredweight, while premium bread costs $65 per hundredweight. The bakery has available $7,200 of capital to make bread. Each hundredweight of regular bread requires 1 h of work, and each hundredweight of premium bread requires 2.5 h. The maximum hours available for work are 330. If the bakery expects to make a profit of $130 per hundredweight of regular bread and $180 per hundredweight of premium breads, how many hundredweight go to regular and premium bread, respectively, to maximize the benefit to the bakery?

Solution

Step I

Variable definition and codification

R: Hundredweight of flour for regular bread

P: Hundredweight of flour for premium bread

B: Total benefit to bakery in dollars

Step II

Formulation of objective function

The total benefits (B) will be $B = 130 \times R + 180 \times P$.

Step III

Constraints

We have three constraints, 140 hundredweights of flour, $7,200 of capital, and 330 h of labor:

Flour (hundredweights):	$R + P \leq 140$,
Capital [$]:	$35 \times R + 65 \times P \leq 7{,}200$,
Labor [h]:	$R + 2.5 \times P \leq 330$.

In addition, all decision variables should be integers and equal to or greater than 0 (R and P integers and ≥ 0).

Step IV

As detailed and explained in the warm-up example (11.5.5) we will use the Solver tool from Microsoft Excel. The following screen shows the objective function (cell L6), the variables R and P (cells I9 and I10, respectively), and the constraints for Flour, Capital and Hundredweights (cells D9, D10 and D11 respectively). We tentatively start with initial values of $R = 50$ and $P = 30$ (Fig. 11.35).

Then, as shown in the following screen, in the Solver tool box we include objective function, changing variables and constraints (Fig. 11.36).

Clicking on Solve we obtain (Fig. 11.37) where the optimum solution is $R = 63.3$ hundredweight for regular bread and $P = 76.7$ hundredweight for premium bread, and with a total benefit of $B = \$22{,}033$.

5. Catalyst [5]. Chemical reactions must be catalyzed, and it has been found that the best combination of catalysts is to use at least 1.5 [mg] of catalyst C_1 and 8.5 [mg] of catalyst C_2. Two products (P_1 and P_2) are sold that contain these two catalysts. The following table provides information on the concentrations of catalysts C_1 and C_2 per gram of each product and, in addition, the cost of these products. Because the products are very expensive, you are asked to determine how many grams must be purchased of each product to minimize the total cost?

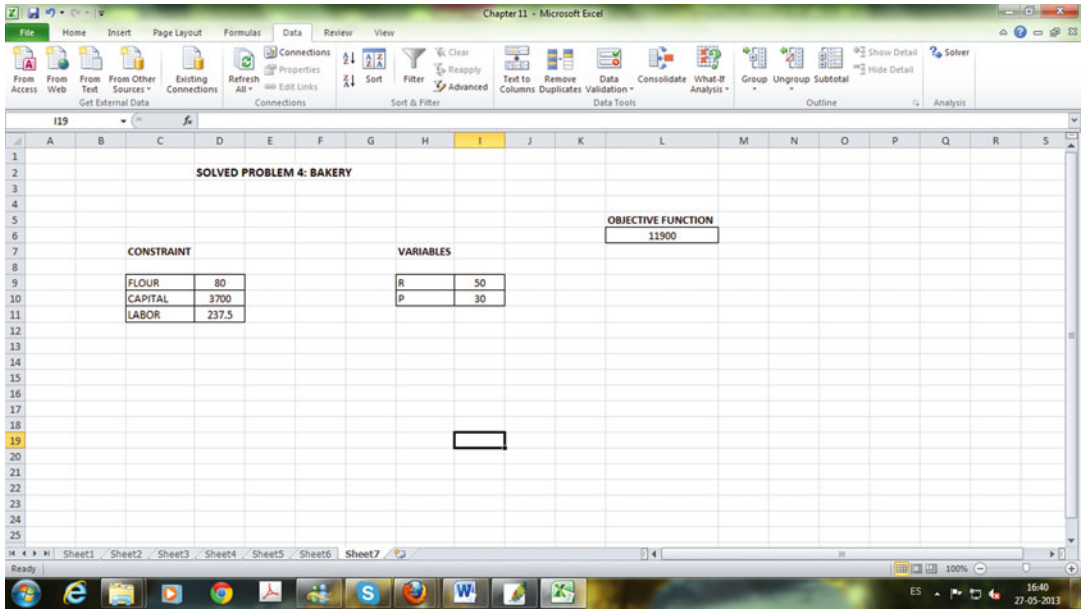

Fig. 11.35 Microsoft Excel screen showing the Objective Function (cell L6), the variables R and P (cells I9 and I10 respectively) and the constraints for Flour, Capital and Hundredweights (cells D9, D10 and D11 respectively)

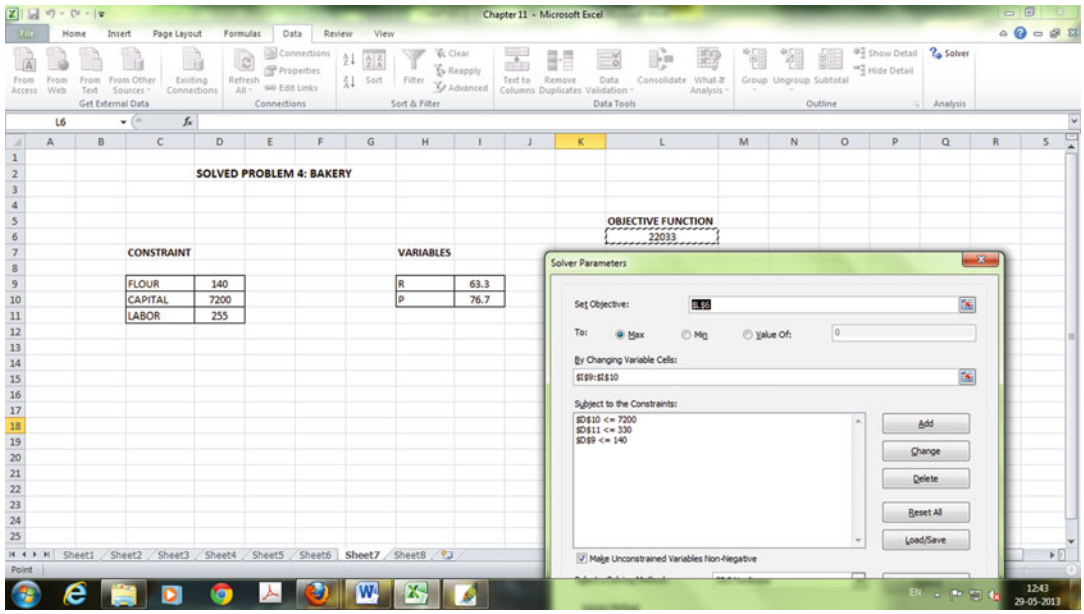

Fig. 11.36 Solver tool box where variables R and P should be integer (I9 and I10 = integer)

	Catalyst C_1 [mg/g]	Catalyst C_2 [mg/g]	Price [$/g]
Product P_1	0.12	0.4	200
Product P_2	0.13	1.4	240

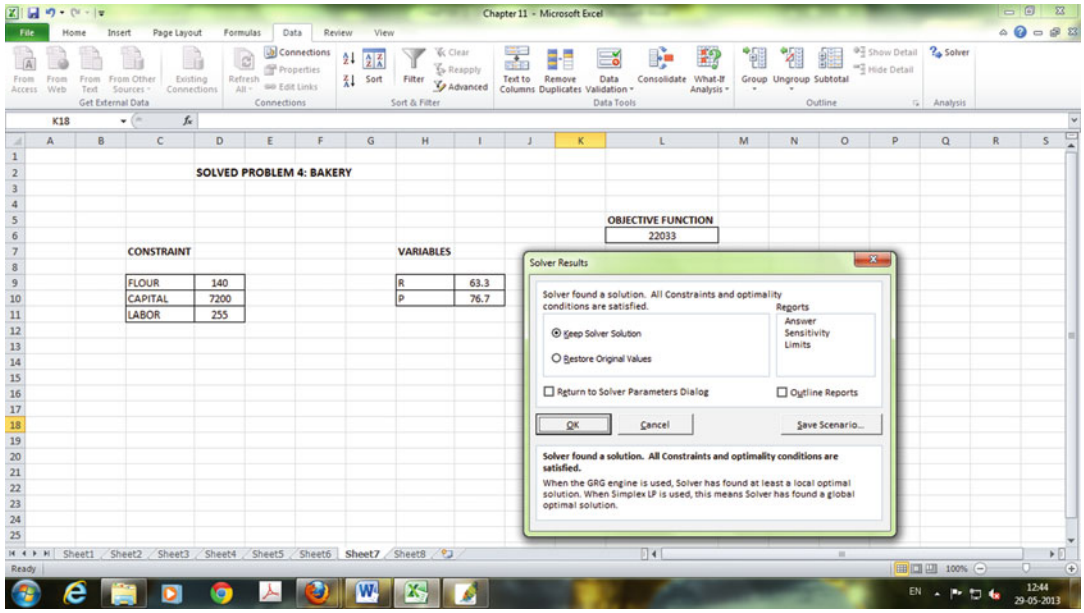

Fig. 11.37 Final screen depicting the solution found by Solver tool

Solution

Step I

Variable definition and codification

G_{P1}: Grams of product P_1 [g]

G_{P2}: Grams of product P_2 [g]

TC: Total cost [$]

Step II

Formulation of objective function

The total cost (TC) will be $\text{TC} = 200 \times G_{P1} + 240 \times G_{P2}$.

Step III

Constraints

We have two constraints, 1.5 [mg] of catalyst C_1 and 8.5 [mg] of catalyst C_2:

$$\text{CATALYST } C_1: \quad 0.12 \times G_{P1} + 0.13 \times G_{P2} \geq 1.5,$$

$$\text{CATALYST } C_2: \quad 0.4 \times G_{P1} + 1.4 \times G_{P2} \geq 8.5.$$

In addition, all decision variables should be equal to or greater than 0 (G_{P1} and $G_{P2} \geq 0$).

Step IV

As detailed and explained in the previous example, we will use the Solver tool from Excel. The following screen shows the objective function (cell J5), the variables G_{P1} and G_{P2} (cells I9 and I10, respectively), and the constraints for catalyst C_1 and catalyst C_2 (cells D11 and D12, respectively). We tentatively start with initial values of $G_{P1} = 8$ [g] and $G_{P2} = 5$ [g] (as shown in the screenshot, this is a feasible solution) (Fig. 11.38):

Then, as shown in the following screen, in the Solver tool box we include the objective function (J5), Min to minimize, I9 and I10 as the changing cells and constraints indicating that catalyst C_1 should be $\geq$1.5 [mg] and catalyst C_2 should be $\geq$8.5 [mg] (Fig. 11.39):

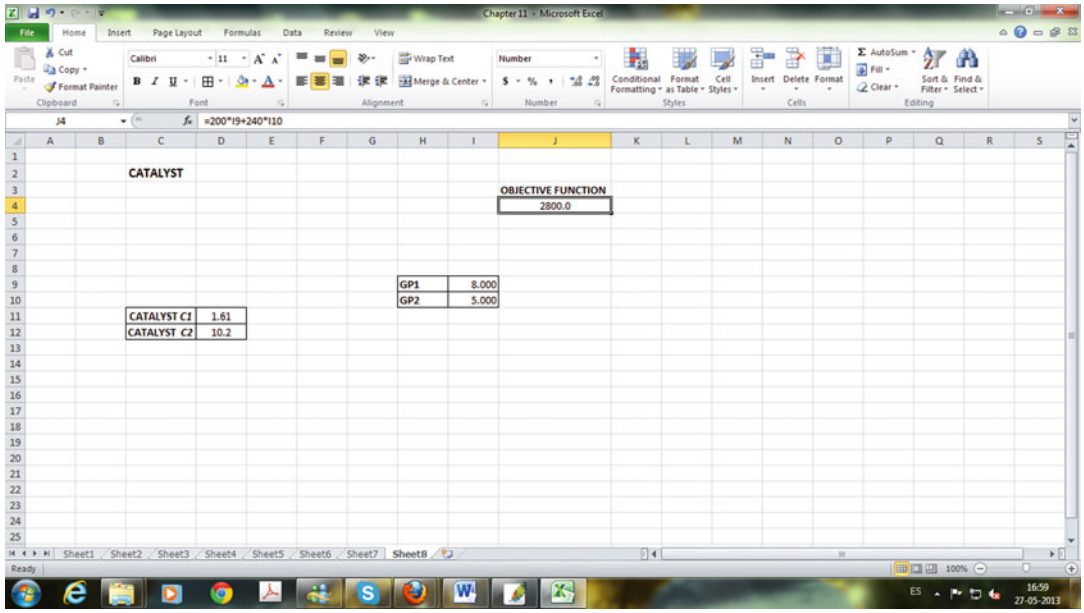

Fig. 11.38 Microsoft Excel screen showing the Objective Function (cell J5), the variables G_{P1} and G_{P2} (cells I9 and I10 respectively) and the constraints for catalyst C_1 and catalyst C_2 (cells D11 and D12 respectively)

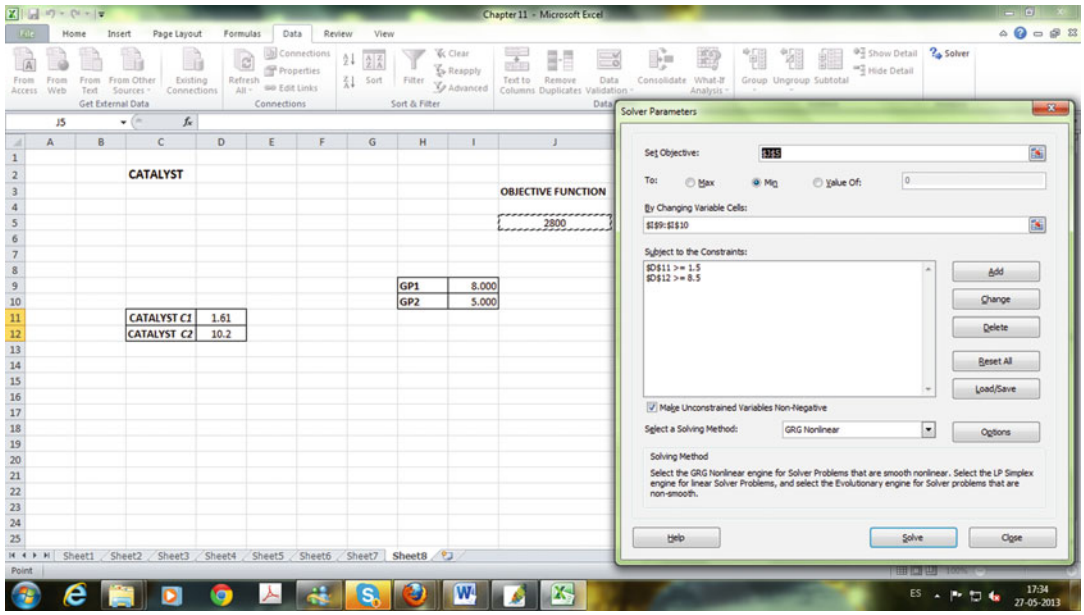

Fig. 11.39 Microsoft Excel screen including the Solver window

Clicking on Solve we obtain (Fig. 11.40)

Where the optimum solution is $G_{P1} = 8.578$ [g] and $G_{P2} = 3.621$ [g] and the minimum total cost is TC = $2,584.5.

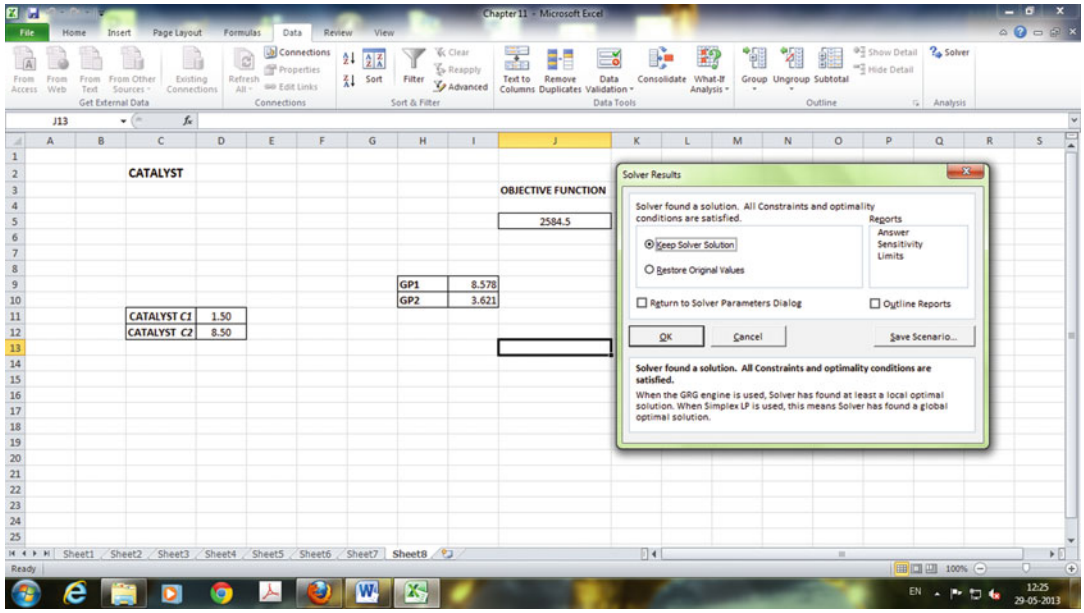
Chapter11 - Microsoft Excel
CATALYST
OBJECTIVE FUNCTION
2584.5
GP1 8.578
GP2 3.621
CATALYST C1 1.50
CATALYST C2 8.50
Solver Results
Solver found a solution. All Constraints and optimality conditions are satisfied.
Reports
Answer
Sensitivity
Limits
Keep Solver Solution
Restore Original Values
Return to Solver Parameters Dialog
Outline Reports
OK
Cancel
Save Scenario...
Solver found a solution. All Constraints and optimality conditions are satisfied.
When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.

Fig. 11.40 Final screen depicting the solution found by Solver tool

11.8 Proposed Problems

11.8.1 Maximum, Minimum, and Applications

1. **Maximizing and minimizing *Y* [4]**. Check for different values of a, b, and c that for the following equation the value of X that maximizes Y is equal to $-b/2a$ when a is negative and minimizes Y when a is positive:

$$Y = aX^2 + bX + c.$$

2. **Finding *X* [3]**. Find the value of X that minimizes the value of Y in the following equations: (a) $Y = 5X^2 - 20X + 15$; (b) $Y = 7X^2 - 35X + 18$; (c) $Y = 9X^2 - 9X + 10$.
   A: (a) $X = 2$; (b) $X = 2.5$; (c) $X = 0.5$
3. **Finding *X* [3]**. Find the value of X that maximizes the value of Y in the following equations: (a) $Y = 12 + 6X - 6X^2$; (b) $Y = 12 + 10X - 5X^2$; (c) $Y = 19 + 36X - 6X^2$.
   A: (a) $X = 0.5$; (b) $X = 1$; (c) $X = 3$
4. **Maximum product [4]**. Find two positive real numbers whose sum is 100 and whose product is a maximum.
   A: 50 and 50
5. **Airplane [6]**. A cargo plane is arriving in Santiago from southern Chile at a speed of 600 [km/h]. When the cargo plane is 200 [km] from Santiago, a Cessna airplane departs from Santiago to Argentina at a speed of 300 [km/h]. When will the minimum distance be reached between the cargo and the Cessna airplanes after the departure of the Cessna airplane? **Assumption**: The two airplanes are moving at an angle of 90° from each other.
   A: After 16 min (0.267 h)

6. **Pencils [5]**. A company that sells pencils is going through a critical time. The manager of the company, Mr. Johnson, decides to hire an engineer to help resolve the problem. Based on an economic analysis, the engineer determines the functions of sales and costs for the company as follows:

$$Q = 5,000 - 100P,$$
$$C = 35,000 + 15Q,$$

where Q is the quantity of pencils produced, P is the selling price of each pencil ($), and C is the total costs of producing Q pencils ($).
Using these equations, the engineer determines that the company is operating in suboptimal conditions and suggests that a new price and quantity should be established by the company. (a) What is the optimum price for the pencils? (b) What is the optimum number of pencils to manufacture? (c) Could the company earn a profit? **Hint**: Profits = Input − Output.
A: (a) $17.5; (b) 3,250 pencils; (c) Yes, $21,612.50.

7. **Optimum speed [10]**. The owner of an aircraft wants to start doing commercial trips. For this he needs to hire a pilot, who requires a monthly salary of $10,000 plus an allowance per flight hour of $30. We also know that legally, the pilot cannot fly more than 100 h/month.
For each flight, the plane needs maintenance to verify proper operation, which means an average cost of $2,000 per trip. The aircraft has a fuel consumption that is proportional to the flying speed, where 0.1 + v/2,000 [L/km], determined under windless weather conditions, and the price for the fuel is $3/L. The owner plans to do regular trips between Columbus, Ohio, and New York City, a distance of 600 [km]. What is the optimal flying speed on the flight to New York City, given that, on average, the wind has a speed of 15 [km/h] and moves from east to west?
A: 205.7 [km/h]
8. **Cartons [10⁺]**. You are assigned to construct cartons having a volume of 1 [m^3]. You have a machine that performs cuts of 0.2 × 0.2 [m] in the corners of the carton. What dimensions of the carton sheet would minimize consumption of cardboard?
A: 2.636 [m] each side
9. **Canned food [6]**. You have been charged with designing a cylindrical jar for a new canned food product. Try out different can volumes (e.g., 0.3, 0.5, and 0.8 [L]) whereby the optimal ratio between height (H) and diameter (D), in order to minimize the amount of material (area including both lids), is when $H = D$.
10. **Storage tank [6]**. You have been assigned to design a stainless steel cylindrical storage tank of 200 [m^3] without a lid. What are the optimal dimensions [diameter (D) and height (H)] of the tank to minimize the amount of stainless steel used?
A: $D \sim 4$ [m] and $H \sim 3.98$ [m]
11. **Editorial [7]**. An editorial company wishes to publish a booklet of 25,000 words. According to the technical department, two words occupy a space of 1 [cm^2] of printed material. Standard policy of the editorial company is that each page should have margins of 1 in per side and 1.5 in at the top and bottom of the page. If the editorial wants to print the booklet in 200 pages, what are the optimum dimensions of each page in order to minimize the consumption of paper?
A: 8.60 and 13.18 [in.]
12. **Riverside [5]**. A chemical plant is located by a river that is 1 [km] wide. In addition, 2 [km] upstream is a power station. A cable needs to be run from the chemical plant to the power station at minimum cost. It costs $4 to place each meter of cable through the land and $5 through the river.

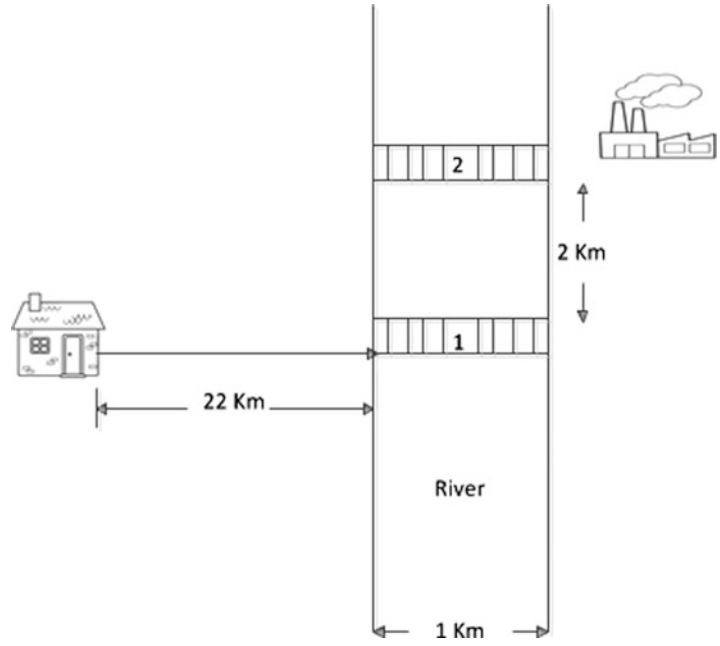

Fig. 11.41 Schematic representation of the bioprocessing plant that is located at the other side of the river

How much cable should you run through the land and the river? **Assumption**: Ignore the cost of the cable.

A: 2/3 [km] through the land and 1.67 [km] through the river

13. **Going to work [5]**. Every day, you take the highway to work at the bioprocessing plant. The plant is located on the other side of the river (Fig. 11.41). Your average speed is 100 [km/h], and it takes you 15 [min] to get to the processing plant. In addition, there are two bridges you could take to get to work; call them bridge 1 and bridge 2. Because you have an all-wheel-drive vehicle, at any time on the way to work you can bypass bridge 1 through the country and go directly to bridge 2, but the speed limit along that route is just 20 [km/h]. One day, bridge 1 is under construction and you are forced to take bridge 2 to get to work. (a) What is the optimum route? (b) How long will the trip take?

    A: (a) You should go approximately 21.6 [km] on the highway, then use the back roads (on a diagonal; see Fig. 11.42) to reach bridge 2. Go one more kilometer to cross the river. (b) Approximately 20 min 2 s.

14. **Businessman [6]**. An important businessman is planning a long car trip from Columbus, Ohio, to Auburn, Alabama (approximately 1,200 [km]). The businessman rents a car and is told by the car rental company that the car's gas consumption is directly proportional to the average speed of the car. At an average speed of 90 [km/h], the fuel efficiency is 12 [km/L] and at 100 [km/h], it is 10 [km/L]. In addition, the businessman will hire a driver who charges $15/h of driving. The rental car company charges the businessman $100 to deliver the car at the destination in Auburn. Also, the businessman must pay the driver a fixed amount for the hotel, meals, and trip back to Columbus. If the speed limit on the highway is 110 [km/h], what is the optimum average speed to minimize the total cost of the trip?

    A: 95.1 [km/h]

15. **Designing a new flag [9]**. Oregon State University is designing a new, small, and modern flag for its basketball team. As a designer, you receive the following instructions: (a) The flag should

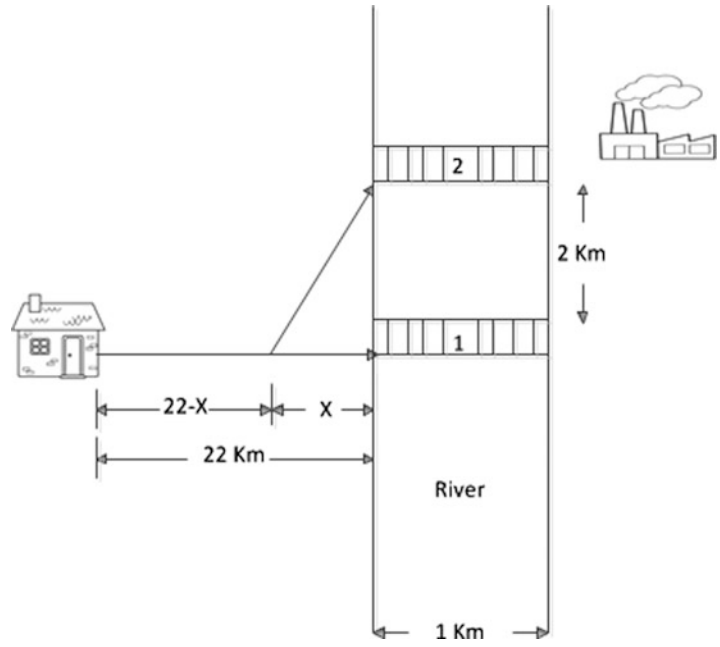

Fig. 11.42 Potential optimal route using bridge 2

have three vertical stripes, two orange (same dimensions) and one black. (b) The perimeter of the flag should be 40 [cm]. (c) The two orange stripes should have a total area of 20 [cm^2]. What should the dimensions of the stripes be to minimize the amount of fabric?
A: Two orange stripes of 10 × 1 [cm] and one black stripe of 10 × 8 [cm]

16. **Piece of wire [9]**. A piece of wire will be cut in two to make a square and a circumference. The total area (square plus the circumference) should be 16 [cm^2]. What is the maximum length (L) of wire that can be used?
A: $L = 21.39$ [cm]
17. **University openings [7]**. The planning department of a nonprofit university is trying to find the number of spots to open up next year and at the same to minimize tuition costs per student. This year, the university had 6,000 students, but because of graduation and students dismissed as a result of poor performance, it will be finishing the year with 5,400 students. The state government gives $30 million annually to the university. On the other hand, the fixed costs are $50 million annually. The monthly salary of professors is $6,000 and university policy is to have 1 professor for every 25 students. Additionally, there is a cost for support of $1/professor × student per month. (a) What should be the price of the tuition? (b) What will be the number vacancies to offer next year? (c) What will be the number of professors (must be integer)? (d) What will be the university budget?
A: (a) $7,256; (b) 1,671; (c) 283 [(5,400 + 1,671)/25]; (d) ~$81.3 million
18. **Silo [5]**. You are designing and building a silo of 1,000 [m^3]. The silo will be cylindrical with a hemispherical cap. Due to construction techniques, the hemispherical cap costs twice as much as the cylindrical part per square meter. (a) What radius (R) and height (H) of the cylinder would minimize the cost of construction? (b) If the cost of the cylinder and the hemispherical cap were the same, then would the optimal radius be lower or higher than in answer (a)? (c) What radius and height of the cylinder would minimize the cost of construction in question (b)? (d) Explain why, when the cost of the hemispherical cap is double, the radius of the silo is shorter.

A: (a) $R = 4.3$ [m], $H = 17.2$ [m]. (b) Higher. (c) $R = 5.42$ [m], $H = 10.84$ [m]. (d) Because the higher the cost of the hemispherical cap, the more the silo will be such as to minimize the amount of material needed in this part of the construction (shorter radius).

19. **Math test [7]**. You love math and, in addition, you have a natural talent for it, but unfortunately you are also a little bit anxious about it. There is no doubt that with the right amount of effort, you can get an A^+ (close to 100), but if you study too much (all day and night), you get a bit anxious about not visiting your devoted girlfriend, and your grade drops. Because of your vast experience taking tests and with your girlfriend, you have developed the following equation that relates your test grades (G, from 0 to 100) as a function of your effort (E, days of study) and anxiety (A): $G = E - A + C$, where C is a constant indicating that if you decide not to study ($t = 0$), you will get a score of $G = 30$.
In addition, as mentioned, effort (E) and anxiety (A) are related to time (t) as follows:

$$E = 28t \text{ and } A = 3t^2.$$

At this time you have 7 days to study and to visit your girlfriend. (a) How many days will you study to maximize your test score and what will your grade be? (b) Regarding your loving girlfriend, what is the best option?
A: (a) 4.67 days and $G = 95.34$. (b) A good option for getting a good grade and having the necessary time to share with your loving girlfriend is to study 4 days and get a 94!

20. **Apples [4]**. The owner of some land with apple trees estimates that if he plants 45 trees per hectare, each tree will produce 550 apples annually. For every additional tree that is planted per hectare, the number of apples produced per tree will decrease by eight. How many trees (integer) should be planted per hectare to maximize the production of apples?
A: 57 trees

21. **Microorganisms [6]**. The microbial growth rate can be expressed by the following rate equation:

$$v_r = kN(N_m - N), \text{ where } k = 0.4[\text{g cfu/h}] \text{ and } N_m = 10^7[\text{cfu/g}].$$

(a) What is the maximum growth rate (V_r)? (b) What is the microorganism's concentration (N) at the maximum growth rate?
A: (a) $V_r = 10^{13}$ [cfu/g h]. (b) 0.5×10^7 [cfu/g] ($N_m/2$)

22. **Tubular reactor [8]**. You have been commissioned to design and determine the dimension sizing of a pilot plant cylindrical tubular reactor with semispherical lids. Preliminary estimates indicate that the total volume should be 50 [L]. If we want to minimize the amount of steel used: (a) What should the dimensions of the reactor be (radius and height)? (b) What should the dimensions of the reactor be if the lids cost four times as much as the cylindrical part due to construction technicalities and fitting devices?
A: (a) Radius = 0.363 [m] and height = 0. Interestingly, according to the results, the reactor is spherical. This is because the geometric figure that encloses the minimum area per unit volume is the sphere. (b) Radius = 0.21 [m] and height = 1.13 [m]

23. **Two or one pressure tanks [9]?** You are responsible for the purchase of a pressure storage tank of 100 [m^3]. The manufacturer indicates that it is much more convenient to buy two tanks of 50 [m^3] each. Although the two 50 [m^3] tanks are more expensive than one tank of 100 [m^3], the manufacturer indicates that maintenance costs for these two tanks will be significantly lower, so the offer is very appealing. You have some doubts because it is known that this manufacturer has no pressure tanks of 100 [m^3]. But, in fact, you have information that the maintenance costs of

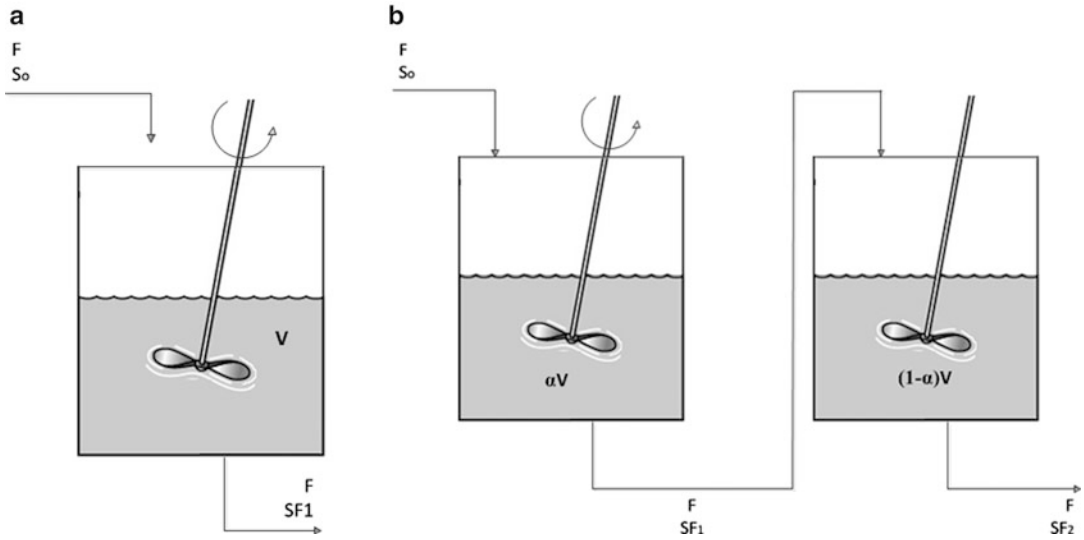

Fig. 11.43 (**a**) Continuous stirred-tank reactor (CSTR) (**b**) Two Continuous stirred-tank reactors (CSTR) in series

these two tanks will be lower than for one tank of 100 [m^3]. Therefore, to be clear, you ask the manufacturer for information regarding the price of the tanks according to the volume and about maintenance costs. The manufacturer gives you complete information as detailed below:
Pressure tank price (P): $P = 10{,}000 \times (V)^a$, where V is volume in cubic meters and a is a constant equal to 0.6. The shelf life of the pressure tank is 10 years.
Maintenance costs (M_c): $M_c = 18 \times (V)^b$, where V is volume in cubic meters and b is a constant equal to 2. The maintenance is for the whole shelf life of the pressure tank (10 years).
After receiving all the information, you carry out an optimization analysis to decide what to do. Your options are to buy one pressure tank of 100 [m^3], two pressure tanks of 50 [m^3] each, or two pressure tanks of different volumes, but with the condition that the total volume be 100 [m^3]. What will you do to minimize your costs?
A: The best option is to buy two pressure tanks of equal size (50 [m^3] each).

24. **Two reactors? [9]**. A young engineer is proposing to replace an old continuous stirred tank reactor (CSTR) of volume V for two CSTR reactors (of volume $V/2$ each) arranged in series. The manager agrees to replace the old reactor but disagrees with your proposition. The manager, who has no training in chemical engineering, wants to replace the old one for a similar one, but with up-to-date technology, instead of buying two new ones of volume $V/2$ each. According to Chap. 8, and considering that the disappearance of the substrate follows a first-order reaction, you derive the following equations for one reactor of volume V (Fig. 11.43a) and for two reactors (total volume V) in series (Fig. 11.43b):

$$1 \text{ reactor}: \qquad S_{F1} = FS_0/(F + kV), \tag{11.5}$$

$$2 \text{ reactors in series}: \qquad S_{F2} = F^2 S_0/[(F + \alpha kV)(F + (1-\alpha)kV)], \text{where } 0 < \alpha < 1, \tag{11.6}$$

where $F = 20$ [L/h], $S_0 = 12$ [g/L], $k = 0.4$ [1/h], $V = 100$ [L], and S_{F1} and S_{F2} are the outputs for one reactor and two reactors in series, respectively. With a simple calculation, say $\alpha = 0.5$, you can check that $S_{F2} < S_{F1}$. Therefore, it is better to have two CSTR reactors in series than just

one big CSTR reactor. (a) Derive and check (11.5) and (11.6). (b) Determine the value of α that maximizes the disappearance of the substrate (S_F minimum).
A: $\alpha = 0.5$

25. **Continuous culture [6]**. The biomass productivity (Q_x) of a continuous culture is calculated using the following equation:

$$Q_x = \left[y_{x/s} \left(S_0 - \frac{DK_s}{\mu_m - D} \right) \right] D.$$

Determine the value of D that maximizes the biomass productivity using the following data: $Y_{x/s} = 0.42$, $S_0 = 20$ [g/L], $K_s = 0.01$ [g/L], and $\mu_m = 0.5$ [1/h]. Assume that the dilution rate, D, has the restriction that it must be less than the maximum specific growth rate ($D < \mu_m$).
A: $D = 0.4888$ [1/h]

26. **Extraction process [6]**. In an extraction process, the extract concentration (g/L) can be described by the following equation as a function of time (t):

$$C = -3.75t^2 + 13.75t + 30.$$

Furthermore, the flow rate (L/min) can be expressed by

$$Q = 15t.$$

After how much time does it reach the maximum rate of product extracted ($C \times Q$ [g/min])?
A: 3.26 min

27. **Fuel mixture [7]**. A specialized laboratory conducts studies to determine the cost per kilometer of a fuel that is made from a mixture of diesel and bioethanol. Project engineers estimate that the model that describes the behavior of the cost against the fraction of each component is

$$C = 55.2D^3 + 65.2B^2 + 24.8,$$

where D ($0 < D < 1$) is the fraction of diesel and B ($0 < B < 1$) the fraction of ethanol. Determine the fraction of diesel (D) and ethanol (B) that minimizes the cost per kilometer of fuel. Remember: $D + B = 1$.
A: $D = 0.58$ and $B = 0.42$

28. **Piping [10⁺]**. A long pipe is going to be installed to carry a compound from building B1, where it is made, to building B2, where it will be used. Both buildings are constructed in parallel beside each other with a separation of 8 [m] between them, and each one has a length of 60 [m]. In addition, both buildings are oriented north–south. The output of the pipe in B1 is located 5 [m] to the south end of the building, and the beginning of the process line in B2 *is* located 5 [m] from the north end of building B2. The pipe installed between and outside the two buildings requires an additional coating; therefore, the cost of installation is 75 % higher than for one installed inside the buildings. Engineers have proposed the following diagram for the pipe installation (Fig. 11.44). Determine the angle (θ) that minimizes the cost of installation.
A: ~34.85°

29. **New package design [5]**. A natural-juice factory seeks to minimize the amount of material to use in its design of parallelepiped-shaped boxes. The designer has set one condition: one side must be

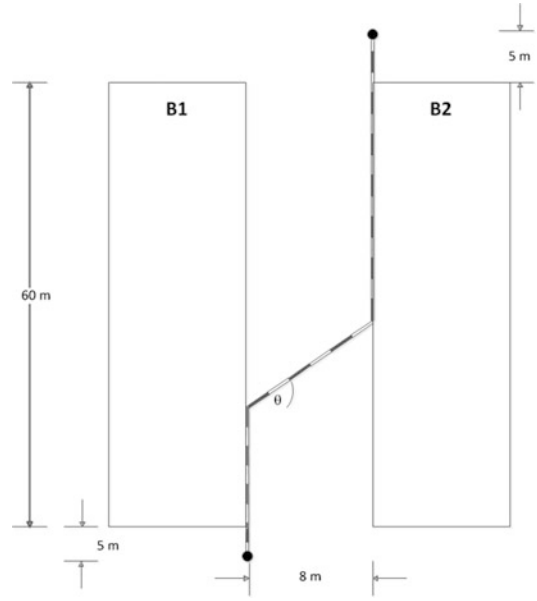

Fig. 11.44 Long pipe installed to carry a compound from the building *B1* where it is made to the building *B2* where it will be used

twice the length of another side and, in addition, the volume should be 1 [L]. What are the dimensions that minimize the use of material?
A: 7.211 [cm], 14.422 [cm], and 9.615 [cm]

30. **Activated enzyme [6]**. In an enzyme recovery process, engineers were able to model the recovery of protein [$P(t)$ %] and the enzyme activity [$\alpha(t)$ UI/mg] as a function of time with the following equations:

$$P(t) = 0.001t^3 - 0.207t^2 + 7.508t + 5.387,$$

$$\alpha(t) = 0.001t^2 - 0.0166t + 9.945,$$

where t is in minutes.

What extraction time maximizes the product of these two functions (known as activated enzyme)?
A: 22.27 min

31. **Pipe insulation [8]**. A pipe 15 [m] long and 3 [in.] in diameter carries steam, and the process engineer recommends insulating the pipe to reduce heat loss and, thus, reduce steam condensation. The cost of steam condensation is 1 [MU/kg*]. The insulation has a cost of 0.012 [MU/cm^3] and a thermal conductivity (k) of 0.04 [W/m K]. Neglecting the thickness of the pipe, determine the thickness of the insulation material that delivers the lowest annual cost. Because the cost of insulation is a fixed cost (once), it should be multiplied by 0.16 to be expressed as an annual cost. Therefore, the total cost to be minimized is

$$\begin{aligned}\text{Annual cost} = {} & (0.16 \times \text{insulation cost})\ [\text{MU/year}] \\ & + \text{Annual cost of steam condensation}\ [\text{MU/year}].\end{aligned}$$

The following equation can be used to determine steam loss (kg/s):

$$\mathrm{SL} = \frac{2\pi L r \Delta T k}{x\omega},$$

where SL is steam losses (kg/s), L is the length of the pipe (m), r is the radius of the pipe (m), ΔT is the temperature difference (80 K), k is the thermal conductivity (W/m K), ω is the heat of vaporization of water $2{,}257 \times 10^3$ (J/kg), and x is the thickness of insulation (m).
The plant operates 24 h/day, 330 days a year. What thickness of the insulation material (cm) would minimize the annual cost?
A: 8.2 [cm]

32. **Canned food design [9]**. A small cannery is developing a new seafood product to be packed in a cylindrical can of 400 [cm^3]. The designer wants to minimize the distance from the center of the can to the center of the lid plus the distance from the center of the can to the wall. (a) What are the radius and height of the can? (b) What should the radius and height of the can be if the distance from the center of the can to the middle of the lid plus the distance from the center of the can to the border of the lid should be minimized?
A: (a) Radius = height = 5.031 [cm]. (b) Radius = 5.65 [cm], height = 3.9885 [cm]
33. **Extraction costs [6]**. In a pilot plant the engineer estimates the cost (MU) of a concentrated product obtained from roots. The model is expressed as a function of time through the following equation:

$$\mathrm{Cost} = \left(-0.0012t^3 + 0.04t^2 + 0.9167t\right)/(10t). \text{ (time in min)}$$

If the concentration, C ($\mathrm{kg/m}^3$) against time is

$$C = -0.0012t^3 + 0.04t^2 + 0.9167t,$$

then: (a) What is the extraction time at which the extract is obtained at a lower cost? (b) What is the concentration of the extract?
A: (a) 16.67 [min], (b) ~ 20.84 [$\mathrm{kg/m}^3$]
34. **Free energy of formation [10⁺]**. The free energy of formation of a spherical crystal (ΔG_T) can be described as the change in free energy of the liquid to the solid phase (ΔG_P) multiplied by the volume of the spherical crystal plus the free energy for creating the solid–liquid interface (ΔG_i). Thus,

$$\Delta G_\mathrm{T} = V\Delta G_\mathrm{P} + \Delta G_\mathrm{i},$$

where $\Delta G_\mathrm{i} = A\gamma$, $\Delta G_\mathrm{P} = \Delta H_\mathrm{f} \times \dfrac{\Delta T}{T_\mathrm{f}}$, A is the area of the sphere (m^2), γ is the surface free energy ($\mathrm{J/m}^2$) ($\gamma_{(\mathrm{H_2O})} = 0.072$ [$\mathrm{J/m}^2$]), ΔH_f is the heat of fusion ($\mathrm{J/m}^3$) ($\Delta H_{\mathrm{fH_2O}} = 3.344 \times 10^8$[$\mathrm{J/m}^3$]), ΔT is the degrees of subcooling (K), V is the volume of the spherical crystal (m^3), and T_f is the temperature of fusion (K) ($T_{\mathrm{fH_2O}} = 273$[K]).
The radius of the crystal to where the free energy of formation of the spherical crystal (ΔG_T) is at its maximum is known as the critical radius; this proves to be the minimum size to begin the formation of a crystal. Determine the critical radius for water with 10C ($\Delta T = 10$ [K]) of subcooling. **HINT:** Search around 1.1×10^{-8} and 1.2×10^{-8} [m].
A: 1.1756×10^{-8} [m] (0.011756 [μm])

11.8.2 Operation Research Problems

35. **Trucks [6]**. A food distribution company has three types of trucks, T_1, T_2, and T_3. T_1 has no refrigeration capacity and a volumetric capacity of 25 [m^3]. T_2 has a modern refrigeration system and a total capacity of 45 [m^3], but just 35 [m^3] are equipped with the cooling and the remaining 10 [m^3] are not refrigerated. Finally, T_3 is a smaller truck with a capacity of 15 [m^3] and without a refrigeration system. Wal-Mart is located 100 [km] from the distribution company and is ready to launch a mega store and places a big order of 2,500 [m^3] of refrigerated foods and 3,800 [m^3] of nonrefrigerated foods. If the cost per kilometer of using each truck is \$0.20 for the T_1 truck, \$0.26 for T_2, and \$0.23 for T_3, then in order to minimize the distribution costs, how many trucks of each type must use the distributor?
    A: 124 T_1 trucks, 72 T_2 trucks, and no T_3 trucks.
36. **A bus trip [7]**. Our chemical and bioprocess engineering department is preparing a trip to a scientific meeting to be held 250 [miles] away. The department contacted a bus company that provides good service at a reasonable price. We think 420 students will attend meeting. The bus company has eight buses with 45 seats (B_1) and six buses with 35 seats (B_2). In addition, the company has just 11 drivers available. If the company charges \$950 and \$750 for the large and small buses, respectively, then in order to minimize costs, how many buses of each type should the department hire?
    A: Seven large buses, three small buses.
37. **Chemical products [10]**. You are a sales manager of a small chemical company. You are interested in selling the last stock of two chemicals, A_1 and B_Z. Each chemical is stored in 1 [kg] jars. The inventory indicates that you have 200 jars of chemical A_1 and 100 jars of chemical B_Z. Hoping to sell the entire inventory you decide to offer two types of packages. The first package is one jar of A_1 and one jar of B_Z for \$60. The second package is three jars of A_1 and one jar of B_Z for \$100. You expect to sell at least 30 packages and, in addition, that you will sell twice as many of the first package as the second package. To maximize your sales, how many packages will you need to sell of each type?
    A: 66 of the first type and 33 of the second type
38. **Candies and chocolates [9]**. A candy company receives, out of the blue, an urgent order for its most popular candies, Candit and Choc. Table 11.3 shows the current availability of each of these products in the manufacturing plant.
    To fulfill this order, two trucks are used with the following characteristics (Table 11.4):
    The trucks are expected to arrive at the destination 3 h late, so each truck has to carry at least 2 [tons] of each product. How much Candit and Choc does each truck have to carry to maximize profits?
    A: Truck 1: 2 [tons] of Candit and 13 [tons] of Choc; Truck 2: 6 [tons] of Candit and 6 [ton] of Choc.

Table 11.3

Product	Amount [ton]	Specific volume [m^3/ton]	Profit [\$/ton]
Candit	22	4	550
Choc	24	5.2	650

Table 11.4

Truck	Weight capacity [ton]	Volume capacity [m^3]
1	15	70
2	12	55

39. **Small food processing company [6]**. A small company has two food processing plants and has received two orders, one for 450 [kg] of product from Corvallis, Oregon, and the other for 300 [kg] of product from Olympia, Washington. The company has one processing plant in Portland, Oregon, and a second one in Salem, Oregon. The product availability in Portland is 500 [kg] and in Salem 400 [kg]. It costs \$0.50/kg to ship the product from Salem to Corvallis but \$1.50/kg to ship it to Olympia. On the other hand, it costs \$0.80/kg to ship the product from Portland to Corvallis and \$1.20/kg to ship it from Portland to Olympia. The minimum order to be shipped from each processing plant is 10 [kg]. How many kilograms of product should the company ship from each plant to Corvallis and Olympia to minimize the cost of the order?
    A: To minimize the cost of the order, from the plant in Portland the company must send 61 [kg] to Corvallis and 290 [kg] to Olympia; from the plant in Salem it must send 389 [kg] to Corvallis and 10 [kg] to Olympia.
40. **Pet food [7]**. Although you are young and also a new engineer at the Pet Food Company you are sure that the cost of the pellets can be minimized. So far, the pellets (1 [g]) are manufactured from three base products, A, B, and C, with unit costs of \$0.50, \$1.25, and \$2.25 per kilogram, respectively. Each pellet must contain at least 3 [mg] of vitamin M and 4 [mg] of vitamin N. It is known that for every gram the base products A, B, and C contain 1, 4, and 4 [mg] of vitamin M and 1, 2, and 6 [mg] of vitamin N, respectively. Your argument to change the actual recipe is that the current product is not using a filler or excipient. The filler does not contain vitamins N and M but is very cheap (\$0.0625/kg). (a) What is the actual cost per kilogram and composition of the pellets? (b) What is the cost per kilogram and composition of the pellets if you include filler? (c) What is the minimum cost of the pellets if they must meet the vitamins requirements but can weigh less than 1 [g]?
    A: (a) ~\$1.5834 and using 0.34 [g] of A, 0.0834 [g] of B and 0.5834 [g] of C per pellet; (b) \$1.5781 and using 0 [g] of A, 0.125 [g] of B, 0.625 [g] of C, and 0.25 [g] of filler per pellet. So you were right, using filler reduces the cost, but just slightly. (c) \$1.5625 and using 0 [g] of A, 0.125 [g] of B, 0.625 [g] of C, and 0 [g] of filler per pellet.
41. **4 × 100 [m] relay [10]**. Following the team's defeat in the 4 × 100 [m] relay at the Olympics in London, the US men's trainer has been conducting an interesting study to determine the best way to arrange each sprinter in the relay. The performance of each sprinter at the different stages of the 4 × 100 [m] relay has been carefully measured by the trainer. The following table shows the time of each sprinter in each stage of the relay.

	Stage 1	Stage 2	Stage 3	Stage 4
Justin Gatlin	9.9	9.3	9.5	9.2
Tyson Gay	9.7	9.5	9.3	9.5
Darvis Patton	10.0	9.3	9.2	9.1
Jeff Demps	9.8	9.2	9.1	9.3

    In addition, the trainer has noticed that the way he arranges his sprinters really matters. For example, if he puts Justin Gatlin first, Tyson Gay second, Darvis Patton third, and Jeff Demps last, the time for the 4 × 100 [m] relay is 37.9 s. On the other hand, if Tyson Gay goes first, Justin Gatlin second, Darvis Patton third, and again Jeff Demps last, the time is 37.5 s. The trainer is smart and knows that there must be an order that minimizes the time. Help him find the best order of runners and determine what the time will be for this optimum arrangement.
    A: The optimum arrangement is Tyson Gay first, Justin Gatlin second, Jeff Demps third, and Darvis Patton last, and the time is 37.2 s

42. **Feeding pigs [8]**. Some farmers are planning to feed their pigs with a mixture of three products: crushed meal, a special food, and vitamins. They have several thousand pigs, and each pig should eat at least 8 [kg] of the mixture per day. The daily dietary requirements and associated costs are presented in the following table:

	Calories	Vitamin (mg)			
Minimum daily requirement	3,500	A	B	C	
		6	10	22	
Content per product					Costs
Crushed meal [kg]	450	0.5	–	–	0.70 [$/kg]
Special food [kg]	290	0.5	–	1	0.80 [$/kg]
Vitamin concentrate[a] [bottle]	–	0.5	7	14	1.5 [($/Bottle)]

[a]The vitamin concentrate weighs 100 [g] per bottle. In addition, the number of bottles could be fractional.

(a) What is the optimal mixture of the three products to minimize the cost of the diet? (b) What is the cost of the daily diet per pig? (c) Your veterinarian argues that for health and productivity reasons, it is very important to restrict the amount of the mixture to no more than 10 [kg/day]. What is the optimum result if you follow the veterinarian's recommendation? (d) What is the cost of this new daily diet per pig?

A: (a) 10.429 [kg] of crushed meal and 1.571 bottles of vitamin concentrate, (b) $9.657, (c) 9.778 [kg] of crushed meal and 2.222 bottles of vitamin concentrate, (d) $10.178.

43. **Optimizing distribution [9]**. A company that manufactures animal feed has two processing plants. To fulfill a contract with three supermarkets, it produces 400,000 bags of food a week at plant 1 and 200,000 at plant 2. These bags will be transported to two packaging companies whose capacities are 300,000 bags each. Finally, the bags must be sent to the three supermarkets (200,000 bags to each supermarket). The following table shows the transport costs (arbitrary units) per bag of food. How many packages should be sent to each packaging company and to each supermarket to minimize costs?

	Packaging company 1	Packaging company 2	Supermarket 1	Supermarket 2	Supermarket 3
Plant 1	50	90	–	–	–
Plant 2	120	150	–	–	–
Packaging company 1	–	–	200	–	165
Packaging company 2	–	–	–	180	195

A: The following table shows the number of packages sent to each packaging store and to each supermarket.

	Packaging company 1	Packaging company 2	Supermarket 1	Supermarket 2	Supermarket 3
Plant 1	300,000	100,000	–	–	–
Plant 2	–	200,000	–	–	–
Packaging company 1	–	–	200,000	–	100,000
Packaging company 2	–	–	–	200,000	100,000

44. **Optimal investment [10]**. A large chemical company is considering the implementation of four projects, designated P1, P2, P3, and P4. The following table shows the capital requirements for each of these projects for the next 3 years. The same table shows the NPV (net present value) of each project and the annual availability of capital that the company will have over the next 3 years.

In addition, the table uses an arbitrary currency and the money values corresponding to the year in question.

	NPV (at beginning of year 1)	Annual capital requirements per project			
		Year 1	Year 2	Year 3	Observation
P1	61	−22	−5	−21	
P2	77	−31	−31	0	Runs during first 2 years
P3	103	0	−38	−37	Runs in second and third years
P4	120	−20	−12	−40	
Available capital per year		65	60	82	

Moreover, projects P2 and P4 are mutually exclusive, i.e., if one of them is carried out, the other is not, although perhaps neither is carried out. The policy of the board and the CEO is to run more than one project. Finally, P1 can be executed only if P3 is executed. (a) Which projects should the company go with? (b) Based on the answer to (a), do you have any comments?

A: (a) Projects P3 and P4. (b) If each year some money will be made available (45, 10, and 5, respectively), then after the first year, the board and CEO might reevaluate the situation (new projects?).

45. **Cookies [6]**. An industrial bakery receives from a supermarket chain an urgent order for 1,000 [kg] of high-protein cookies. The cost of the ingredients should be minimized and the mixture must meet the following minimum requirements: 360 [kg] protein, 225 [kg] fat, 240 [kg] carbohydrates, and 45 [kg] sugar. In addition, the moisture content of the cookies should be lower than 3.5 % w/w.

The cookies are made based on the mixture of four ingredients R, S, T, and U, with associated costs of $2, $0.50, $0.75, and $0.25/kg, respectively. The composition of the ingredients is given in the following table:

	Protein [% w/w]	Fat [%w/w]	Carbohydrate [% w/w]	Sugar [% w/w]	H_2O [% w/w]	Filler [% w/w]
R	48	30	14	5	3	0
S	10	15	48	13	4	10
T	28	5	29	10	3	25
U	0	5	5	28	2	60

How many kilograms of each ingredient do you need to use to minimize the cost?

A: R = 607.69 [kg], S = 230.77, T = 161.54, U = 0.

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Basic Economic Principles and Deciding Among Alternatives

12[3]

In the business world, the rearview mirror is always clearer than the windshield.

Warren Buffett

In economics, the majority is always wrong.

John Kenneth Galbraith

Art is the beautiful way of doing things. Science is the effective way of doing things. Business is the economic way of doing things.

Elbert Hubbard

12.1 Chapter Purpose

The purpose of this chapter is to familiarize you with the concept of interest rates (or time value of money) and then give you the capability to manage simple but engaging decision-alternative problems related to real-life situations and process engineering. We will keep it simple in terms of economic concepts, but you will face interesting situations from everyday life and, additionally, stimulating and thought-provoking problems related to chemical and bioprocess engineering. We will continue to learn about and engage with these broad fields. Through the problems, you will become familiar with different loan options and learn how to compare them and improve your effectiveness as a rational consumer. The core of the chapter is comprised of problems where you, as an engineer, will need to compare different alternatives and make decisions mainly based on economic considerations. You will face questions where your technical expertise and knowledge of economics will be challenged to compare, for example, offers from two or more companies to replace reactors at a processing plant. At this stage, the problems will focus on economic analysis. For us, though, as future process and bioprocess engineers, the technical aspects of the project are more relevant, so we will leave this matter for future courses.

Our challenge is to teach you just a few concepts, as has been our approach throughout the book, and add maybe just one more (interest rates). Then we will present stimulating and motivating examples from everyday life and, more importantly, from situations one might encounter in chemical and bioprocess engineering.

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12.2 Understanding What Economics Is

Economics is a social science. As Professor Paul A. Samuelson (1915–2009) rightly stated, economics is the oldest of the arts, the newest of the sciences, in fact, the queen of the social sciences. It is difficult to briefly define economics. Quoting, again, Professor Samuelson, the economics concept can be generalized through the following definition:

> Economics is the study of how men and society end up choosing, with or without money, employment of productive resources (scarce) that may have different uses to produce various commodities and distribute them for consumption, present or future, between various individuals and groups that make up society, analyzing the costs or benefits of improving patterns of resource distribution.

For the purposes of this book, we will be concerned with understanding and familiarizing you with the concept of economics and focus on what is of interest to future process and bioprocesses engineers. The scheme presented below gives us the opportunity to understand all the interrelationships in the economy and identify what aspect we will focus on in this first year as novice prospective engineers.

12.3 What You Will Be Learning

Within this big picture (Fig. 12.1) our humble objective is to provide you with the capability to approach real-life and engineering problems, situations where it is necessary to decide among different alternatives that are technically equivalent. In real-life engineering you will face several situations where the final decision will depend on your technical and economic analysis. For example, a manufacturer is offering you more efficient and automatic equipment to replace your current

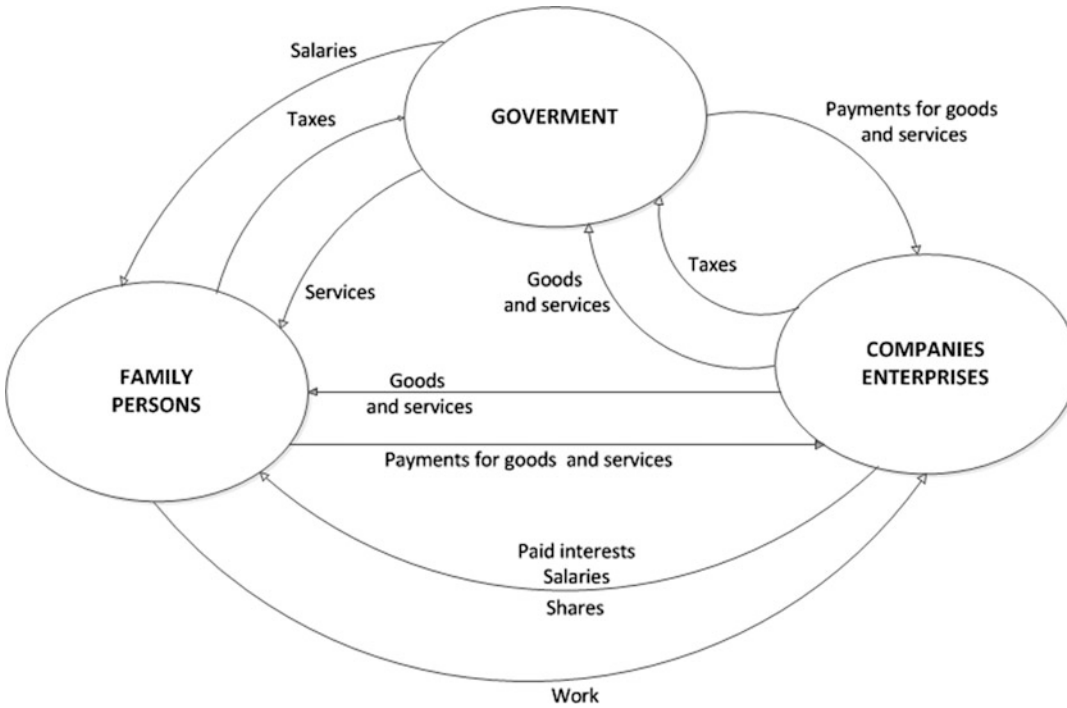

Fig. 12.1 The scheme presents all the inter-relationships of the economy in our society

equipment. What do you do? First, you carry out a technical analysis and check all the specifications given by the manufacturer and how they fit and work with the existing process and quantify how the new process will improve the existing process. Second, you perform a careful economic evaluation among the alternatives before making your final decision.

At this stage, we will introduce and illustrate the concept of interest rate (compound interest rate) and, in addition, provide a few elementary formulas of financial mathematics (Sect. 12.5) to empower and give you the capacity to solve all the problems presented in this chapter.

12.4 What Is the Interest Rate? Some Simple Calculations

The interest rate represents the monetary value of the ability to use money over time. As an analogy, when you use a house, a car, equipment, or other item that you do not own, you pay a fee: in this context, it is called rent. In simple terms, interest is what I need to pay in order to be able to use some specific amount of money by a certain time. Although many people have a negative perception of the interest rate, in reality, for the same people, it is natural to pay rent for an apartment, a car, a piece of equipment, etc. (which in turn have an equivalent cash value). Therefore, it is logical that we should pay "rent" for using an amount of money that we borrowed from, in most cases, a bank (or equivalent institution). In the same way, it is logical to receive money when you rent out your house or when you lend money to someone. The question is how much to charge. At this point, it is important to introduce the concept of opportunity cost. For example, if someone borrows $1,000 from you for 1 month, how much will you charge the person to use your money for this period of time? Assuming that the only alternative is to deposit the money in a bank account, your opportunity cost is the amount that the bank is willing to pay you to use your money. In general, opportunity cost refers to the best alternative use of your resource (e.g., house, land, money, time).

As mentioned, money has value throughout time. If you want a loan for some specific time period—say 1 month—then you need to pay interest to use, for your own benefit, this money for 1 month. For example, if you want a loan of $\$L$ for 1 month and the interest rate is r % per month (compound interest), then after 1 month you need to return $\$L_{\mathrm{F}}$ to the lender, where the financial mathematical equivalence between L and L_{F} is

$$L_{\mathrm{F}} = L\left(1 + \frac{r}{100}\right)^{1}, \tag{12.1}$$

where L_{F} is the future value ($), L is the present value ($), and r is the monthly interest rate (%).

In words, in this case, the future value (L_{F}) is equivalent to the present value (L) multiplied by 1 plus the interest rate divided by 100 (because the interest rate was given as a percentage) with an exponent equal to 1 because the loan was for 1 month. If the interest rate is given in decimal form, then the financial mathematical equivalence between L_{F} and L is

$$L_{\mathrm{F}} = L(1 + r_{\mathrm{d}})^{1}, \tag{12.2}$$

where L_{F} is the future value ($), L is the present value ($), and r_{d} is the monthly interest rate in decimal form (where $r_{\mathrm{d}} = r/100$).

If the loan is for n months, then after n months you need to return

$$L_{\mathrm{F}} = L\left(1 + \frac{r}{100}\right)^{n} \quad \text{or} \quad L_{\mathrm{F}} = L(1 + r_{\mathrm{d}})^{n}. \tag{12.3}$$

Simple and compound interest

It is necessary to distinguish between simple and compound interest. Simple interest is paid only on the deposit (L), whereas compound interest is where the bank pays interest on the principal deposit and the accumulated interest on the principal. Equation (12.3) is the future value of L calculated with compound interest.

Henceforth, all calculations in this chapter are carried out assuming compound interest.

Warm-up example 1. You go to a bank and apply for a $1,000.00 loan. (a) How much you will need to pay the bank back after 6 months if the monthly interest rate is 1.5 %? (b) Is it worth taking out the loan? (c) How does your answer in (a) change if another bank offers you a monthly interest rate of 1 %?

Solution

(a) If we call L_F the amount of money that you need to pay the bank after 6 months, then

$$L_F = 1,000\left(1 + \frac{1.5}{100}\right)^6 = 1,093.44.$$

The result shows you that the use of $1,000 for a period of 6 months (at $r = 1.5$ %) has a cost of $93.44.

(b) It depends; maybe a monthly interest rate of 1.5 % is a bit too high. There are different ways to determine whether or not it is worth it, but, for example, if you are investing this money in a project and the profits from the project are expected to be much higher than $93.44, then it is worth applying for this loan, unless you have another option from a different bank with an interest rate lower than 1.5 % [see answer (c)].

(c)
$$L_F = 1,000\left(1 + \frac{1.0}{100}\right)^6 = 1,061.52$$

Warm-up example 2. You are looking for a loan of $1,000 from a bank with the constraint that you need to return the money to the bank within 1 year (12 months). The bank's loan officer tells you that you are an excellent client and you deserve a very low interest rate. So the loan officer tells you that credit has been approved for you and that you need to pay the bank $1,100 within 1 year. (a) What is the monthly interest rate? (b) What is the annual interest rate? (c) Find the equivalence between monthly interest rate (r_m) and annual interest rate (r_a).

Solution

(a) Based on (12.3), we can write

$$1,100 = 1,000\left(1 + \frac{r_m}{100}\right)^{12}, \tag{12.4}$$

where r_m is the monthly interest rate and the unknown in (12.4). As was explained in the previous chapter for similar calculations, we can use an Excel spreadsheet to obtain the value of r_m. It could be done graphically or using the Solver tool. Using the Solver tool we got r_m approximately equal to 0.8 %.

(b) In this case we can write

$$1,100 = 1,000\left(1 + \frac{r_y}{100}\right)^1, \tag{12.5}$$

where r_y is the annual interest rate and, at the same time, the unknown. In this case we get r_y analytically. Thus, $r_y = 10\ \%$.

(c) From (12.4) and (12.5) we get a general relationship between annual and monthly interest rate, as follows:

$$\left(1 + \frac{r_m}{100}\right)^{12} = 1 + \frac{r_y}{100}. \tag{12.6}$$

From (12.6) you can check that a 0.8 % monthly interest rate is equivalent to an approximately 10 % annual interest rate.

12.5 Financial Mathematical Equivalences

Keeping things simple, we will basically use two financial mathematical equivalences and the concepts of net present value (sometimes called net present worth) and annual equivalent benefits (or annual equivalent cost) (Sect. 12.6) to do all calculations for real-life problems and project evaluations. Although it represents a limitation (not using more concepts and equations), you will be amazed at the wide variety of real-life and engineering problems that can be solved with these few rather elementary tools. Thus, we continue to meet our objectives, which are to (a) teach just a few concepts and (b) captivate you with the diversity of problematic situations that arise in process and bioprocess engineering.

As mentioned, here we present two financial mathematical relationships and from both obtain a third one.

We set the following definitions:

F_V: Future value ($)
P_V: Present value ($)
S: Equal payments ($)
r: Interest rate per period (%) (could be months or years, depending on how the period is defined)
r_d: Interest rate per period in decimal form (could be months or years, depending on how the period is defined)
n: Number of periods (each period could be months or years, for example)

Relationship between future value (F_V) and present value (P_V)

Using the nomenclature defined in this section we can rewrite (12.3) and get

$$F_V = P_V\left(1 + \frac{r}{100}\right)^n \text{or, with the decimal form of interest rate, } F_V = P_V(1 + r_d)^n. \tag{12.7}$$

Relationship between present value (P_V) and equal payments (S)

$$P_V = \frac{S\left(\left(1 + \frac{r}{100}\right)^n - 1\right)}{\frac{r}{100}\left(1 + \frac{r}{100}\right)^n} \quad \text{or, with the decimal form of interest rate,}$$
$$P_V = \frac{S((1 + r_d)^n - 1)}{r_d(1 + r_d)^n}. \tag{12.8}$$

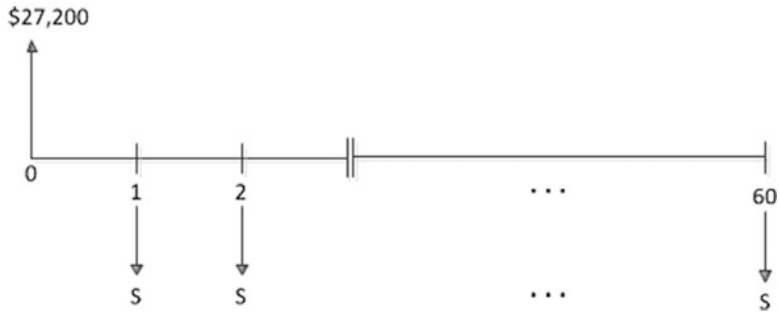

Fig. 12.2 Graphic representation (cash flow) of the Present value ($P_V = \$27{,}200$) and 60 equal payments (S)

From (12.7) and (12.8) we get a relationship between future value (F_V) and equal payments (S), as follows:

$$S = F_V \frac{\frac{r}{100}}{\left(\left(1+\frac{r}{100}\right)^n - 1\right)} \quad \text{or, with the decimal form of interest rate,}$$
$$S = F_V \frac{r_d}{\left(\left(1+r_d\right)^n - 1\right)}. \tag{12.9}$$

Warm-up example 3. You just graduated with a degree in chemical engineering (Universidad Técnica Federico Santa María, Chile) and found a good job in a large petrochemical company. Now, you are a bit restless, and you are going to want to buy a new car soon. So after a quick search you decide that the best option is a Ford Fusion SE hybrid sedan, which has a price tag of $27,200. Of course, you do not have the money to pay cash, but because of your new position in the petrochemical company, you qualify for a car loan from a bank. The loan officer at the bank offers you the loan ($27,200). You will need to make 60 equal payments with an interest rate of 1 % per month. (a) Draw a graphical representation of the problem. (b) What is the amount in dollars of each payment?

Solution

(a) Graphical representation (cash flow) (Fig. 12.2)

(b) As depicted in the graphical representation, we know the present value ($P_V = \$27{,}200$), the interest rate ($r = 1$ % per month), and the number of periods ($n = 60$ months), but we do not know the value of each payment (S). Therefore, substituting P_V, r and n into (12.8) we get

$$27,200 = \frac{S\left(\left(1+\frac{1}{100}\right)^{60} - 1\right)}{\frac{1}{100}\left(1+\frac{1}{100}\right)^{60}},$$

so $S = \$605.05$ per month.

Warm-up example 4. You need to buy a new refrigerator. Based on your initial research, you go to The Economy store. The advertised price on the fridge you are interested in is $1,000.00, and the seller gives you the option to buy the refrigerator with 24 equal monthly payments at an interest rate of 3.5 % per month. As a good Introduction to Engineering student, you analyze other options before taking a decision. Unfortunately, as a student, you have encountered several difficulties in

obtaining a loan from the bank. Finally, you find a bank that offers a very affordable interest rate (1 % per month), it but forces you to take out an insurance policy for $12 a month. (a) What is the value of the payments in The Economy store? (b) What is the value of the payments at the bank (excluding the insurance)? (c) What is more appealing: the loan from the bank (including the insurance) or the store?

Solution

(a) The Economy: here we know the present value ($P_V = \$1{,}000.00$), the interest rate ($r = 3.5$ % per month), and the number of periods ($n = 24$ months), but we do not know the value of each payment (S_{CE}). Thus, substituting into (12.8) we get

$$1,000 = \frac{S_{CE}\left(\left(1+\frac{3.5}{100}\right)^{24}-1\right)}{\frac{3.5}{100}\left(1+\frac{3.5}{100}\right)^{24}},$$

where S_{CE} represents the equal payments in The Economy store.
Thus, $S_{CE} = \$62.27$ per month.

(b) Bank (without insurance). Here we know the present value ($P_V = \$1{,}000.00$), the interest rate ($r = 1$ % per month), and the number of periods ($n = 24$ months), but we do not know the value of each payment (S_B). Thus, substituting into (12.8) we get

$$1,000 = \frac{S_{B}\left(\left(1+\frac{1}{100}\right)^{24}-1\right)}{\frac{1}{100}\left(1+\frac{1}{100}\right)^{24}}$$

Thus, $S_B = \$47.07$ per month.

(c) The loan from the bank is more appealing because you need to pay $47.07 per month plus the insurance ($12 per month) giving a total of $59.07 per month (lower than $62.27 per month in the commercial store). Notice that the comparison was made possible by the fact that the number of payments was the same in both alternatives.

12.6 Net Present Value (NPV) and Annual Equivalent Benefits/Cost (AEB or AEC)

We have already described and analyzed all the financial mathematical tools needed for our calculations to fulfill the purpose of the chapter. Using these basic tools, we now introduce two concepts that are important for comparison and decision among alternatives (real-life situations and engineering projects). First, the net present value is one of the most common tools to economically evaluate projects. One of the limitations of NPV is when you are comparing two or more projects with different lifespans. Although this difficulty can be remedied, it could be cumbersome. Second, the annual equivalent benefits/costs has the advantage over NPV that you can compare projects with different lifespans.

Although project evaluation is a whole branch of engineering, here we will limit its use to some elementary concepts to simply show potential implications and applications in process and bioprocess engineering.

Fig. 12.3 Cash flow for *NPV*

12.6.1 Net Present Value

The NPV represents all cash flows of a project (incoming and outgoing) evaluated at present ($t = 0$). If the sum of all cash inflows/outflows evaluated at present is positive, then the project is economically attractive. Typically when you are in the process of economically evaluating a project, you will have different cash flows (incoming and outgoing) at different times and with their own characteristics. The following schematic representation shows a typical project and then the financial mathematical equation to calculate its value through the NPV concept (Fig. 12.3).

$$\text{NPV} = -I + \frac{\text{CF}_1}{\left(1+\frac{r}{100}\right)^1} + \frac{\text{CF}_2}{\left(1+\frac{r}{100}\right)^2} + \cdots + \frac{\text{CF}_n}{\left(1+\frac{r}{100}\right)^n} = -I + \sum_{i=1}^{i=n} \frac{\text{CF}_i}{\left(1+\frac{r}{100}\right)^i}, \tag{12.10}$$

where NPV = net present value ($), I is investment ($), CF_1, CF_2, …, CF_n are the flow for each period (not necessarily positive) ($), and r is the discount rate (%).

In the particular case where $\text{CF}_1 = \text{CF}_2 = \ldots = \text{CF}_n = \text{CF}$, (12.10) can be written as

$$\text{NPV} = -I + \frac{\text{CF}\left(\left(1+\frac{r}{100}\right)^n - 1\right)}{\frac{r}{100}\left(1+\frac{r}{100}\right)^n}. \tag{12.11}$$

Warm-up example 5. Your friend Werner has an excellent idea but does not have the money to invest in this potential and novel project. After a careful analysis of all the cash flows, he explains to you that he needs $1.5 million. According to his estimations the annual cost of the project will be $70,000, with expected annual income of $320,000. At the end of the lifespan (10 years) the equipment items can be sold for $200,000. Would you be willing to invest in Werner's project? Consider an annual discount rate of 10 %.

Solution

To decide if the project is attractive, we will calculate NPV, where NPV ≥ 0 implies that the project is economically attractive.

Step I

Schematic representation

Step II

NPV calculation

First, the annual benefits are

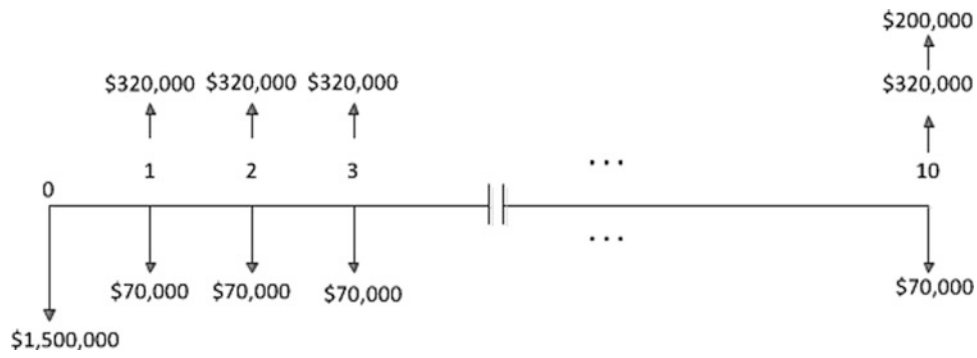

Fig. 12.4 Cash flow including all data

$$\mathrm{CF_i} = \$320,000 - \$70,000 = \$250,000.$$

Given that, in this case $\mathrm{CF}_1 = \mathrm{CF}_2 = \ldots = \mathrm{CF}_n = 250{,}000$, we can write the NPV equation (12.11) as follows:

$$\mathrm{NPV} = -1,500,000 + \frac{250,000\left((1.1)^{10} - 1\right)}{0.1(1.1)^{10}} + \frac{200,000}{(1.1)^{10}} = \$113,250, \tag{12.12}$$

where the third term in (12.12) corresponds to the present value of the equipment items sold at the end of the project (year 10). (NPV could easily be calculated using a spreadsheet.)

Step III

Analysis

As long as NPV ≥ 0, the project is attractive. In real-life problems it is advisable to do a sensitivity analysis answering questions like: what if the discount rate is 12 % (higher than the supposed 10 %)? What if the real investment turns out to be $1.7 million (higher than the estimated $1.5 million)? etc. For example, in Werner's project, if the required investment is greater than $1,613,250, then the NPV will be negative. In addition, if the discount rate is 12 %, the NPV will again be negative. With the data provided by Werner, small variations in his assumptions would put his project at risk.

Finally, as your business advisor I would recommend that you be cautious with respect to Werner's project!

12.6.2 Annual Equivalent Benefits/Cost

In a similar manner to NPV where all the cash flows were sent to the present ($t = 0$), here the idea is to obtain an amount of equal benefits/costs for each period of the project. An easy and straightforward way to obtain the annual equivalent benefit/cost is to first calculated the NPV and then, using (12.8), calculate the annual equivalent benefits/costs.

To learn more about the annual equivalent benefit/cost (AEB or AEC) calculation, we will refer to warm-up example 5 to calculate AEB. As mentioned, the first step is to obtain the NPV value. Thus, Fig. 12.4 shows the cash flows of warm-up example 5, and (12.12) shows that NPV = 113,250. Hence, rearranging (12.8) and using the appropriate nomenclature, we get

$$\text{AEB} = \frac{\text{NPV}\left(1+\frac{r}{100}\right)^{n}\frac{r}{100}}{\left(\left(1+\frac{r}{100}\right)^{n}-1\right)}. \tag{12.13}$$

Substituting into (12.13), NPV = \$113,250; $r = 10\,\%$; $n = 10$ years; and we get AEB = \$18,431 (the project is attractive because AEB ≥ 0). In (12.13), if NPV is negative, then AEB will be negative. As with NPV, the project will be attractive if AEB ≥ 0.

As mentioned, the advantage of AEB over NPV is that you can compare projects with different lifespans.

12.7 Comparing and Deciding Among Different Alternatives: Warm-Up Example

In the following example, we are invited to give our opinion and decide which project is better. Although both projects could be attractive, the required investment limits us to choose one of the two.

Warm-up example 6. A small pharmaceutical processing plant producing antibiotic pills wants to buy an automatic packaging machine, and company executives are analyzing two alternatives.

Determine which machine should be selected using an annual discount rate of 12 %.

Solution

As shown in Table 12.1, the lifespans of machines A and B are different. Thus, as suggested, we will use the AEC concept to decide which machine is better in economic terms. It is assumed that both machines have equal packaging capabilities.

Step I

Schematic representation (Fig. 12.5a, b)

Step II

AEC calculation

First, we will calculate the NPV per each alternative (machine A and B):

$$\text{NPV}_{\text{A}} = -200,000 - \frac{20,000\left((1.12)^{7}-1\right)}{0.12(1.12)^{7}} + \frac{30,000}{(1.12)^{7}} = -\$277,705;$$

$$\text{NPV}_{\text{B}} = -300,000 - \frac{6,000\left((1.12)^{10}-1\right)}{0.12(1.12)^{10}} + \frac{90,000}{(1.12)^{10}} = -\$304,924.$$

Table 12.1 Price and characteristics of machines A and B

	Machine A	Machine B
Price of machine (\$)	200,000	300,000
Annual maintenance cost (\$)	20,000	6,000
Residual value (\$)	30,000	90,000
Lifespan (years)	7	10

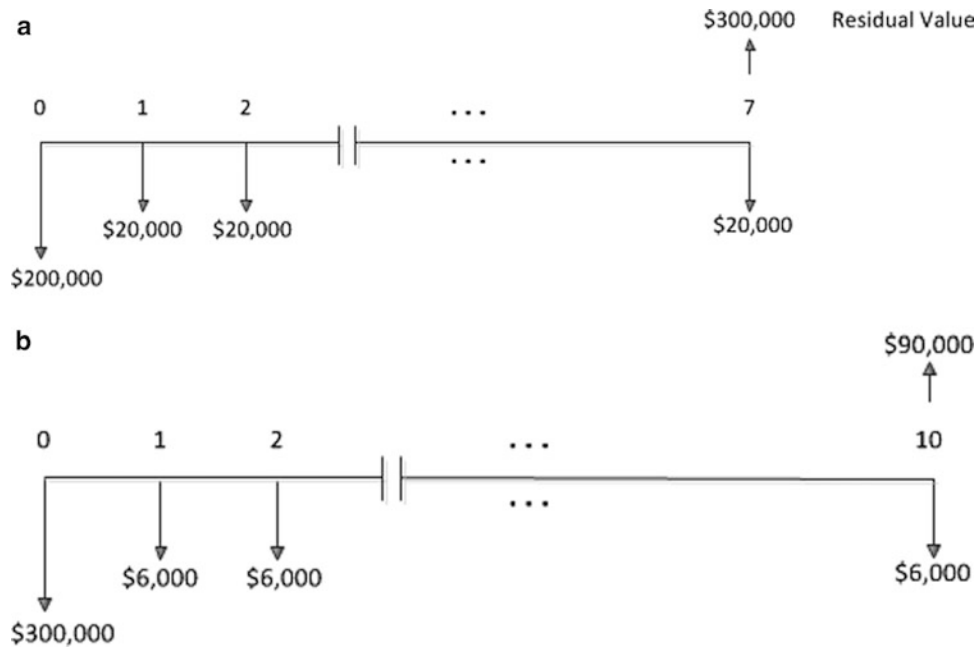

Fig. 12.5 (**a**) Cash flow for machine A. (**b**) Cash flow for machine B

Now using (12.13) we get the AEC for each machine:

$$\mathrm{AEC_A} = \frac{\mathrm{NPV_A}\left(1+\frac{12}{100}\right)^7 \frac{12}{100}}{\left(\left(1+\frac{12}{100}\right)^7 - 1\right)} = -\$60,850.10;$$

$$\mathrm{AEC_B} = \frac{\mathrm{NPV_B}\left(1+\frac{12}{100}\right)^{10} \frac{12}{100}}{\left(\left(1+\frac{12}{100}\right)^{10} - 1\right)} = -\$53,966.70.$$

Step III

Analysis

According to the AEC criterion, machine B is a better alternative than machine A because $\mathrm{AEC_B} > \mathrm{AEC_A}$, or less negative. It is interesting to note that if you directly use the data provided by the NPV calculations, machine A is better than machine B. Why? Although you can use NPV too, the requirement is that the projects should have the same lifespan or in some way equalize the lifespan of the evaluation. For example, if you carry out the first alternative ten consecutive times (70 years) and the second alternative seven consecutive times (70 years), now both evaluations show the same lifespan, 70 years (but, at least in this case, this is too cumbersome!).

12.8 Solved Problems

12.8.1 Real-Life Problems

1. **BMW [3]**. You are a recent Ph.D. graduate. As a specialist in mass transfer and process control a large international petrochemical company has an interest in your particular capabilities. Your starting annual salary will be $90,000. Now you are eager to fulfill one of your long-time dreams:

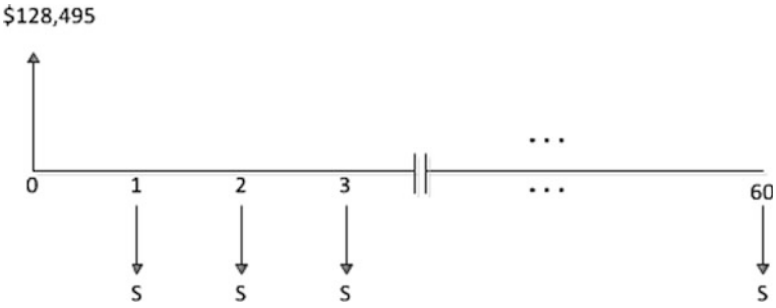

Fig. 12.6 Cash flow for the bank loan

to buy a new 2014 BMW Alpina B7. Of course, you cannot pay cash, but because of your new position at the petrochemical company, any bank will be very happy to give you a loan. The list price of the Alpina is $128,495. What will your monthly payments be if you get a loan that is payable in 60 months with an interest rate of 1 % per month?

Solution
The available data are the present value of the car ($P_V = \$128{,}495$), the monthly interest rate ($r = 1$ %), and the number of periods ($n = 60$). Then we can rearrange (12.8) and calculate the monthly payment (S).

Step I
Schematic representation (Fig. 12.6)

Step II
Payment calculation
Rearranging (12.8), we obtain

$$S = \frac{P_V\left(1+\frac{r}{100}\right)^n \frac{r}{100}}{\left(\left(1+\frac{r}{100}\right)^n - 1\right)} = \frac{128,495(1.1)^{60}0.1}{\left((1.1)^{60}-1\right)} = \$2,858.30/\text{month}.$$

Step III
Analysis
My only concern is that $2,858.30 per month will represent an significant portion of your monthly salary (almost 40 %).

12.8.2 Engineering Problems

2. **Wine cellar [4]**. A medium-sized wine cellar wants to replace an old bioreactor with a new and modern one. So far, it has received three offers:

At a similar annual cost the company prefers bioreactor C (because C is a modern and more reliable bioreactor). Similar means that if bioreactor C is no more than $2,000/year more expensive than the cheapest alternative, the company will buy bioreactor C. If the annual discount rate is 10 %, which of the bioreactors will the company buy?

Table 12.2

Equipment	Bioreactor A	Bioreactor B	Bioreactor C
Price ($)	80,000	120,000	150,000
Annual labor cost ($)	16,000	10,000	11,000
Annual maintenance ($)	4,000	3,000	–
Residual value ($)	20,000	40,000	80,000
Lifespan (years)	10	8	7

Solution

As shown in Table 12.2, the lifespan of bioreactors A, B, and C are different. Thus, as suggested, we will use the AEC (annual equivalent cost) concept to decide which bioreactor is better in economic terms. Consider that in this case it is not assumed that the bioreactors are equal, so to compare the alternatives, we will discount $2,000 to the AEC of bioreactor C.

Step I

Schematic representation (Fig. 12.7a–c)

Step II

AEB calculation

First, we calculate the NPV for each alternative (bioreactors A, B, and C):

$$\mathrm{NPV_A} = -80,000 - \frac{20,000\left((1.1)^{10}-1\right)}{0.1(1.1)^{10}} + \frac{20,000}{(1.1)^{10}} = -\$195,180,$$

$$\mathrm{NPV_B} = -120,000 - \frac{13,000\left((1.1)^{8}-1\right)}{0.1(1.1)^{8}} + \frac{40,000}{(1.1)^{8}} = -\$170.694,$$

$$\mathrm{NPV_C} = -150,000 - \frac{11,000\left((1.1)^{7}-1\right)}{0.1(1.1)^{7}} + \frac{80,000}{(1.1)^{7}} = -\$162,500.$$

Now using (12.13) we get the AEB for each bioreactor:

$\mathrm{AEC_A} = -\$31{,}764.7$/year,

$\mathrm{AEC_B} = -\$31{,}995.5$/year,

$\mathrm{AEC_C} = -\$33{,}378.4$/year.

If we discount $2,000 per year for bioreactor C, then the best alternative is bioreactor C.

Step III

Analysis

Again, as in warm-up example 6, if you directly use the data provided by the NPV calculations, then the best options in order are first C, then B, and finally A, contrary to the correct procedure using the AEC criterion, where the order is A, then B, and finally C. In conclusion, bioreactor C will be selected because, according to the problem statement, we can discount $2,000/year as bioreactor C is a more reliable piece of equipment.

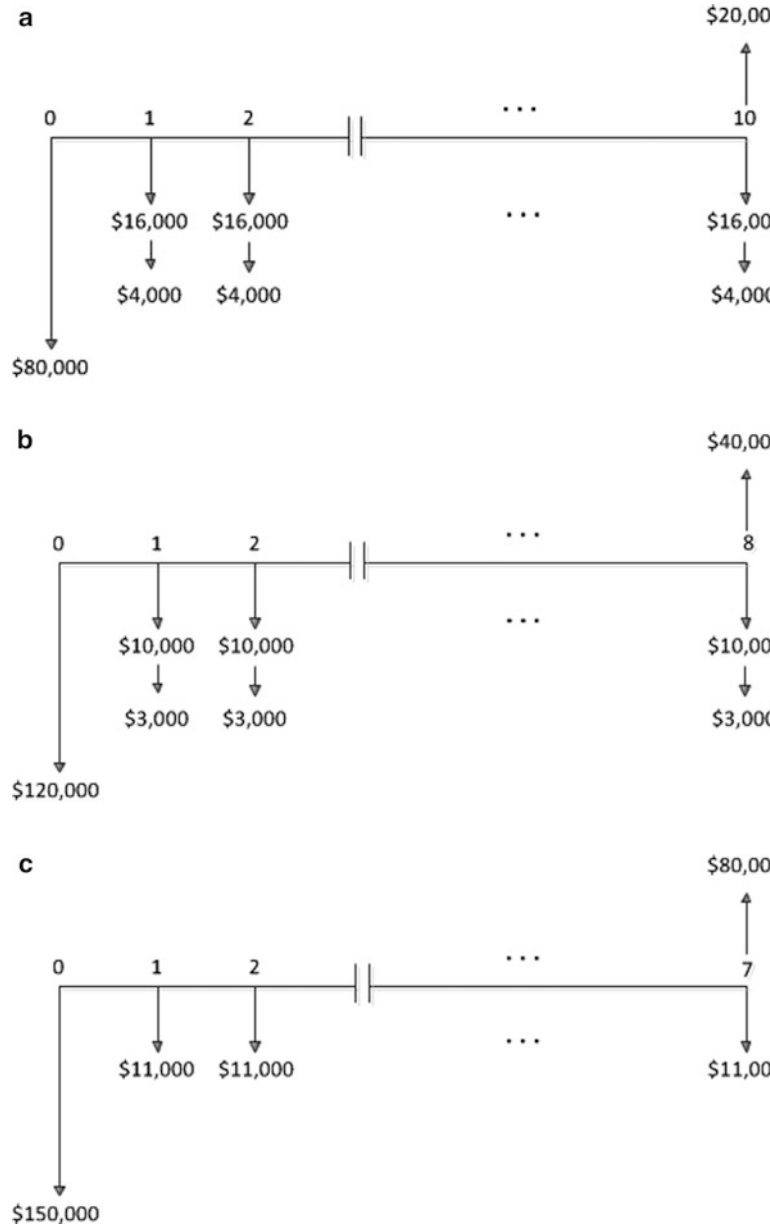

Fig. 12.7 (**a**) Cash flow for Bioreactor A. (**b**) Cash flow for Bioreactor B. (**c**) Cash flow for Bioreactor C

12.9 Proposed Problems

12.9.1 Real-Life Problems

1. **Superscreen TV [3]**. You have your eye on a 55″ HD TV, but you cannot afford its $1,500 price tag. The salesman offers you payment plans of 12, 24, or even 36 equal payments. The monthly interest rate in this mega store is 2.5 %. What is the monthly payment in each case: 12, 24, and 36 payments?
   A: $146.23, $83.87, and $63.68, respectively

2. **Superscreen TV 2 [3]**. Although you can make 36 payments of $63.68, before making a deal with the salesman, you go to the bank for a consumer loan. The executive tells you that because you are an old customer, you will receive a loan with an exclusive monthly interest rate of 0.6 %. (a) What is the monthly payment to the bank in each case of 12, 24, and 36 payments? (b) Any comments?
   A: (a) $129.9, $67.29, and $46.45, respectively. (b) First, normally it is much better to avoid payments. However, if you need a loan, the bank is normally the superior option.
3. **Interest rate [8]**. (a) Find a function that relates annual interest rate to monthly interest rate (decimal form). (b) Find a function to relate annual interest rate to semiannual interest rate (decimal). (c) Find a function to relate semiannual interest rate to monthly interest rate (decimal form).
   A: (a) $i_a = [(1 + i_m)^{12} - 1]$, (b) $i_a = [(1 + i_s)^2 - 1]$, (c) $i_s = [(1 + i_m)^6 - 1]$, where i_a, i_s and i_m are the annual, semiannual, and monthly interest rates, respectively.
4. **Interest rate 2 [4]**. (a) If the annual interest rate is 12.00 %, what is the monthly interest rate? (b) If the semiannual interest rate is 6.00 %, what is the annual interest rate? (c) If the semiannual interest rate is 6.00 %, what is the monthly interest rate?
   A: (a) ~ 0.95 % (0.950), (b) 12.4 %. (c) 0.976 %
5. **New house [4]**. You and your wife have found the house of your dreams. The only problem is that the price is $200,000. Because you are a good customer of a big bank, you can get a 20-year mortgage. The loan officer offers you a mortgage rate of $1,200/month for 20 years. What is the monthly interest rate that the bank is charging?
   A: ~0.32 %
6. **New house 2 [4]**. In relation to the previous problem, a $1,200 payment is a bit too high for you and your wife, but you have $40,000 in a savings account. Therefore, you will require a loan of $160,000. What is the monthly payment in this case?
   A: $961
7. **Money, savings, and interest rate [3]**. (a) How much money will you have in your savings account in 12 years if you deposit $350,000 today at an annual interest rate of 7 %? (b) How much money will you have in your savings account in 12 years if you deposit $350,000 today at an annual interest rate of 10 %? (c) How much money will you have in your savings account in 20 years if you deposit $350,000 today at an annual interest rate of, again, 10 %? (d) What do you think about these results?
   A: (a) $788,267.0. (b) $1,098,450. (c) $2,354,625. (d) One way to look at it is that the interest rate and the time you keep the savings have a big impact on your future savings. Just as we have continually recommended being steady and persistent with your study efforts, this also applies to your savings.
8. **Big machine [5]**. A company needs to buy a machine that costs $500,000. The bank offers the company a loan to be paid back in six equal payments every 6 months. If the annual interest rate is 12 %, what is the amount of each installment?
   A: $101,139
9. **Ferrari Testarossa [3]**. Your dad is dreaming of exchanging his old Ford Maverick (1973) for a new Ferrari Testarossa (2013). Because the Ferrari is very expensive, in his dreams, he has a detailed plan to secure a loan from the bank and pay it back in 8 years. The price of a used Testarossa is $80,000. Your father thinks he can afford that and goes to the bank to negotiate an $80,000 loan. After extensive paperwork and exhaustive revision of your father's records, the loan officer agrees to give him the loan through monthly payments (for 8 years) with an extraordinary annual interest rate of just 8 %. What is the monthly payment?
   A: ~ $1,120 (1,119.60)

10. **The power of savings [3]**. Imagine that your mom has been depositing money for you from the moment you were born (18 years ago). Regularly, each month, she goes to the bank and makes a deposit of $100.00. (a) How much money do you have in your savings account if the monthly interest rate has always been 0.8 %? (b) How much money do you have in your savings account if the monthly interest rate has always been 1 %?
    A: (a) ~ $57,385, (b) $75,786
11. **Mr. Smith [4]**. Mr. Smith is a very cautious and methodical man. He has been saving money for his retirement over the last 14 years. Every month he goes to the bank and makes a deposit of $1,290.00. How much money does Mr. Smith have in his savings account after 14 years if the monthly interest rate has been 0.7 %?
    A: $410,613
12. **Mr. Smith's son [4]**. John Smith, Mr. Smith's son, is really impressed with his dad's savings, and he would like to start saving for his retirement. Little John is ambitious and wants to have $1 million for his retirement within 40 years. What should be his monthly deposit at the same interest rate (0.7 %)?
    A: Modestly ~$255
13. **Loan for a friend [3]**. A friend of yours needs $1,500 and asks you for help. He promises to pay you back $2,500 in 5 years. What is the monthly interest rate on the loan?
    A: 0.855 %

12.9.2 Personal Finance

14. **To study or not to study, a simple estimation [5]**. You are finishing high school and your parents expect you to apply to The Ohio State University to pursue a bachelor's degree in chemical engineering. As Ohio State is an excellent university, and you are living in Upper Arlington, Columbus appears to be, if not the best, at least a very interesting option. Unfortunately for your parents, you are tempted to start working right away because recently you received a job offer for $30,000 a year. Your dad and mom, as professionals, are very concerned and they try to convince you to go to the university and at least get a bachelor's degree. Both mom and dad have a Ph.D. So far, they are not trying to convince you to get a Ph.D., but at least a bachelor's degree. Your dad, as an engineer, has done some calculations and demonstrates to you the advantages (in economic terms) of getting a degree. In addition, he thinks that it would be more rewarding for you as a person. For his calculation, your dad assumes that if you start working today you will work for 47 years (starting today, 47 + 18 = 65, age of retirement) and you will always receive the same salary. On the other hand, if you decide to get a bachelor's degree (4 years), instead of receiving $30,000 a year, you will spend $30,000 a year (university tuition and living). (a) What is the minimum salary with a bachelor's degree to justify getting the degree? (b) What do you think, is it worthwhile to study? Your dad assumes an annual interest rate of 10 %.
    A: (a) $58,316. (b) Although a very simple calculation, it shows us that study is not only rewarding, but also the best investment. Recalling the Benjamin Franklin quotation cited in Chap. 11: "An investment in knowledge always pays the best interest." Possibly, and rightly, you do not think that the assumption of a constant salary for 47 years is correct. Indeed, the assumption is a bad one; however, it is a terrible assumption if you get a degree. Why? Because in both cases, the salary will increase over time, but it would likely increase much more if you have a degree.

15. **To study or not to study 2, a more detailed estimation [7]**. Based on salaries in the USA, we have developed the following estimation table:

Year	Salary without bachelor's degree	Salary (cost) with bachelor's degree
0	–	–
1	$30,000	−$30,000
2	$30,000	−$30,000
3	$30,000	−$30,000
4	$30,000	−$30,000
5	$40,000 (first promotion)	$60,000
...	$40,000	$60,000
10	$40,000 (second promotion)	$60,000 − $80,000 (getting an MBA) + $100,000 (bonus) (first promotion)
11	$45,000	$90,000
...	$45,000	$90,000
20	$45,000 (third and final promotion)	$90,000 (promoted to plant manager) + $150,000 (bonus) (second promotion)
...	$50,000	$120,000
30	$50,000	$120,000 (last promotion) + $200,000 (bonus) (last promotion)
...	$50,000	$150,000
	$50,000	$150,000
47	$50,000	$150,000 + 300,000 (retirement bonus)

What is the total amount of money that you will have earned at the moment of your retirement (future value)? Assume an annual interest rate of 10 %.

A: Without a bachelor's degree, approximately $34.34 million; with a bachelor's degree and MBA, approximately $45.9 million.

12.9.3 Engineering Problems

16. **Small and medium-sized enterprises (SMEs) [6]**. An SME is investing $1.4 million to buy a new and modern machine that will increase the process efficiency. What should the monthly savings be to recover the investment in 2.5 years (NPV = 0) if the annual interest rate is 12.68 %?

    A: $54,247.40

17. **Chemical company [8]**. A chemical company that manufactures oil products buys a semiautomatic machine for $13,000. The annual maintenance cost and operation are $1,700. Five years later, the company must decide on the purchase of a unit that will fully automate the machine. The manufacturer of the unit has estimated that the company's annual maintenance cost will be reduced to $900 per year, and then 11 years later you can sell the unit for $1,800. The price of the unit is $7,100. (a) Do you recommend buying the unit? (b) If not, how much should the company pay for the unit? (c) If the cost of the unit is still $7,100, how much should the annual maintenance cost be to make buying the unit attractive? Annual interest rate = 9 %.

    A: (a) No. (b) $6,142. (c) $759

18. **Canned fish [6]**. A small food processing plant that produces canned fish wants to buy an automatic packaging machine and they are analyzing two alternatives:

	Machine A	Machine B
Price of machine ($)	150,000	250,000
Annual maintenance cost ($)	16,000	4,000
Residual value ($)	30,000	60,000
Lifespan (years)	7	10

Determine which machine should be selected using an annual interest rate of 12 %.

A: Given that the lifespans of the machines are different, one way to compare both alternatives is through the annual cost. The annual cost of machine A is $45,894, and the annual cost of machine B is $44,827. Therefore, machine B is selected.

19. **Equipment company [5]**. The Alba Equipment Company wants to buy a new machine to build heat exchangers. The expected additional revenue from the machine is $150,000 at the end of the first year, $120,000 at the end of the second year, $90,000 at the end of the third year, and so on. The company expects that the machine will not be operative after year 6; therefore, its residual value is zero. If the interest rate is 15 % per year, how much should the company be willing to pay for the machine?

A: $329,569

20. **Company expansion [6]**. A company that is planning to expand has deposits $700,000 annually for 8 years. Starting with the ninth year, the company increases the deposit to $1.2 million for 5 years. How much money does the company have in its account immediately after making the last deposit if the average interest rate for the capitalization of the funds is 5 % per year?

A: $15.162 million

21. **Old house [9]**. A builder dedicated to restoring old houses and selling them acquires a nice, big old house, but in bad condition, for $1.7 million. At the end of the first month, he has invested $300,000. Immediately after he repairs the house, a couple looking for a house offers him $2.4 million for the old and now beautifully restored house. After considering the offer, the builder decides to keep the house and rent it for $20,000 per month, receiving the first 2 months' rent after purchase. He rents the house for 15 months and then sells it for $2.2 million. If the interest rate is 1 % per month, how much extra money did the builder win or lose by not selling the house immediately after the remodeling?

A: He lost approximately $264,400 (money at month 16)

22. **High or low pressure? [8]**. A process engineer is trying to decide between two operating pressures for a waste water irrigation system. Using high pressure, the system would require less piping and sprinklers, but pumping costs would be higher. Alternatively, the process engineer can use lower pressure but more irrigators. The pumping cost is $15 psi million m^3 of wastewater. If he decides to use a pressure of 80 psi, the system will need 25 sprinklers at a cost of $220 per unit. In addition, the system will require 4,000 m of aluminum pipe at a cost of $28 per meter. A lower pressure of 50 psi would require 85 sprinklers and 13,000 m of pipe. The aluminum tubing has a lifespan of 10 years, and the sprinklers have a lifespan of 4 years. If you expect a volume of wastewater of 120 million m^3 per year: (a) What pressure should be selected? (b) What pressure should be selected if the cost of pumping increases to $20 psi*million m^3 of wastewater? (c) At what price are both pumping alternatives equivalent? The annual interest rate is 20 % per year.

A: (a) High pressure (the annual cost of high pressure is ~$172,800 and for low pressure ~ $184,046). (b) Low pressure (the annual cost of high pressure is ~$220,800 and for low pressure ~$214,046). (c) ~$18.1 psi*million m^3 of wastewater

23. **Sausage factory [10]**. A sausage factory needs a new cooling system. The company manager asks you as an engineer for your expertise in food refrigeration. You must decide between two alternatives. The spray method showers water on the hams until the temperature drops to 15 °C. This method requires L_S liters of water per ham. However, an immersion method that only requires L_I liters of water per ham ($L_I < L_S$) might be better. However, this method would require an additional initial investment of \$AI with additional repair costs of \$RC per year, and the equipment has a lifespan of 10 years. The company processes N_J million hams per year and pays $\$C_W$ per liter of cold water. The company also must pay $\$R_W$ per 1,000 [L] to remove the sewage. If the annual interest rate is 15 %, find an expression where the annual cost of each alternative is the same.
    A: $L_S \times N_J \times C_W + R_W \times L_S \times N_J/1{,}000 = L_I \times N_J \times C_W + [(AI \times 0.15 \times (1.15)^{10})/((1.15)^{10} - 1)] + RC + R_W \times L_I \times N_J/1{,}000$
24. **High-tech laboratory [10⁺]**. A highly specialized biotechnology company is considering installing a laboratory at its plant to avoid having to send samples to independent laboratories for analysis. If a full laboratory were installed, the initial investment would be \$250,000. It would require a specialized biotechnologist at a cost of \$130,000 per year. The cost of utilities, chemicals, etc. would be \$50 per sample. If the laboratory were only partially installed, the initial investment would be \$100,000 and would require a specialized biotechnologist working half time with a salary of \$50,000 a year. The cost of the sample analyzed in this laboratory would be \$30, but because not all tests can be performed in the laboratory, the services of an outside laboratory would be required at a cost of \$200 a sample. If the company prefers to continue with the current system, each sample will cost \$550. The laboratory equipment has a lifespan of 12 years and the annual interest rate is 10 %. How many samples must be analyzed each year to justify the installation of a (a) full or (a) partial laboratory in relation to the external laboratory? (c) If the company expects to analyze 175 samples per year, which of the three alternatives should be selected?
    A*: (a) >333 samples per year. (b) >202 samples per year. (c) External laboratory analysis.
    *It is necessary to add that for 333 samples or more, the complete laboratory is better than the current situation (external laboratory), but at this number of samples, the best option is the partial laboratory. In fact, from 0 to 202 samples the best option is the external laboratory, from 202 to 570 the partial laboratory, and from 570 samples, the best option is the full laboratory.
25. **High-tech laboratory 2 [9]**. In relation to the previous problem, the external laboratory, knowing that the biotech company is looking into the possibility of having its own laboratory facility, decides to lower the price of its analyses. The external laboratory does not believe that the biotech company would ever send out more than 250 samples for analysis per year, so the laboratory decides to lower its price to \$475 per sample. Analyze and discuss the price set by the external laboratory. Is it sufficient to go down to \$475 per sample?
    A: When the price per sample was \$550, the external laboratory was better than the partial laboratory as long as the number of samples was less than 202. By lowering the price to \$475, the external laboratory will be a good option as long as the number of samples is less than 263, and so it is a good move because the number of samples will not be more than 250.
26. **Sports car [9]**. The owner of a car is deciding between buying four radial tires or four retreaded tires. The radial tires cost \$200 each and would last 42,000 [miles]. The retreaded tires cost \$70 each but would last only 12,000 [miles]. Since this is a sports car, the owner uses it only on weekends, putting on 6,000 [miles/year]. Assuming that the cost of gasoline is \$1 per liter and the car consumes a gallon every 20 [km], what kind of tires should the car owner buy if the interest rate is 6 % per year?

A: As the lifespan of each kind of tire is different (7 years for the radial and just 2 years for the retreaded tires), an easy way to compare both alternatives is by calculating the annual cost of each kind of tire. The annual cost of the radial tires is $143.30 and for the retreaded tires it is $152.70. The radial tires are not only a bit cheaper per year but also safer.

27. **Canning company [7]**. A canning company wants to expand its product line, for which will it buy a new fruit grinder. An equipment manufacturer offers two options for machinery whose costs are presented in the following table:

	Machine A	Machine B
Price of machine ($)	260,000	360,000
Annual maintenance cost ($)	10,000	5,000
Annual labor cost ($)	110,000	70,000
Residual value ($)	20,000	30,000
Lifespan (years)	6	9

If both machines are equally suitable for the production line, which one should the canning company buy? Assume an annual interest rate of 10 %.

A: The annual cost for machine A is $177,105 and for machine B $135,300. Thus, it is better to buy machine B.

28. **Community center [9]**. A planning commission in Columbus, Ohio, is considering two proposals for the development of a new community center. Proposal O requires an initial investment of $12 million and a cost of expansion of $400,000 per year for 10 years. It is expected that the annual operating cost will be $250,000. In addition, the commission is expecting to receive $190,000 the first 4 years from conventions, shows, and other events and then $280,000 until the tenth year. From the 11th year onward, the income would be $350,000 per year.

Proposal H would require an initial investment of $15 million and annual operating costs of $300,000 per year. The income is expected to be $320,000 for the first 7 years. Then, the income will be $440,000 annually.

Determine which proposal the committee should select if the interest rate is 6 % per year, assuming an evaluation horizon of 20 years and a residual value for each alternative of 20 % of the initial investment.

A: The present value of proposal O is approx. −$13.87 million and the present value of proposal H is approx. −$13.1 million. Therefore, proposal H is a cheaper but not necessarily better proposal. Why? In the real world, you need to considerer other factors to make a final decision, even in cases like this, where the difference is very small and the investment is significant for the community.

29. **Storage pond [7]**. A company is buying a water storage pond at a value of $6,000. The production manager is considering the possibility of coating the pond because due to the high concentration of salts in the water, under present conditions, the pond will have an extremely short lifespan. An alternative is to apply a bituminous coating at a cost of $3,000. This would increase the lifespan to 6 years, after which, if the coating is repaired, at a cost of $500, it could last another 3 years. Another alternative would be to apply an epoxy coating, giving a lifespan of 14 years. How much money would the company need to pay for the epoxy coating so that the two alternatives were equivalent? The annual interest rate is 7 %. In addition, at the end of the pond's lifespan, its residual value would be 10 % of the initial cost.

A: ~ $6,380

30. **Chemical company [8]**. A medium-sized chemical company wants to replace an old reactor with a new one. So far they have received three offers, as follows:

Equipment	Reactor A	Reactor B	Reactor C
Price ($)	60,000	100,000	200,000
Annual labor cost ($)	21,000	12,000	5,000
Annual maintenance ($)	–	5,000	–
Maintenance every 3 years (years 3, 6, and 9)	25,000	–	–
Residual value ($)	45,000	–	100,000
Lifespan (years)	12	10	8

At a similar annual cost, the company will prefer reactor A, meaning that if reactor A is no more than $1,000/year compared to the cheapest alternative, the company will buy reactor A. If the annual interest rate is 10 %, which of the reactors will the company buy?

A: The annual cost for the three alternatives is very similar: for reactor A it is $34,085, for reactor B $33,274, and for reactor C $33,744. Therefore, although reactor A it is not the cheapest, it is the one selected according to the criteria established by the company.

31. **Instant coffee [8]**. A company that produces instant coffee needs to replace its old evaporators with new ones. The project engineers are looking at two alternatives. The first alternative has a cost of 2,200 arbitrary monetary units [MU] and retains the current plant capacity of 1,100 [kg/h] of instant coffee. The second alternative has a cost of 2,650 MU and allows for a 5 % increase in plant capacity. In addition, the evaporator of the second alternative can be sold at the end of its lifetime for 700 [MU]. In either case, the profits from selling instant coffee are 0.2 [MU/ton]. The plant operates continuously 300 days a year. Assuming an evaluation horizon of 10 years and an annual interest rate of 7 %, determine which of the two alternatives is more profitable.

A: The second alternative is better. The NPV of the first alternative is 8,925 [MU] and the NPV of the second alternative is 9,388 [MU].

32. **Canning company [6]**. A small canning company decides to apply for certification to meet international requirements, which would allow it to export 55 % of its production to international markets. This would increase the profits of the cannery. The certification process and the adjustments to be made to the plant to meet international standards have a total cost of 770.25 [MU]. How much should the cannery increase its profits per year for a payback in 4 years with an annual interest rate of 10 %?

A: 243 [MU/year]

33. **Natural sweeteners [8]**. A plant that produces natural sweeteners produces 100 [ton/year] running at full capacity, but based on a market research study the managers have been advised to increase production to 300 [ton/year]. The first alternative is to do this by increasing the plant capacity to 300 [ton/year] in one step at the start of the project, which will cost 1,000 [MU]. The second alternative is to do it in two stages, each time increasing the capacity by 100 [ton/year], one at the beginning of the project, at a cost of 600 [MU], and the other in the fifth year at a cost of 500 [MU]. Currently the annual profits are 100 [MU/year], which is directly proportional to the level of production. Determine which alternative is the best option in an evaluation horizon of 10 years and at an annual interest rate of 12 %.

A: The first alternative is better. NPV(1) = 695.1 [MU] and NPV(2) = 450.9 [MU]

34. **Shrimp waste [9]**. A fishery working with crustaceans is considering different alternatives for the disposal of waste. Currently the disposal of 11 tons per month of residual shells has a cost of 180,000 [MU]. The fishery is prepared to offer an 8-year contract for the best proposal. A manufacturer of lime is offering to pay 4,500 [MU/ton], removing the residue from the

company. Also, an innovative company that produces chitosan from crustacean shells is offering to pay 9,000 [MU/ton], but the offer does not include the removal of waste. The fishery estimates that it would have to pay 55,000 [MU] for each 11-ton freight. Since the fishery is interested in the proposal of the innovative chitosan producer, it is looking into whether the company can improve its offer. After a thorough analysis, the chitosan producer says that it can cover the costs of removing the waste from the fishery plant the first 2 years. If the annual interest rate is 12 %, determine which of the two alternatives is most attractive, i.e., the one with the larger NPV.
A: NPV(Lime) = 14,417,937 [MU] and NPV(Chitosan) = 15,247,936 [MU]

35. **Olive oil [8]**. A cooperative with many years of experience working with olives is exploring the possibility of producing olive oil. For this it has developed a project that begins with the construction of a warehouse and installation of the necessary equipment, second hand, all for a cost of 200 [MU]. This would allow it to operate for 3 years with profits of 180 [MU] per year, producing 6,000 [L] of olive oil per year. In the fourth year production is projected to increase by 2,000 [L] annually, maintaining the ratio of profits per liter of oil. For the sixth year, it is projected that the equipment originally purchased will need to be replaced at a cost of 250 [MU], which would allow an oil of better quality to be produced. The old equipment will be sold at a price of 40 [MU]. The cooperative will take advantage of this change to modify the container to a lighter volume, but this would allow the oil to be sold at higher prices on the most exclusive markets, increasing the relative utility per liter of oil by 50 %. This will increase production again to 2,000 [L/year]. Calculate the NPV of the project given an evaluation horizon of 10 years and an interest rate of 12 %.
A: 1,228.7 [MU]
36. **Biogas production [9]**. It has been estimated that the installation of a plant to produce biogas from human waste has a negative NPV of $2.8 million in an evaluation horizon of 8 years and at an annual interest rate of 6 %. To make the installation profitable, the project engineer proposes to sell dry biol as fertilizer, which requires an additional investment of $600,000 for the purchase of two thickeners, $2.2 million for the installation of dryers, and $1,000,000 for the purchase of additional land. Further, it will be necessary to hire an extra worker at a salary of $5,000 per month. This is expected to earn extra revenue of $1.96 million per year. Determine the NPV resulting from the sale of the dry biol fertilizer coupled with biogas production.
A: NPV = $5,198,608
37. **Protein purification [10]**. A certain protein is produced industrially by fermentation, which is then followed by a series of separation processes to obtain the pure protein for packaging. A new plant will be installed and two alternatives have been proposed. The first alternative has an installed cost of 25,000 [MU] and an operating cost of 0.020 [MU/L] (liters of broth fed to the process), making it possible to attain a purity of 95 %. The second alternative has an installed cost of 34,000 [MU] and an operating cost of 0.024 [MU/L], enabling a purity of 99.5 %. A certain continuous fermenter designed to produce the protein at a concentration of 15 [g/L] operates at a flow rate of 100 [L/h]. Assume that the fermenter operates continuously 330 days a year. The protein with 95 % purity is sold for 3.0 [MU/kg] and the protein with 99.5 % purity sold for 4.5 [MU/kg]. Determine which of the two alternatives is more profitable given an evaluation horizon of 10 years and an annual interest rate of 10 %.
A: NPV(1) = 96,662 [MU] and NPV(2) = 177,693 [MU]
38. **Anaerobic digester [8]**. A pig farmer is considering installing an anaerobic digester to treat the waste generated by the pigs. For this the farmer has only one alternative, which is inexpensive but has not been tested elsewhere, so its design is only theoretical and would cost $3.2 million. Estimates indicate that the availability of waste can generate 3 [m^3/day] of biogas, which can be

used to replace the use of LPG in the club nearby. The gas can be sold to the club at \$700.00/m^3, and the operating costs of the plant would be \$39,600 a year in supplies; there would be no labor costs in addition to those related to waste disposal. Since the theoretical model has not been tested, it is highly recommended to evaluate the project under different price scenarios. (a) What is the NPV of the project if the cost of biogas is \$700.00/m^3? (b) What should be the price if the NPV is 0? Use an evaluation horizon of 10 years and an annual interest rate of 8 %.
A: (a) \$1,677,558.20, (b) ~\$471.70/m^3

39. **Microbrewery [10⁺]**. A microbrewery is applying for a loan of at most \$10 million at a 10 % annual interest rate to be paid back within 7 years. The microbrewery has three projects that can be implemented together or separately.
(P1) Make improvements to the analytical laboratory, with an investment of \$1.8 million, allowing the reuse of previous batches, generating savings of \$240,000 per year. This alternative has a residual value of \$500,000 after 7 years.
(P2) Make improvements to the equipment and implement energy-efficiency measures, with an investment of \$5.5 million. The microbrewery would save an estimated \$540,000 yearly in electricity and \$720,000 in liquid gas, but this would entail extra costs of \$55,000 annually to maintain the boilers. This alternative has no residual value.
(P3) Install a new fermenter/soaker, at a cost of \$3,000,000, which could increase profits by \$1,800,000 yearly. This alternative has a residual value after 7 years of \$1,000,000.
Determine the NPV of each project. What is the optimal mix of projects? Assume an evaluation horizon of 7 years and a minimum acceptable rate of return of 10 %. Assume that the loan is paid back in equal annual payments.
A: NPV(P1) = –375,000; NPV(P2) = 366,445; NPV(P3) = 6,276,312; the optimal combination is P2 and P3.

40. **Industrial liquid waste (ILW) [8]**. A company that produces antibiotics is analyzing the possibility of treating the ILW in its own plant (the plant operates 330 days per year). Currently ILW is generated at a rate of 30 [m^3/day], and the company pays a sanitation company 0.11 [MU/m^3] to treat the waste. According to the production team, ILW generation is expected to increase to 55 [m^3/day] from year 6. The company is considering two alternatives:
(A_1) Install a plant to process 30 [m^3/day] for the first 5 years at a cost of 1,000 [MU] and then at the end of year 5 install an additional plant, also at a cost of 1,000 [MU] to expand the processing capacity to 55 [m^3/day]. The cost of operating the plant would be 0.1 [MU/m^3].
(A_2) The same as (A1), with the difference that in year 5 instead of installing an additional plant, install enzymatic reactors at a cost of 3,000 [MU], but this would reduce the generation of ILW to a third of its total.
Assuming an annual interest rate of 5 % and a 10-year evaluation horizon, what is the best alternative, A1, A2, or continuing with the sanitation company?
A: The best alternative is A_2, where NPV(A_1) = –12,227 [MU]; NPV(A_2) = –9,689 [MU]; NPV(Sanitation Company) = –11,487 [MU]

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